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## NUMERICAL STUDY OF Cr<sup>4+</sup>:YAG PASSIVELY Q-SWITCHED Nd:GdVO<sub>4</sub> LASER

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### Abstract

Theoretical investigation is presented to analyze and optimize the performance of passively Q-switched Nd:GdVO<sub>4</sub> laser with Cr<sup>4+</sup>:YAG as a solid state saturable absorber. A simple model is developed based on numerically solving the passive Q-switching rate equations, where the impact of the saturable absorber excited states photon absorption is included. The effect of both the saturable absorber doping concentration and output coupler reflectivity on the output laser characteristics are studied. The simulation results show reasonable agreement with those obtained experimentally by other research groups.

### INTRODUCTION

High energy laser pulses of short durations may be obtained by Q-switching techniques, where energy is stored in the gain medium through optical pumping while the quality factor of the laser resonator is decreased to prevent laser oscillation. Consequently, these techniques have attracted much research regarding their theory and design [1-18]. Compared with active Q-switching, passive Q-switching is more economical and practical because of the modest requirement of optical elements inside the laser cavity. The use of solid state saturable absorber (SA) as a passive Q-switching of diode pumped solid state lasers allows for the development of compact, miniature (microchips), stable and highly efficient laser sources of short bursts and high peak power densities which are desirable for military applications especially for airborne range finders. The good thermal, mechanical and optical properties of Cr<sup>4+</sup>:YAG crystal in addition to its low saturating intensity and high damage threshold allow it to be the most promising Q-switcher SA for many Nd:hosted lasers because of its near infrared absorption band (0.9-1.06  $\mu\text{m}$ ) [1-4]. Since 1992 Nd:GdVO<sub>4</sub> crystal appeared as a new solid state laser gain medium suitable for direct laser diode pumping and excellent for operation as a continuous wave laser and active Q-switching emission [5-9]. In 2000, Passive Q-switching operation of Nd:GdVO<sub>4</sub> laser with Cr<sup>4+</sup>:YAG SA was demonstrated by Li et al. [10] but very low values of pulse energy and peak powers were reported. Very recently, two different research groups [11,12] succeeded in achieved efficient passive Q-switching pulses of larger single pulse energy and higher peak powers using only simple plane-concave resonator.

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In this paper, a simple simulation model is presented based on solving numerically the passive Q-switching rate equations which has been modified to include the influence of the excited states SA photon absorption. The model is applied to analyze and optimize the performance of passively Q-switching Nd:GdVO<sub>4</sub>/Cr<sup>4+</sup>:YAG laser. The effects of the SA concentration and the output coupler reflectivity on optimizing the Q-switching laser characteristics are examined.

### RATE EQUATIONS

Considering the photon absorption of the SA excited states as a photon loss process and neglecting their spontaneous emission contribution and also the spontaneous emission of the gain medium, the rate equations for the laser photon density, gain medium population inversion and SA inversion [16] are modified here to the following three-coupled equations:

$$\frac{d\Phi}{dt} = \frac{\Phi}{t_R} \{2\sigma L N - [2L_S(\sigma_S n + \sum_x \sigma_{Sx} n_x) + \ln(\frac{1}{R}) + \Gamma]\} \quad (1)$$

$$\frac{dN}{dt} = PK - \gamma c \sigma N \Phi - \frac{N}{\tau} \quad (2)$$

$$\frac{dn}{dt} = -\frac{A}{A_S} c \sigma_S n \Phi + \sum_x \frac{n_x}{\tau_{Sx}} \quad (3)$$

where  $\Phi$  is the photon density;  $N$  is the population inversion of the laser gain medium;  $n$ ,  $n_x$  are the population of the ground state and the excited state "x" of the SA, respectively;  $\sigma$  is the laser emission cross section;  $\sigma_S$  and  $\sigma_{Sx}$  are the absorption cross-sections of the ground state and the excited state of the SA;  $t_R$  is the cavity round trip transit time;  $1/\tau$  is the decay rate of the upper laser level;  $\tau_{Sx}$  is the life time of the excited state "x";  $L$  is the length of the laser gain medium;  $L_S$  is the length of the SA crystal;  $\gamma$  corresponds to the net reduction in the population inversion resulting from the stimulated emission of a single photon where  $\gamma = 1$  for an ideal four-level laser system and  $\gamma = 2$  for an ideal three-level laser system;  $R$  is the output coupler reflectivity;  $\Gamma$  is the remaining round-trip cavity dissipative loss;  $P$  is the absorbed pump power;  $K$  is the pumping rate constant [13];  $A$  is the effective laser beam area and  $A_S$  is the effective area of the SA.

The energy levels diagram of the SA "Cr<sup>4+</sup>:YAG" crystal is shown in figure 1 [1-4]. Noting that the transition between the ground state and the first excited state <sup>3</sup>T<sub>2</sub> is solely responsible for the optical bleaching of the crystal [4]. The total population of the SA atoms is  $n_0 = n + \sum n_x = n + n_T + n_E + n_{T1} + n_{A1}$ . Because of the fast decay from levels  $T_1$  and  $A_1$  ( $\tau_{T1}$  and  $\tau_{A1} < 10$ ps [4]), it is assumed that the total population is only  $n_0 = n + n_T + n_E$ . At the beginning of pumping, the photon density in the laser cavity is small and almost all the saturable absorber population is in the ground state (i.e.  $n \approx n_0$ ) with a small signal transmittance  $T_0$ . Setting the left hand side of equation (1) equal to zero, the corresponding initial population inversion  $N_i$ , before saturation of the SA, can be approximated to,

$$N_i \cong \frac{1}{2\sigma L} (2\sigma_S L_S n_0 + \ln(\frac{1}{R}) + \Gamma) = \frac{1}{2\sigma L} \left( \ln(\frac{1}{T_0}) + \ln(\frac{1}{R}) + \Gamma \right) \quad (4)$$

Continuing pumping, the number of photons within the cavity increases rapidly and results in saturation of the saturable absorber ( $n \approx 0$ ) with a corresponding threshold value of the population inversion  $N_{th}$  approximated to,

$$N_{th} \cong \frac{1}{2\sigma L} \left( 2\sigma_{ST} L_S n_T + 2\sigma_{SE} L_S n_E + \ln\left(\frac{1}{R}\right) + \Gamma \right) \quad (5)$$

Let  $\sigma_{ST} = \sigma_{SE} = 3 \times 10^{-19} \text{cm}^2$  [4] and  $n_T + n_E = n_0 - n$ , therefore eq. (5) is simplified to,

$$N_{th} \cong \frac{1}{2\sigma L} \left( 2\sigma_{ST} L_S n_0 + \ln\left(\frac{1}{R}\right) + \Gamma \right) = \frac{1}{2\sigma L} \left( \delta \ln\left(\frac{1}{T^2}\right) + \ln\left(\frac{1}{R}\right) + \Gamma \right) \quad (5b)$$

where  $\delta = \sigma_{ST} / \sigma_S$ . As  $N_i \gg N_{th}$ , a giant Q-switched laser pulse is generated with a corresponding sharp reduction in the population inversion to a low value  $N_f$ . If the duration of pumping is long enough a second and a third laser pulses can be formed.

Since the duration of the generated giant laser pulse is very short compared to the pumping and the relaxation times of the laser gain medium and the SA, therefore we can simplify equations (2),(3) to,

$$\frac{dN}{dt} = -c\sigma\gamma\Phi N \quad (6)$$

$$\frac{dn}{dt} = -\frac{A}{A_s} c\sigma_s \Phi n \quad (7)$$

Dividing (6) on (7) and integrating, we get;

$$\frac{n}{n_0} = \left( \frac{N}{N_i} \right)^{\left( \frac{\sigma_s A}{\gamma \alpha A_s} \right)} = \left( \frac{N}{N_i} \right)^\alpha \quad (8)$$

Substituting (5b) into (1) and dividing the result by (6), then substituting from (8) we get,

$$\frac{d\Phi}{dN} = \frac{L}{L_C \gamma} \left[ 1 - \frac{(1-\delta)\ln(1/T_0^2)}{2\sigma L N} \left( \frac{N}{N_i} \right)^\alpha - \frac{\ln(1/R) + \delta \ln(1/T_0^2) + L}{2\sigma L N} \right] \quad (9)$$

This equation can be integrated to yield

$$\Phi(N) = \frac{L}{L_C \gamma} \left[ N_i - N - N_{th} \ln\left(\frac{N_i}{N}\right) - \frac{1-\delta}{2\sigma L \alpha} \ln\left(\frac{1}{T_0^2}\right) \left( 1 - \left( \frac{N}{N_i} \right)^\alpha \right) \right] \quad (10)$$

At  $N = N_{th}$ , the photon density reaches its maximum, then an expression for  $\Phi_{max}$  can be obtained from (10). After releasing the giant Q-switched pulse the photon number in the cavity becomes very low ( $\Phi \approx 0$ ) corresponding to a low inversion value  $N_f$ . At this point equation (10) can be rewritten as;

$$N_i - N_f \cong N_{th} \ln\left(\frac{N_i}{N_f}\right) + \frac{N_i}{\alpha} \left(1 - \frac{N_{th}}{N_i}\right) \left(1 - \left(\frac{N_f}{N_i}\right)^\alpha\right) \quad (11)$$

Expression (10) is transcendental and can be solved numerically. When  $N_i$  and  $N_f$  are known, the output energy  $E_{out}$ , the average power  $P_{av}$  and the peak power  $P_{peak}$  of the Q-switched laser pulse can be calculated.

### NUMERICAL SIMULATION

Equation 1-3 has been solved numerically using Runge-Kutta method and a simple computer model has been developed (which include the numerical solution) to investigate the passive Q-switching performance of Nd:GdVO<sub>4</sub> laser with Cr<sup>4+</sup>:YAG as a SA. The experimental parameters used in this simulation are as follows [11]: the pump wavelength = 808 nm; the laser wavelength = 1064 nm; the laser cavity length = 60 mm; the laser rod dimensions = 3.5x3.5x6 mm<sup>3</sup>;  $\sigma = 7.6 \times 10^{-19}$  cm<sup>2</sup>;  $\sigma(\text{absorption}) = 4.9 \times 10^{-19}$  cm<sup>2</sup>;  $\tau = 90$   $\mu$ s; the SA crystal dimensions = 5x5x1.74 mm<sup>3</sup>;  $\sigma_S = 3 \times 10^{-18}$  cm<sup>2</sup>;  $\sigma_{ST} = \sigma_{SE} = 3 \times 10^{-19}$  cm<sup>2</sup>;  $\tau_{ST} = 3$   $\mu$ s;  $\gamma = 1$ ;  $\Gamma = 0.02$  and  $A = A_S$ .

The numerical results of the time dependencies of the laser population inversion  $N$ , the SA ground state population  $n$  and the photons density  $\Phi$  are presented in Figure 2 (a, b), for initial transmission of Cr<sup>4+</sup>:YAG  $T_0 = 80\%$ , output coupler reflectivity  $R = 80\%$  and pump power  $P = 10$ W. Figure 2a presents a general view of these time dependent laser parameters, where the first giant pulse is developed at nearly 54  $\mu$ s after the pumping starts and the photon density peaks are developed with a time spacing of  $\sim 29$   $\mu$ s corresponding to pulse repetition frequency PRF  $\approx 30$  kHz. Figure 2b focuses on the details of the temporal variations of these parameters near a peak of the photon density. In this figure, the photon density reaches a maximum value of  $5.7 \times 10^{15}$  cm<sup>-3</sup> with a pulse width of  $\sim 18$ ns (FWHM) and pulse energy of  $\sim 70$   $\mu$ J.

The effect of the initial transmission  $T_0$  of Cr<sup>4+</sup>:YAG on the Q-switch laser pulse energy, pulse width, average and peak powers are presented in Figure 3 (a, b) at output coupler reflectivity  $R = 80\%$ . It is obviously seen from these figures that as  $T_0$  decreases the pulse energy and the peak power increase, whereas the pulse width and the average power decrease. This means that a narrower Q-switched pulse of a higher peak power and energy and lower average power is developed when a thicker or a higher doped concentration of the SA is used.

The calculated values of the pulse energy, pulse width, average and peak powers as functions of the output coupler reflectivity are presented in Figure 4 (a,b) for  $T_0 = 80\%$  and compared to the corresponding experimental results of Liu et al. [11] (solid symbols). It is clear from these two figures that as the reflectivity of the output coupler decreases, the pulse energy and the peak power increase while the average power and the pulse width decreased in accordance with the experimental data of [11,12]. Reasonable agreement is observed between the calculated values and the corresponding experimental ones.

Enhancement of the output pulse energy is one of the main targets in optimizing the performance of Q-switched laser systems. Therefore in figure 5 the curves plot some defined pulse energies as a function of the output coupler reflectivity "R" and the

small-signal SA transmission " $T_0$ ". The appropriate values of  $T_0$  and  $R$  for various required output pulse energies can be immediately identified from this figure.

The dependence of the calculated pulse repetition frequency and pulse width on the pump power are presented in Figure 6(a,b) for  $T_0 = 80\%$  and  $R = 60\%$ . The experimental results of Liu et al. [11] and Ng et al. [12] (solid and empty symbols, respectively) are also presented in the same figure for comparison. It is obvious that increasing the pump power yields shorter pulse widths with higher repetition frequency in accordance with the experimental results [10-12]. These calculations are repeated for two different values of  $T_0 = 80\%$ ,  $R = 80\%$  and  $T_0 = 60\%$ ,  $R = 60\%$  and the results are presented in Figure 6c as a direct relationship between the pulse width and the pulse repetition frequency (at the same value of pump power). Figure 6c shows clearly that shortening the pulse width is accompanied with increasing the repetition frequency, at defined values of  $T_0$  and  $R$ . Also the lower values of SA initial transmissions or output coupler reflectivity exhibit shorter pulse width at the same repetition frequency and slower repetition frequency for the same pulse width.

### CONCLUSIONS

The passive Q-switching performance of Nd:GdVO<sub>4</sub> laser with Cr<sup>4+</sup>:YAG solid state saturable absorber has been studied by solving numerically the passive Q-switching rate equations using Runge-Kutta method. The characteristics of the Q-switched laser parameters such as the pulse energy, the pulse width (FWHM), the average and peak powers and pulse repetition frequency have been explored as functions of the SA doping concentration and the reflectivity of the output coupler. Various combinations of the output coupler reflectivity and the small signal saturable absorber that can be used to enhance the output pulse energy are presented. The ranges of the obtained numerical results are in reasonable agreement with those obtained experimentally by other researcher [11,12].

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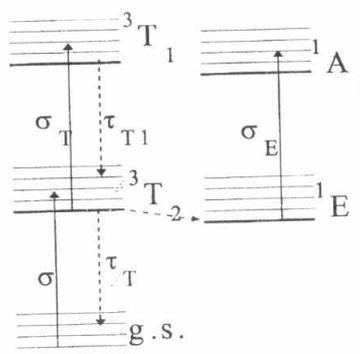
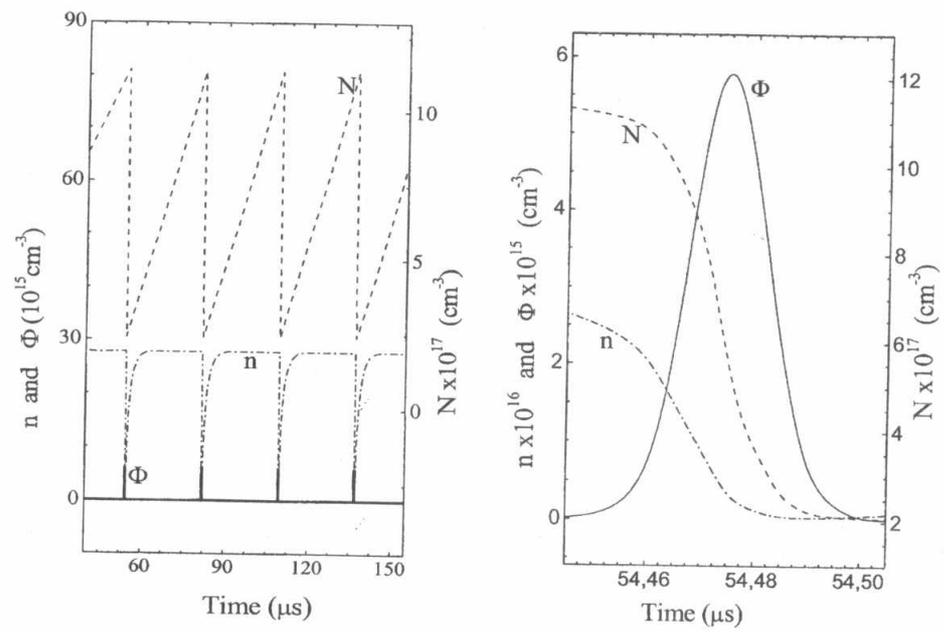


Fig. 1. Energy levels diagram of Cr<sup>4+</sup>:YAG.



**Fig. 2.** Numerical simulation of passively Q-switching rate equations. (a) General view and (b) details of temporal profiles for Nd:GdVO<sub>4</sub> laser population inversion " $N$ ", Cr<sup>4+</sup>:YAG ground state population " $n$ " and cavity photon density " $\Phi$ " at  $T_0 = 80\%$  and  $R = 80\%$ .

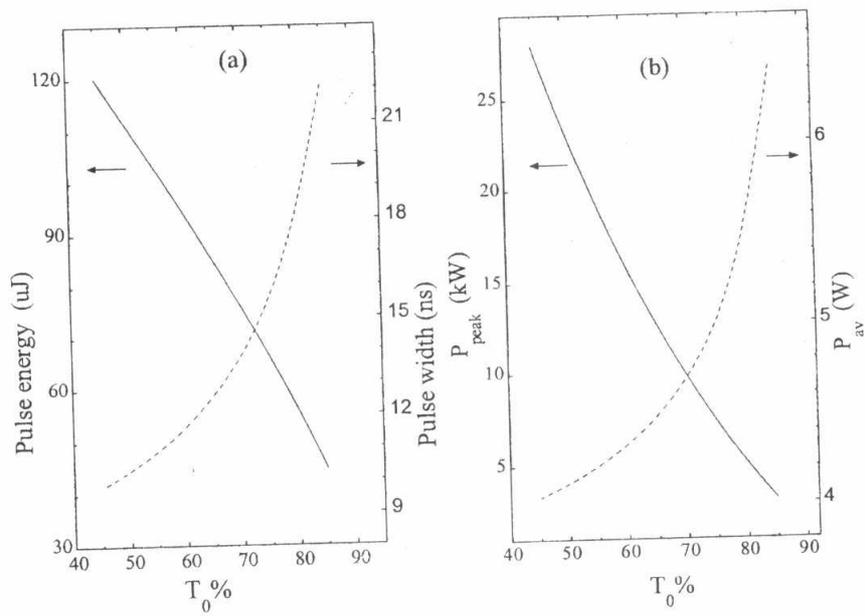


Fig. 3. Dependence of the Q-switched laser parameters: (a) output pulse energy and pulse width (FWHM) (b) Average and peak powers, on the initial transmission of  $\text{Cr}^{4+}$ :YAG at  $R = 80\%$ .

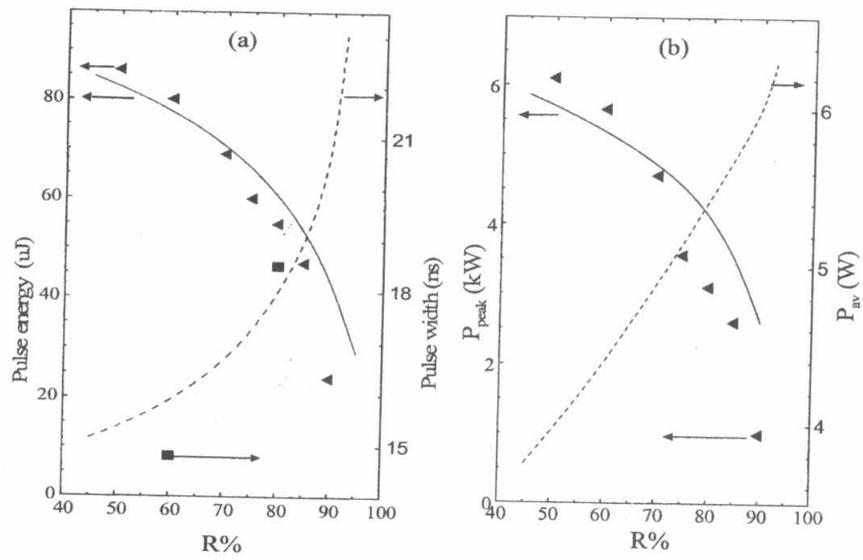


Fig. 4. Variation of (a) output pulse energy and pulse width (b) average and peak powers as functions of output coupler reflectivity at  $T_0 = 80\%$ . The symbols represent the experimental results of Liu et al. [11].

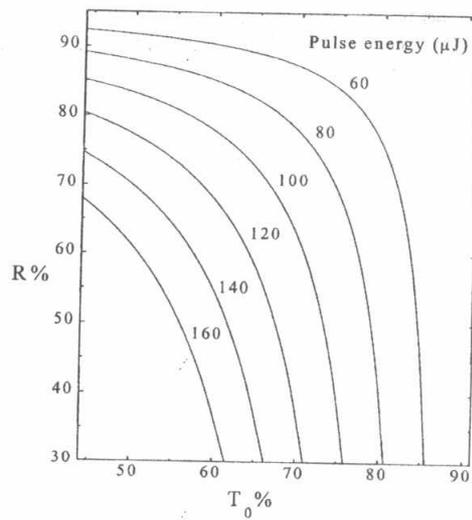


Fig. 5. Plots for some defined values of output pulse energies yielded from various combinations between  $T_0$  and R.

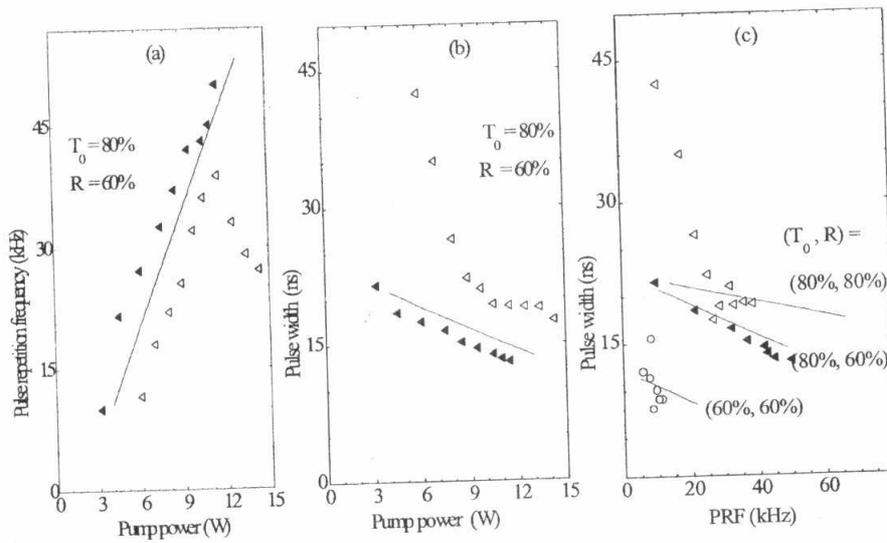


Fig. 6. Dependencies of (a) the pulse width and (b) pulse repetition frequency on the pump power for  $T_0=80\%$  and  $R=60\%$ . The symbols are the experimental data of Liu et al. [11] (empty symbols) and Ng et al. [12] (solid symbols). (c) Variation of the pulse width against the pulse repetition frequency for: i)  $T_0=80\%$ ,  $R=80\%$  ii)  $T_0=80\%$ ,  $R=60\%$  and compared to the experimental ones of [11] iii)  $T_0=60\%$  and  $R=60\%$  and compared to the experimental ones of [12].