PERFORMANCE EVALUATION OF FREQUENCY AGILE RADAR IN PRESENCE OF CHIRP JAMMING

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ABSTRACT

This paper studies the performance of frequency agile radar (FAR) under the effect of chirp jamming technique. Also, the improvement in the barrage jammer range when using the chirp jamming technique is studied. The considered radar is assumed to be a search radar, and the jammer is stand off jammer. A simple simulation model is used to generate the jamming signal in both time and frequency domain. This model is applied to simulate both chirp jamming and barrage jamming signals. The effect of the proposed chirp jamming on the detecting range of FAR is studied. Moreover, the comparison between the effect of barrage jamming and chirp jamming techniques on the jammer effective range and on the FAR detection range is studied.

KEY WORDS: Jamming, and Radar systems

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I. Introduction

Barrage jamming is used to jam a frequency band instead of a single frequency as in the case of spot jamming. The width of this frequency band affects the barrage jamming capability. The effect of barrage jamming on FAR is decreased due to the spreading of the jamming power on a wideband of frequencies. In this paper, a proposed jamming technique to optimize the management of the jamming power to accommodate this wideband of frequencies is introduced. This technique aims to maximize the degradation of the FAR detection range for a given jamming power. The signal used in this technique is a chirp signal, i.e., a linear frequency modulation signal of narrow band noise generated within certain time interval ($\delta$) and repeated every time ($\tau$). The resulting spectrum of this signal consists of multiple spectral lines with same amplitude separated by $(1/\tau)$ and with bandwidth $(2/\delta)$. The number of these spectral lines and the separation between them can be adjusted due the FAR activities.

This paper is organized as follows. In section II, the mathematical representation of the chirp jamming signal is introduced. In section III, the performance of chirp jamming technique is discussed. This section is divided into two subsections, the first one discusses the degradation of FAR detecting range due to the effect of chirp jamming, and the second one explains the improvement of the jammer range due to chirp jamming against FAR. Conclusions is presented in section IV.

II. Mathematical Representation of Chirp Jamming Signal

Chirp jamming is a barrage noise jamming such that its frequency is generated using the linear frequency modulation of a narrow band noise to cover the required band of frequency. Consider a linear frequency modulated signal (LFM) that is modulated by a single saw tooth signal with interval $T$. The time domain representation of this signal is shown in Fig. 1 and it is expressed as

$$f(t) = a \left( \frac{t}{T} \right), \quad \frac{-T}{2} \leq t \leq \frac{T}{2}$$

(1)

where $a$ is the slope of linear FM signal. The frequency domain representation of this modulated signal can be written in general form as [3]

$$\Phi_{lm}(w) = A \sqrt{\frac{2\pi}{b}} \text{Re} \left[ e^{-j\pi/2} \left\{ \frac{C(x_1) + C(x_i)}{\sqrt{2}} \right\} \right]$$

(2)

Where $b = w_{\text{max}} - w_{\text{min}}$, $A$ is the signal amplitude, $w_{\text{max}}$ is the maximum angular frequency, and $w_{\text{min}}$ is the minimum angular frequency. The functions $C(x)$ and $S(x)$ are given respectively by

$$C(x) = \int_{\delta}^{\pi} \cos \left( \frac{\pi}{2} \frac{v^2}{\delta} \right) d\varepsilon$$

(3)
\[ S(x) = \int_{0}^{x} \sin \left( \frac{\pi \xi}{2} \right) d\xi \]  \hspace{1cm} (4)

For large \( x \), we have

\[ \lim_{x \to \infty} C(x) = \frac{1}{2}, \text{ and } \lim_{x \to \infty} S(x) = \frac{1}{2} \]  \hspace{1cm} (5)

Let us define an infinite duration signal which is frequency modulated by a repetitive saw tooth signal with repetition interval \( T \). This signal is expressed as

\[ y_{FM}(t) = \sum_{i=1}^{N} \phi_{FM}(t - iT) \]  \hspace{1cm} (6)

The repetition in time domain leads to discrete spectrum in the frequency domain and therefore \( Y_{FM}(f) \) can be expressed as [4]

\[ Y_{FM}(f) = \Phi_{FM}(f) \sum_{i=0}^{N} \delta \left( f - \frac{2\pi i}{T} \right) \]  \hspace{1cm} (7)

The magnitude of the spectrum of a repeated linear frequency modulation (LFM) signal is shown in Fig. 2. In such case, the jamming power is distributed on these lobed spectral lines only. Practically the bandwidth of the FAR is \([-b : b]\) with center frequency \( f_0 \) as shown in Fig. 3. In this figure, the spectral lines have a small bandwidth because the FM modulation is performed for narrow band noise not for a single frequency signal. If the sweep period \( T \) is selected to be equal to \( 1/\Delta f \), where \( \Delta f \) is the frequency separation between two successive frequencies of FAR, the spectrum of the jamming signal is repeated every \( \Delta f \), i.e. the spectrum of the jamming signal lies only inside the matched filter frequency band and there is no any power losses between the channels due to the redistribution of the jamming power spectrum. In case of barrage jamming, the jamming power within this band is uniformly distributed all over the FAR band. In this case, the number of the lobed spectral lines \( v \) is given by

\[ v = \frac{2b}{\Delta f} \]  \hspace{1cm} (8)

If the radar receiver is exposed to two jamming signals: one of them is barrage jamming and the other is chirp jamming, the output of the matched filter (MF) depends on the MF bandwidth \( B_r \). If MF (it is equal to the separation between any
two successive channels it is equal to the sweeping frequency \(1/T\) in the chirp jamming), there is no difference between the effect of the two types of jamming on the radar [5], [6]. If the MF bandwidth is less than the \(\Delta f\), a part of the barrage noise does not be received by the MF. In this case, the MF output due to the chirp jamming is more than the barrage jamming case by a ratio equal to separation between the successive frequencies to the MF bandwidth. The obtained gain when using chirp jamming can be expressed as:

\[
G_e = \frac{1}{TB_i}
\]  

(9)

A simple simulation example is used to generate the proposed jamming signal in both time and frequency domain. This model is applied to simulate chirp jamming signal and also barrage jamming signal. The parameters used in the simulation example are: linear FM signal with sweep period =0.1 sec., and repeated at period=1 sec., the frequency of the chirp signal start from 0 to 100 Hz within the pulse duration as shown in Fig.4 the details of the linear change of frequency is shown in Fig.5. The spectrum of the chirp signal is shown in Fig.6. It has a bandwidth equal to \(2*(\text{stop frequency- start frequency})= 200\) Hz, and it contains 200 spectral lines with separation 1 Hz as shown in Fig.7.

As an example of near real case, to generate a chirp jamming signal with bandwidth=100 MHz, with spectral lines with separation 2 MHz, linear FM signal with sweep period =0.01 micro sec., and repeated at period=0.5 micro sec. the frequency of the chirp signal start from 0 to 50 MHz within the pulse duration is required as shown in Fig.8.

### III. Performance of Chirp Jamming Technique Against FAR

In this section the performance of the chirp jamming technique is studied. The performance is evaluated from two points of view: the first one is the degradation of the FAR detection range due to chirp jamming and the second is the improvement of the jammer range when using chirp jamming technique.

#### A. Degradation of The FAR Detection Range

The signal to jamming ratio at the input of the FAR radar is given by [1], [2]:

\[
\frac{S}{J} = \frac{P_p \cdot G_p \cdot G_r \cdot \sigma \cdot R_j^2}{4\pi \cdot R \cdot R_j^{4}}
\]  

(10)

where \(\frac{S}{J}\) is the signal to jamming ratio at the FAR receiver, \(P_p\) is radar transmitted power, \(G_r\) is the transmitter radar gain, \(G_r\) is the receiver radar gain, \(\sigma\) is the target cross section, \(R_j\) is the jammer bandwidth, \(R\) is the distance between the jammer and the target.
and radar, \( P_y \) is the jammer transmitted power, \( G_j \) is the jammer transmitter gain, \( G_r \) is the receiver gain in the jammer direction, \( B_r \) is the receiver bandwidth, and \( R_t \) is the target range. Equation (10) can be rewritten as:

\[
R_t = B \sqrt{\frac{P_x}{P_y}} \tag{11}
\]

where \( B = \frac{\pi R^2 G_j G_r \sigma B_r}{(\pi f) G_y G_y 4\pi B_y} \). The radar detection range in the presence of barrage and chirp jamming is given respectively by:

\[
R_{ji} = B \sqrt{\frac{P_x}{P_{ji}}} \tag{12}
\]

\[
R_{j2} = B \sqrt{\frac{P_x}{P_{j2}}} \tag{13}
\]

where \( P_{ji} \) is the power of barrage jamming signal and \( P_{j2} \) is the power of chirp jamming signal. Using (13) and (12), we have

\[
R_{j2} = 4 \frac{1}{G_c} R_{ji} \tag{14}
\]

where \( G_c = \frac{P_{j2}}{P_{ji}} \) is the chirp gain. The relation between the FAR detection range under barrage jamming and the FAR detection range under chirp jamming is shown in Fig.9 at different values of chirp gain. The figure shows that, at \( G_c = 1 \) the radar detection range under barrage jamming is equal to the FAR detection range under chirp jamming, when \( G_c > 1 \) the chirp jamming signal is more effective in decreasing the FAR detection range. As a result, the target protection can be increased by the jammer that uses the chirp signal instead of the barrage signal.

### B. Improvement of The Jammer Range Against FAR

Equation (10) can be rewritten as:

\[
R_j^2 = F P_y \tag{15}
\]

where \( F = \frac{\pi \sigma B \sqrt{G_y G_y 4\pi B_y R_t^4}}{G_y G_y \sigma B_y P_y} \). The barrage jammer range \( R_{ji} \), can be expressed as

\[
R_{ji} = F \sqrt{\frac{P_y}{P_{ji}}} \tag{16}
\]
where $P_{o1}$ is the barrage jamming signal power. The chirp jammer range $R_{j2}$, can be expressed as

$$R_{j2} = F \sqrt{P_{o2}}$$  \hspace{1cm} (17)

where $P_{o2}$ is the chirp jamming signal power $P_{o1}$. Then from (16) and (17), we have

$$R_{j2} = \sqrt{G} R_{j1}$$  \hspace{1cm} (18)

The relation between the jammer ranges when it uses barrage and chirp jamming is shown in Fig.10 at different values of chirp gain. As shown in this figure, at $G = 1$ the jammer ranges are equal, when $G > 1$ the jammer which uses chirp signal has larger effective range.

IV. Conclusion

The effect of barrage jamming signal on FAR is decreased due the usage of wideband frequencies. Jamming signal reforming is essential to increase its effectiveness to face this wideband frequency by the generation of chirp signal. when $G > 1$ the chirp jamming signal is succeeded in decreasing the FAR detection range. As a result, if the matched filter bandwidth is less than the FAR channel width $\Delta f$, the effectiveness of the chirp jamming exceeds that of the barrage jamming of the same power. This means that the FAR detection range is decreased more, and the effective jammer range is larger.

V. REFERENCES

Fig. 1. Linear FM.

Fig. 2. Spectrum magnitude of Linear FM signal.

Fig. 3. Spectrum magnitude of Linear FM signal, repetitive.
Fig. 4. Chirp signal in time domain

Fig. 5. The linear change in frequency during pulse width
Fig. 6. FFT of chirp signal with right up band = 100 Hz

Fig. 7. Spectral lines with separation = 1 Hz

Fig. 8. Spectral lines with separation = 10 MHz
Fig. 9. The relation between the FAR detection range under barrage jamming and the FAR detection range under chirp jamming.

Fig. 10. The relation between the jammer range uses barrage signal and chirp signal.