CHARACTERISTICS OF SEMICONDUCTOR LASER WITH EXTERNAL FEEDBACK FOR MULTIPLE REFLECTION MODEL

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ABSTRACT

Chaotic fluctuation of laser output is obtained at a relatively wide range of laser parameters. The dynamical behavior of semiconductor lasers with optical feedback from an external mirror was described by the rate equations modeled by Lang and Kobayashi. Some models were numerically analyzed this behavior but in our work we extended the previous results by allowing two reflections, and multiple reflections in the extended cavity. It is found that the chaotic state be very sensitive to the higher order reflections when the reflection coefficient is large.

I-INTRODUCTION

Laser diodes with a short cavity lifetime are sensitive to the feedback light from outside the cavity. Many intensive studies regarding the nonlinearities in laser diodes have been conducted in order to better understand the mechanisms of complex dynamics such as mode hopping, coherence collapse, and bifurcation to chaos [1],[2],[3]. The characteristics of semiconductor laser diodes are strongly affected by the external optical feedback that takes place when a portion of the output coupled back into the laser cavity after being reflected at an external surface. Even a relatively small amount of feedback can modify the dynamical and the spectral behaviour of injection laser diodes [4]. On one hand, it is potentially of practical use. For example, the external feedback enhances the longitudinal mode selection, which can be used to narrow the emission spectrum width. It has been used to reduce waveform distortion in the modulated output.

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On the other hand, serious problem arise in practice because of unintentional external feedback. For example reflection at a fiber facet in a diode-to-fiber optical coupling circuit degrades the modulation response characteristics and increases intensity noise [5]. The semiconductor laser with optical feedback is an excellent model for nonlinear optical systems and includes a rich variety of non-linear dynamics. When the light reflected from an external reflector couples with the original field in the laser cavity, the laser oscillation is affected considerably[1]. At weak to moderate optical feedback which is very important in actual applications of a semiconductor laser, the output power of the compound cavity semiconductor laser shows interesting dynamical behavior such as stable state, periodic and quasi-periodic oscillations, and chaos with the change of the system parameter values[4]. The dynamic behaviors of a semiconductor laser with optical feedback are mainly influenced by three parameters in the system, which include the reflectivity of the external mirror, the length of the external cavity, and the injection current [6]. With respect to the reflectivity we have two models, single reflection model (weak to moderate feedback) and multiple-reflection model (strong feedback). The key to the dynamics is the relaxation oscillation of the laser which remains undamped due to the effect of external feedback. It has been proved that not only the frequency corresponding to the external cavity length but also the relaxation oscillation frequency plays an important role for the dynamic behavior of the output power in a semiconductor laser with optical feedback [7]. With nonvanishing relaxation oscillation, the laser output behaves like periodic oscillation and evolves into quasi-periodic and, finally chaotic oscillations with the increase of the feedback level. All previous calculations, for simplicity, only consider the case of a single reflection. We extend previous results by allowing for multiple reflections in the extended cavity. It is found that the chaotic state be very sensitive to the higher order reflections when the reflection coefficient is large.

II-NUMERICAL MODEL FOR SINGLE REFLECTION

![Cavity model for semiconductor laser with external optical feedback.](image)

Figure (1) Cavity model for semiconductor laser with external optical feedback.

The dynamics of semiconductor laser with optical feedback is described by the rate equations modeled by Lang and Kobayashi[1]. Assume that the laser oscillates in a single longitudinal mode with an angular frequency $\Omega$, the complex electric field in the active region is written as,
\[ E(t) = E_o(t) \exp(i(\omega t + \Phi(t))), \] where \( \Phi(t) \) is the phase change due to the feedback effect. The complex rate equations of compound cavity in case of single reflection according to Lang and Kobayashi is,

\[ \frac{dE_o(t)}{dt} = \frac{1}{2} \{ G_\alpha [N(t) - N_e] - \frac{k}{\tau_\alpha} E_o(t - \tau) \cos \theta(t). \]  
\[ \frac{d\phi(t)}{dt} = \frac{\alpha}{2} \{ G_\alpha [N(t) - N_e] \} - \frac{k}{\tau_\alpha} E_o(t - \tau) \sin \theta(t). \]  
\[ \frac{dN(t)}{dt} = \frac{R_p - G_\alpha [N(t) - N_e]}{\tau_s} \]  
\[ \theta(t) = \Omega t + \phi(t) - \phi(t - \tau) \]  

where \( R_p \) is the electric pumping given by \( R_p = j/e/d \), with \( J \) injected current density, \( e \) the electron charge, \( d \) the cavity height, \( k \) is the feedback parameter described as \( k = \frac{r_2}{r_1} \), \( G_\alpha \) is the gain constant, \( N_e \) is the carrier density at transparency, \( N(t) \) is the average carrier density in the active region; \( N_{th} \) is the carrier density at threshold, \( \alpha \) is the linewidth enhancement factor, \( \tau_\alpha = 2n_1/c \) is the diode cavity round trip time, \( L \) is the cavity length, \( n \) is the refractive index, \( \tau_s \) is the carrier life time, and \( \tau = 2L/c \) is the external cavity roundtrip \([1]\).

We have numerically calculated (1) by employing a fourth order Runge-Kutta algorithm and investigated various cases for steady, periodic, quasi-periodic, and chaotic states. We will take the strength feedback parameter \( k \) as a control parameter to determine the dynamical behavior of the external-cavity laser diode. Fig. (2a)-(5a) show the result of temporal variations for \( J = 1.2J_{th} \) and \( L = 9cm \), where \( J_{th} = N_{th}/\tau_s \) is the current density at threshold. Table (I) shows the parameters values for the laser diode used in the numerical simulations. As the external reflectivity \( r_1 \) is increased from 1% to 10% we observed a continuous bifurcation from stationary state to periodic Fig 1, quasi-periodic Fig. (3a), and finally chaotic state Fig. (4a) and Fig. (5a). Their corresponding phase portrait, namely, limit cycle, torus, and chaotic attractor shown. The figure also show the Fourier transform in the four cases and the relation between the phase difference \( [\phi(t) - \phi(t - \tau)] \) and the excess carrier density \( n(t) \).
III-NUMERICAL MODEL FOR TWO REFLECTIONS

For single reflection model, the dynamics of semiconductor laser with optical feedback is described by the rate equations modeled by Lang and Kobayashi. In case of two reflections model we will add a new term represent the effect of the second reflection. Thus the rate equations take the following form:

\[ \frac{dE_0(t)}{dt} = \frac{1}{2} \left[ G_s[N(t) - N_e] - \frac{1}{\tau_p}\right] E_0(t) + \frac{k_1}{\tau_m} E_0(t - r) \cos \theta_1(t) + \frac{k_2}{\tau_m} E_0(t - 2r) \cos \theta_2(t) \]

\[ \frac{d\phi(t)}{dt} = \frac{\alpha}{2} G_s[N(t) - N_e] - \frac{1}{\tau_p} \left[ \frac{k_1}{\tau_m} E_0(t - r) \sin \theta_1(t) - \frac{k_2}{\tau_m} E_0(t - 2r) \sin \theta_2(t) \right] \]

\[ \frac{dN(t)}{dt} = R_p - \frac{N(t)}{\tau_i} - G_s[N(t) - N_e] \|E(t)\|^2. \]

Where \( \theta_1(t) = \Omega r + \phi(t) - \phi(t - r) \), \( \theta_2(t) = 2\Omega r + \phi(t) - \phi(t - 2r) \) are the phase coupling between the original light in the cavity and the delayed light from the external reflector. And, \( k_1 = \frac{(1 - r^2_j)^2}{r_j} \), and, \( k_2 = (1 - r^2_j)r^3_j \), are the feedback parameters for single and two reflections, respectively.

We have numerically calculated equation (2) by employing a fourth order Runge-Kutta algorithm and investigating various cases for steady, periodic, quasi-periodic, and chaotic states. We will take the strength feedback parameter \( (k) \) as a control parameter in determining the dynamical behavior of the external-cavity laser diode. Fig. (2b)-(5b) shows the result of temporal variations for \( J = 1.2 J_n \) and \( L = 9 \text{cm} \), where \( J_n = N_{th} / \tau_i \) is the current density at threshold. Table I shows the parameters values for the laser diode used in the numerical simulations. As the external reflectivity \( r_j \) is increased from 1\% to 10\% we observed a continuous bifurcation from stationary state to periodic one. Fig. (2b), exactly like single reflection model, quasi-periodic (3b), finally chaotic state Fig. (4b) and (5b). We observed that there is a great difference between a single reflection model Fig (4a) and (5a) compared with Fig (4b) and (5b). Their corresponding phase portrait shown. The figures also show the Fourier transform in the four cases and the relation between the phase difference \( [\phi(t) - \phi(t - r)] \) and the excess carrier density \( n(t) \) as in single reflection model. Thus, when the reflectivity of the external mirror increases, we must take into consideration the effect of higher order reflections.
V-CONCLUSION

We have numerically studied the dynamical behavior of the semiconductor laser diode with external optical feedback. We saw the effect of feedback strength change on the dynamical behavior of the semiconductor laser diode in case of single reflection. We observed that by increasing the feedback strength the dynamical behavior of the semiconductor laser diode changed from periodic to quasi-periodic and finally chaotic state. We showed the dynamical behaviors of the semiconductor laser diode under the influence of external optical feedback. Chaotic pulsing was found at certain feedback levels of the external target. The chaotic pulses in this system had both chaotic peak intensities as appear in the phase portrait and chaotic pulse intervals as appear in the power spectra (FFT). A detailed comparison between the phase portraits and the fast Fourier transform was carried out among the laser dynamic states obtained at different feedback levels. The laser is found to follow a quasi-periodic route to chaos. We also showed the dynamical behaviors of the semiconductor laser diode under the influence of external optical feedback for the case of two reflections. We found that the chaotic state was very sensitive to the higher order reflections especially when the reflectivity of the external target was high.

VI-REFERENCES


Table (I)

Some parameter value for the laser Diode used in the numerical simulations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$G_N$</td>
<td>Gain coefficient</td>
<td>$8.4 \times 10^{-13} m^3 s^{-1}$</td>
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<tr>
<td>$\alpha$</td>
<td>Line-width enhancement factor</td>
<td>3</td>
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<tr>
<td>$r_s$</td>
<td>Facet reflectivity</td>
<td>0.556</td>
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<tr>
<td>$N_{th}$</td>
<td>Threshold carrier density</td>
<td>$2.018 \times 10^{24} m^{-3}$</td>
</tr>
<tr>
<td>$N_e$</td>
<td>carrier density at transparency</td>
<td>$1.400 \times 10^{24} m^{-3}$</td>
</tr>
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<td>$\tau_c$</td>
<td>Life time of carrier</td>
<td>2.04 ns</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>Life time of photon</td>
<td>1.927 ps</td>
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<tr>
<td>$\tau_w$</td>
<td>Round trip time in laser cavity</td>
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<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
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<tr>
<td>$l$</td>
<td>Laser cavity length</td>
<td>90 mm</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Laser cavity volume</td>
<td>$1.2 \times 10^{-16} m^3$</td>
</tr>
<tr>
<td>$R_{sp}$</td>
<td>Spontaneous emission factor</td>
<td>0</td>
</tr>
</tbody>
</table>
$r = 1\%$

**Single Reflection**

Fig. 2(a)
Fig. 2(b)
r = 1.5%

Single Reflection

Fig. 3(a)
r = 1.5%  

**Two Reflections**

Fig. 3 (b)
$r = 2\%$

**Single Reflection**

![Graphs and charts related to single reflection](image)

Fig. 4(a)
r = 2%

two Reflection

Fig. 4(b)
$r = 10\%$

**Single Reflection**

Fig. 5(a)
r = 10%

Two Reflections

Fig. 5(b)