A STABLE OPERATION REGION WITH NO LIMIT CYCLES USING
THE METHOD OF DOMAINS SEPARATION

H. Yasin

M. Bishr

ABSTRACT

In determining the region of stable operation in the plane of two parameters, the issue of limit cycles need be studied carefully. The resonance problem will be raised if the mechanical resonance frequency of power systems' is equal to the limit cycle frequency. Despite the stability of each point in the stable region, it is crucial to determine the boundaries in which one can choose the Automatic voltage Regulator (AVR) parameters in such a way that the global stability with no limit cycle is guaranteed.

Separation of such boundaries to dangerous and safety borders have been investigated in the last few years. Many techniques have been devoted to enhance the safe region such that one can choose the AVR parameters in a better manner.

In this paper, the stable region will be separated to a non-limit cycle region and a limit cycle one. The non-limit cycle region is the region in which the adjustable coefficients can be chosen with guarantee that the resonance will never happen.

Two different techniques are used here to solve such a problem. Firstly, by scanning of the whole region; secondly by analyzing the describing function. In spite of using the digital computer in both cases, the second technique is much easier and is a time saver than the first one.

* Lecturer in the Department of Electrical Power Engineering and Machines, Faculty of Engineering and Technology, Menoufia University, Shebin El-Kom, Egypt.
To formulate the transfer functions of concern directly, the above three equations are divided by $AV_f(s)$. By investigating equations 1, 2 and 3, it has been noticed that they are three equations in five unknowns. To solve such set of equations, two more equations must be added.

The transient behavior of the transmission line of infinite busbar can be described by the following linearized equations:

$$\frac{\Delta V(s)}{AV_f(s)} = \frac{\Delta I_d(s)}{AV_f(s)} + \frac{\Delta I_q(s)}{AV_f(s)} = \psi \frac{\Delta \omega(s)}{AV_f(s)} + S y(s) = 0$$

$$\frac{\Delta q(s)}{AV_f(s)} - X_d(s) \frac{\Delta I_d(s)}{AV_f(s)} + [r + S X_d(s)] \frac{\Delta I_q(s)}{AV_f(s)} - \psi \frac{\Delta \omega(s)}{AV_f(s)} - y(s) = 0$$

Similarly, the linearized rotor dynamic equation can be written as:

$$[I_{qo} X_d(s) - \psi qo] \frac{\Delta I_d(s)}{AV_f(s)} + [\psi do - I_{do} X_d(s)] \frac{\Delta I_q(s)}{AV_f(s)} + \omega M S \frac{\Delta \omega(s)}{AV_f(s)} + I_{qo} y(s) = 0$$

To formulate the transfer functions of concern directly, the above three equations are divided by $AV_f(s)$.

By investigating equations 1, 2 and 3, it has been noticed that they are three equations in five unknowns. To solve such set of equations, two more equations must be added.

The transient behavior of the transmission line of infinite busbar can be described by the following linearized equations:

$$\frac{\Delta V(s)}{AV_f(s)} = \frac{\Delta I_d(s)}{AV_f(s)} - (r + S X_d(s)) \frac{\Delta I_q(s)}{AV_f(s)} - X_d \frac{\Delta q(s)}{AV_f(s)} + \frac{1}{S} [V_{dc}$$

$$+ S X_d I_{qo}] \frac{\Delta \omega(s)}{AV_f(s)} = 0$$

$$\frac{\Delta q(s)}{AV_f(s)} + X \frac{\Delta I_d(s)}{AV_f(s)} - (r + S X_d) \frac{\Delta I_q(s)}{AV_f(s)} - \frac{1}{S} [V_{dc}$$

$$- S X_d I_{do}] \frac{\Delta \omega(s)}{AV_f(s)} = 0$$
Note that Equations 4 and 5 are described by a coordinate system consisting of a d-axis on the field winding axis and a q-axis fixed in quadrature w.r.t the d-axis.

For convenience, equations (1-5) can be written in the matrix form as follows:

\[
\begin{bmatrix}
1 & 0 & v_{q0} & [r+sx_d(s)] & x_q(s) \\
0 & 1 & -v_{do} & -x_d(s) & [r+sx_d(s)] \\
0 & 0 & w_{oNS} & [1 - x_d(s) - q_0] & \{d - t d \phi q(s)\} \\
1 & 0 & \frac{v_{q0} + sx_k}{l} & -(r_k + sx_k) & x_k \\
1 & 0 & \frac{v_{dc} - sx_k d}{l} & -(r_k + sx_k) & x_k \\
\end{bmatrix}
\begin{bmatrix}
\Delta V_d(s) \\
\Delta V_q(s) \\
\Delta \omega(s) \\
\Delta I_d(s) \\
\Delta I_q(s) \\
\end{bmatrix}
= \begin{bmatrix}
-sy(s) \\
y(s) \\
0 \\
-1 q_0^y(s) \\
0 \\
\end{bmatrix}
\]

From equation 5, the transfer function vector

\[X^T = \begin{bmatrix}
\Delta V_f^d(s) \\
\Delta V_f^q(s) \\
\Delta \omega^f(s) \\
\Delta I_d^f(s) \\
\Delta I_q^f(s)
\end{bmatrix}^T
\]

can easily be computed at different values of \(\omega\) within a desired range.

The voltage and current transfer functions, expressed in its direct and quadrature components, are:

\[
\begin{align*}
\frac{\Delta V(s)}{\Delta V_f(s)} &= \frac{\Delta V_d}{\Delta V_f} + \frac{\Delta V_q}{\Delta V_f} \\
\frac{\Delta I(s)}{\Delta V_f(s)} &= \frac{\Delta I_d}{\Delta V_f} + \frac{\Delta I_q}{\Delta V_f}
\end{align*}
\]

**Derivation of the AVR Transfer Function**

The AVR is considered to be a multi-input-single-output device as shown in Fig. 3. Therefore, three transfer functions need be found. The AVR output \(\Delta V_f^f(s)\) will be considered as the single input to the synchronous machine.

![Block diagram representation of AVR](image)

The AVR transfer functions consists of three components, voltage channel, current channel and frequency channel transfer functions [6]:

\[
\begin{align*}
\Delta V &\rightarrow G_V(s) \\
\Delta I &\rightarrow G_I(s) \\
\Delta \omega &\rightarrow G_\omega(s) \\
\end{align*}
\]
Determining of the adjustable parameters of the AVR $(K_{of}, K_{lf})$ is one of the main objectives of the stability analysis in this paper. Such parameters are strongly affecting the system steady-state stability, since they are frequency channel coefficients.

Therefore, it is advantageous to find the region in which these parameters can be chosen so that the robust stability is guaranteed. Fig.4 illustrates the block diagram of the system of concern including the nonlinearity.

For the sake of the nonlinearity analysis, one can write

\[
\frac{\Delta V_{f1}(s)}{\Delta V(s)} = -G_v(s)G(s) \frac{\Delta V(s)}{\Delta V_f(s)} + G_I(s)G(s) \frac{\Delta I(s)}{\Delta V_f(s)} + G_\omega(s)G(s) \frac{\Delta \omega(s)}{\Delta V_f(s)}
\]

or

\[
\frac{\Delta V_{f1}(s)}{\Delta V_f(s)} = \frac{1}{N} = -G_v(s)G(s) \frac{\Delta V(s)}{\Delta V_f(s)} + G_I(s)G(s) \frac{\Delta I(s)}{\Delta V_f(s)} + G_\omega(s)G(s) \frac{\Delta \omega(s)}{V_f(s)}
\]

Now, the adjustable coefficients $(K_{of}, K_{lf})$ can be separated as follows

\[
L = K_of G_1(s) + K_{lf} G_2(s) = \frac{1}{N} + G_0(s)
\]
where \( G_0(s) = G_v(s) \frac{\Delta V(s)}{\Delta V_f(s)} - G_1(s) \frac{\Delta I(s)}{\Delta V_f(s)} \)

\[ K_{of} G_1(s) + K_{if} G_2(s) = G_0(s) \frac{\Delta \omega(s)}{\Delta V_f(s)} \]

Separating the real and imaginary parts and substituting \( s \) by \( j\omega \) yield the following:

\[ G_0(j\omega) = R_1(\omega) + j R_2(\omega) \]
\[ G_1(j\omega) = P_1(\omega) + j Q_1(\omega) \]
\[ G_2(j\omega) = P_2(\omega) + j Q_2(\omega) \]

Then, equation 16 becomes

\[ K_{of} P_1(\omega) + K_{if} P_2(\omega) = R_1(\omega) + \frac{1}{N} \]
\[ K_{of} Q_1(\omega) + K_{if} Q_2(\omega) = R_2(\omega) \]  

First, assume that the system is linear i.e. \( N = 1 \), then vary \( \omega \) from 0 to \( \omega \) and calculate the values of \( K_{of} \) and \( K_{if} \), then construct the main steady state stability region. Despite the stability of the power system whose coefficients were chosen inside the prescribed region, the resonance may take place. This may cause failure to the whole system, therefore, exact models should be used and the describing function analysis must be taken into consideration.

RESULTS

Stability Region Separation Using Scanning Approach:

The main stability region in Fig. 6 has been constructed with the assumption that the system is linear i.e. \( N = 1 \). The first approach used to separate this region to limit-cycle and non-limit cycle ones is the scanning of the whole region. Recently, with the digital computer facilities, exact models can be used and more accurate results can be obtained using such models. In this method, two computer programs are needed, the first one is used to construct the whole stable region with \( N = 1 \), and the second is used to test all of the region operating points at different values of \( \omega \) and determine the region in which the limit cycle will not occur. The drawback of this approach is that, it is a time consumer and it does not assign the non-limit-cycle region precisely but rather a number of points.

Stability Region Separation Using the Describing Function Analysis:

This new approach is more precise in determining the non-limit cycle stability region and moreover it is a time saver technique.

The stability region in Fig. 6 can be separated to a limit-cycle and a non-limit cycle ones using the describing function analysis as follows.
For each value of $N$ in equation (19), one can use the same computer program whose flow chart is shown in Fig. 5, to draw the corresponding stability region. To clarify such a method an example with the realistic data given in Table 1 is solved.

\begin{align*}
\text{DATA} \\
\text{Calculation of normal operation parameters} \\
\text{Determination of transfer functions of the system elements (generator of infinite busbar and AVR)}
\end{align*}

\begin{align*}
N & = 1 \\
\omega & = 0.001 \\
\text{Equation 19} \\
\text{PRINT } K_0, K_1 \\
\omega & = \omega + \Delta \omega \\
\omega & = 30 \\
N & = N - \Delta N \\
N & = 0 \\
\text{STOP}
\end{align*}

Fig. 5. Flow chart of the computer program

It has been noticed that, at specific value of $N$ (in this example $N=0.1$) the sequence of the boundaries portraits starts to change the direction. The intersection between this reversed boundary and the main one with $N=1$ is the non-limit cycle region shown in Fig. 7. To verify that the limit cycle will never take place if the adjustable coefficients are chosen within such a region, five points in the two dimentioned plane are tested. Fig. 6. shows the locations of these points ($a$, $b$, $c$, $d$ and $e$); the Nyquist stability criterion is used to check if there is a limit cycle or not. If the adjustable coefficients $K_0$ and $K_1$ are chosen at $a$, $b$ or $c$, then the stable limit cycle will take place.
(points a', b' and c' in Fig. 8) at equal frequencies and different magnitudes. However, at point d that has been chosen on the border of non-limit cycle stability region, the Nyquist contour will touch the negative real axis at point d'. If the operating point is chosen inside the prescribed non-limit cycle stability region, then the limit cycle will never take place (point e in Fig. 6 and Nyquist contour e in Fig. 8).

### TABLE 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Typical value (p.u)</th>
<th>Parameter</th>
<th>Typical value (p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>0.8</td>
<td>X_T</td>
<td>0.16</td>
</tr>
<tr>
<td>v</td>
<td>1.05</td>
<td>X_c</td>
<td>0.64</td>
</tr>
<tr>
<td>g</td>
<td>0.0079</td>
<td>r_x</td>
<td>0.119</td>
</tr>
<tr>
<td>r</td>
<td>1.53</td>
<td>V_c</td>
<td>1</td>
</tr>
<tr>
<td>X_d</td>
<td>0.97</td>
<td>o</td>
<td>314.118</td>
</tr>
<tr>
<td>X_q</td>
<td>1.88</td>
<td>K_Bv</td>
<td>0.87</td>
</tr>
<tr>
<td>X_f</td>
<td>1.97</td>
<td>K_ov</td>
<td>50</td>
</tr>
<tr>
<td>X_1d</td>
<td>1.17</td>
<td>K_1v</td>
<td>7</td>
</tr>
<tr>
<td>X_1q</td>
<td>1.35</td>
<td>K_o1</td>
<td>2</td>
</tr>
<tr>
<td>X_ad</td>
<td>0.79</td>
<td>K_11</td>
<td>8.1</td>
</tr>
<tr>
<td>X_aq</td>
<td>75</td>
<td>K_21</td>
<td>1.45</td>
</tr>
<tr>
<td>K_s</td>
<td>0.27</td>
<td>K_21</td>
<td>1.45</td>
</tr>
</tbody>
</table>

### APPENDIX A

**AVR Transfer Functions**

\[
G_v(s) = G_{V}(s) \left[ G_{oV}(s) + G_{1v}(s) \right]
\]

\[
G_{BV}(s) = K_{BV}/(1 + 0.0135 + 0.000083 \, S^2)
\]

\[
G_{ov}(s) = K_{ov}/(1 + 0.0026 \, S)
\]

\[
G_{1v}(s) = K_{1v}/(1 + 0.0026 \, S)
\]

\[
g_{o1}(s) = G_{o1}(s) \left[ G_{11}(s) + G_{21}(s) \right]
\]

\[
g_{1f}(s) = S \, K_{1f}/(1 + 0.026 \, S)
\]

\[
g_{21}(s) = 1/(1 + 0.0038 \, S)
\]

\[
g_{oi}(s) = K_{oi}/(1 + 0.036 \, S)
\]

\[
G_{o1}(s) = G_{o1}(s) \left[ G_{11}(s) + G_{21}(s) + G_{o1}(s) \right]
\]

\[
G_{BI}(s) = 1/(1 + 0.0038 \, S)
\]

\[
G_{BL}(s) = K_{o1}/(1 + 0.036 \, S)
\]
\[
G_{1I}(s) = \frac{S K_{1I}}{1 + 0.036 S} \\
G_{2I}(s) = \frac{S^2 K_{2I}}{1 + 0.123 S + 0.0023 S^2} \\
G_E(s) = G_s(s) G_j(s) G_{E}\text{(s)} \\
G_s(s) = \frac{K_s (1 + 0.1 S)}{1 + 0.66 S} \\
G_j(s) = \frac{1}{1 + 0.005 S} \\
G_{EX}(s) = \frac{1}{1 + 0.05 S} \\
H_E(s) = K_{FB}
\]

Fig. 6. Steady state stability region.

Fig. 7. Regions of stable operation at different values of N(1, 0.8, 0.6, 0.4, 0.2, 0.1, 0.08, 0.06).

Fig. 8. Frequency response characteristics.
CONCLUSIONS

Choosing the AVR adjustable coefficients within stable operation region in the plane of two parameters in the past was not the optimum choice, since there was a chance for a limit cycle occurrence.

The stability region has been separated to limit-cycle and non-limit cycle ones. Two techniques have been involved, first by scanning the main region, second by describing function analysis. A real example data has been used so as to compare the two techniques results and prove that the second is a time saver and more precise one.

Enhancing the non-limit cycle region, so that the coefficients can be chosen freely may be the scope of the future research.

REFERENCES

2] Letkinze E.V., Nonlinear Oscillation in regulated power systems, Moscow, Moscow Energy Institute, 1974, pp. 146.

NOMENCLATURE

All quantities in per unit on machine base

$X_d(s), X_q(s)$ Operational impedances (direct and quadrature components)

$y(s)$ Operational admittance

$\psi_{do}, \psi_{qo}$ Steady state fluxes (direct and quadrature components)

$V_{do}, V_{qo}$ Steady state generator voltages (direct and quadrature components)

$I_{do}, I_{qo}$ Steady state generator currents (direct and quadrature components)

$r$ Transmission line resistance

$X_L$ Transmission line reactance

$V_f$ Field voltage

$M$ Mechanical inertia constant

$G_L(s)$ Open loop transfer functions