COM-2 1227

MILITARY TECHNICAL COLLEGE

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A FAST SEPARATION OF SPECTRA ALGORITHM FOR THE DESIGN OF 2-D CIRCULARLY SYMMETRIC DIGITAL FILTERS

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ABSTRACT

The separation of spectra algorithm contributes a simple technique for designing 2-D circularly symmetric digital filters. The calculation of the pole-,and zero-coefficients of a filter of order(p1,p2,q1,q2) needs the solution of two systems of equations.Their solution using Gauss elimination method consumes $O(p2+p1(1+p2))^3 + O(q2+q1(1+q2))^3$ complex operations. For reducing the complexity of calculation,a 2-D Levinson algorithm is deduced and used for solving the two systems of equations. The complexity of calculation is reduced to $O(p2+p1(1+p2))^2 + O(q2+q1(1+q2))^2$-complex operation.The 2-D separation of spectra algorithm which employs the 2-D Levinson algorithm is simple,fast,robust,and efficient.Besides,it has been efficiently used for the design of different kinds of 2-D recursive digital filters.

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I. INTRODUCTION

The design of 2-D digital filters is a very important problem, since 2-D filters are employed in many applications. They are used for enhancement of photographic data, such as weather photos, air photos, and medical X-rays. Also, they are used for processing seismic records and magnetic data. Five approaches for the design of 2-D digital filters exist; the space domain approach [1], the frequency transformation approach [2-3], the optimization approach [4-5], spectral factorization approach [6-7], and the statistical approach [8]. We introduce in this paper a fast version of the separation of spectra algorithm which belongs to the statistical approach. The algorithm is considered as an extention of the separation of spectra algorithm used for the design of 1-D recursive [9] and non recursive [10] digital filters.

The separation of spectra algorithm starts by calculating the 2-D pole-coefficients of the filter as a result of the exact fitting of the correlation function of the desired (ideal) characteristic. Then, the reciprocal of the inverse filtered spectral density function is separated. The correlation of the separated characteristic is then fitted to get the required zero-coefficients of the filter. A filter of order (p1,p2,q1,q2) requires the solution of two systems of equations for calculating its p1xp2 pole-coefficients and q1xq2 zero-coefficients. The algorithm in [8] used Gauss elimination (G.E.) method for solving the two systems of equations. The calculation of pole-, and zero-coefficients consumed $O(p2+p1(1+p2))^3$ and $O(q2+q1(1+q2))^3$ respectively. For reducing the complexity of calculation the 2-D Levinson algorithm proposed in this paper is used. The complexity of calculation of pole-, and zero coefficients is reduced to $O(p2+p1(1+p2))^2$ and $O(q2+q1(1+q2))^2$ respectively. The 2-D Levinson algorithm is similar to the 1-D Levinson one [11]. It solves the system of Toeplitz (or simi-Toeplitz) equations recursively, starting by order (1,1) and increasing the order upto the predetermined one ((p1,p2), or (q1,q2)).

II. 2-D SEPARATION OF SPECTRA ALGORITHM

The 2-D separation of spectra algorithm [8] introduces a simple and efficient method for the design of 2-D circularly symmetric causal recursive filters. It starts by solving the approximation problem for the pole-coefficients (the coefficients of the denominator polynomial of the filter's transfer function). After this step, the algorithm separates the reciprocal of the inverse filtered spectral density characteristic, which is then used for solving the approximation problem for the zero-coefficients (coefficients of the numerator polynomial of the filter's transfer function).

The transfer function of the filter, H(z1,z2), is to be calculated such that its spectral density characteristic approximates a desired (ideal) one (RI(z1,z2)), namely :

$$RI(z1,z2) \approx H(z1,z2)H(z1,z2^{-1})H(z1^{-1},z2)H(z1^{-1},z2^{-1}) = R(z1,z2) \quad (1)$$

where

$$H(z1,z2) = -\frac{\sum_{i=o}^{q1}\sum_{j=o}^{q2} b_{ij}\, z1^{-i}\, z2^{-j}}{\sum_{i=o}^{p1}\sum_{j=o}^{p2} a_{ij}\, z1^{-i}\, z2^{-j}} = \frac{B(z1,z2)}{A(z1,z2)} \quad (2)$$

For solving this approximation problem,the steps of the algorithm are manipulated as follows :

1-calculate the 2-D ideal correlation function as

$$ri(m,n) = IDFT\left\{RI(z1,z2)\right\} \quad ,m=o,1,\ldots,N1-1 \quad ,n=o,1,\ldots,N2-1 \quad (3)$$

2-solve the system of equations to get the pole-coefficients

$$\sum_{\substack{k=o\\(k,l)\neq(o,o)}}^{p1}\sum_{l=o}^{p2} a_{kl}\, ri(m-k,n-j) = -ri(m,n) \quad ,m=q1,q1+1,\ldots,q1+p1 \quad ,n=q2,q2+1,\ldots,q2+p2 \quad ,\text{and } (m,n)\neq(q1,q2) \quad (4)$$

3-check the stability of the filter using the conjecture in [8]
4-calculate the inverse filtered spectral density function as

$$RA(z1,z2) = A(z1,z2)A(z1^{-1},z2^{-1})RI(z1,z2)A(z1^{-1},z2)A(z1,z2^{-1}) \quad (5)$$

5-calculate the correlation of the reciprocal of RA(z1,z2) as

$$rr(m,n) = IDFT\left\{1/RA(z1,z2)\right\} \quad ,m=o,1,\ldots,N1-1 \quad ,n=o,1,\ldots,N2-1 \quad (6)$$

6-solve the system of equations to get the zero-coefficients

$$\sum_{\substack{i=o\\(i,j)\neq(o,o)}}^{q1}\sum_{j=o}^{q2} b_{ij}\, rr(m-i,n-j) = -rr(m,n) \quad ,m=M,M+1,\ldots,M+q1 \quad ,n=N,N+1,\ldots,N+q2 \quad ,\text{and } (m,n)\neq(M,N) \quad (7)$$

where M,N are integers $\geq$ o,and are chosen to change range of fitting the correlation function rr(m,n).

The calculation of the pole-,and zero-coefficients of the filter using (4) and (7) requires $O(p2+p1(1+p2))^3$ and $O(q2+q1(1+q2))^3$ complex operations respectively. For large values of p1, p2, q1, and q2 the complexity of calculation is high. For reducing the complexity of calculation and consequently speeding up the design procedure, a 2-D Levinson algorithm is used for solving the two systems of equations (4) and (7).

III. 2-D LEVINSON ALGORITHM

The system of equations given by :

$$\sum_{i=1}^{p} a_i\, r(n-i) = -r(n) \qquad ,n=q+1,q+2,\ldots,q+p \quad (8)$$

has an interesting property shown when rewritting it in matrix form as follows :

$$R\,A = -r \qquad (9)$$

or equivalently

$$\begin{bmatrix} r(q) & r(q-1) \ldots\ldots\ldots\ldots & r(q+1-p) \\ r(q+1) & r(q) \quad \ldots\ldots\ldots\ldots & r(q+2-p) \\ \vdots & \vdots & \vdots \\ r(q+p-1) & r(q+p-2) \ldots\ldots\ldots & r(q) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} \begin{bmatrix} r(q+1) \\ r(q+2) \\ \vdots \\ r(q+p) \end{bmatrix} \qquad (10)$$

Since r(n) is the correlation function, one can see that the elements along any diagonal of the matrix R are identical. This matrix is called assymmetric Toeplitz. The solution of (10) using the traditional G.E method (matrix inversion) consumes $O(p)^3$ complex operations. Levinson [11] derived an elegent recursive procedure for solving this type of equations. The method requires only $O(p)^2$ complex operations : a big saving from the more general methods. Since equations (4) and (7) comprise the same property of matrix R in Eq.(10), with blocks replace elements, we derive in this section a 2-D generalized Levinson algorithm (GLA) for solving them recursively. In the following we give the algorithm interms of the parameters of Eq.(4).

Analogous to the 1-D Levinson algorithm [11], Marzetta [12] proposed a special 2-D Levinson algorithm for solving the linear prediction problem of the autoregressive (AR) filter. Our 2-D Levinson algorithm modifies the algorithm of Marzetta to be general and useful for solving the linear prediction problem of the autoregressive moving average (ARMA) filter.

The idea of the proposed 2-D Levinson algorithm is to choose the parameters of the forward and backward predictors such that the following equations are satisfied :

$$A_{m+1,n+1}(z1,z2) = A_{m,n}(z1,z2) + k_{m,n}\, D'_{m,n}(z1,z2) \qquad (11)$$

$$D_{m+1,n+1}(z1,z2) = D_{m,n}(z1,z2) + f_{m,n}\, A'_{m,n}(z1,z2) \qquad (12)$$

where the polynomial $A'_{m,n}(z1,z2)$ and $D'_{m,n}(z1,z2)$ are related to the forward and backward predictors as follows :

$$A'_{m,n}(z1,z2) = z1^{-m-1}\, z2^{-n-1}\, A_{m,n}(z1,z2) \qquad (13)$$

$$D'_{m,n}(z1,z2) = z1^{-m-1}\, z2^{-n-1}\, D_{m,n}(z1,z2) \qquad (14)$$

and the forward and backward predictors are given by :

$$A_{m,n}(z1,z2) = \sum_{i=o}^{m} \sum_{j=o}^{n} a_{i,j;m,n}\, z1^{-i}\, z2^{-j} \qquad (15)$$

$$D_{m,n}(z1,z2) = \sum_{i=o}^{m} \sum_{j=o}^{n} d_{i,j;m,n}\, z1^{-i}\, z2^{-j}$$

respectively.The forward and backward partial correlation coefficients, $k_{m,n}$ and $f_{m,n}$ respectively, are chosen such that the (m+1,n+1)th predictors are orthogonal to the following polynomial :

$$W_{k,l}(z1,z2) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} ri(q1+k-i,q2+l-j)\, z1^{-i}\, z2^{-j} \qquad (16)$$

This leads to the proposed algorithm which is written in the following :

Initialization :

$$a_{o,o;m,n} = d_{o,o;m,n} = 1 \quad ,m=o,1,2,\ldots,p1 \quad ,n=o,1,2,\ldots,p2 \qquad (17)$$

$$e_{o,o} = ri(q1,q2) \qquad (18)$$

Recursion :

$$k_{m,n} = -\sum_{i=o}^{m-1} \sum_{j=o}^{n} a_{i,j;m-1,n}\, ri(q1+m-i,q2+n-j)/e_{m-1,n} \qquad (19)$$

$$f_{m,n} = -\sum_{i=o}^{m-1} \sum_{j=o}^{n} d_{i,j;m-1,n}\, ri(q1+m-i,q2+n-j)/e_{m-1,n} \qquad (20)$$

$$e_{m,n} = e_{m-1,n}\,(1-k_{m,n}\,f_{m,n}) \tag{21}$$

$$a_{m,n;m,n} = k_{m,n} \tag{22}$$

$$a_{m,n} = a_{m,n;m,n} \tag{23}$$

for $m=1,2,\ldots,p1$, and $o \leqslant n \leqslant m$

$$a_{i,j;m,n} = a_{i,j;m-1,n} + k_{m,n}\,d_{m-i-1,n-j;m-1,n} \tag{24}$$

$$d_{i,j;m,n} = d_{i,j;m-1,n} + f_{m,n}\,a_{m-i-1,n-j;m-1,n} \tag{25}$$

for $1 \leqslant i \leqslant m-1$, and $o \leqslant j \leqslant n$

Eq's (19) - (26) are used for calculating the coefficients a_{mn} and $m>n$. For calculating them with $m<n$, the following recursion is used :

$$k_{m,n} = -\sum_{i=o}^{m}\sum_{j=o}^{n-1} a_{i,j;m,n-1}\, ri(q1+m-i,q2+n-j)/e_{m,n-1} \tag{26}$$

$$f_{m,n} = -\sum_{i=o}^{m}\sum_{j=o}^{n-1} d_{i,j;m,n-1}\, ri(q1+m-i,q2+n-j)/e_{m,n-1} \tag{27}$$

$$e_{m,n} = e_{m,n-1}\,(1-k_{m,n}\,f_{m,n}) \tag{28}$$

$$a_{m,n;m,n} = e_{m,n} \tag{29}$$

$$a_{mn} = a_{m,n;m,n} \tag{30}$$

for $n=1,2,\ldots,p2$, and $o \leqslant m \leqslant n$

$$a_{i,j;m,n} = a_{i,j;m,n-1} + k_{m,n}\,d_{m-i,n-j-1;m,n-1} \tag{31}$$

$$d_{i,j;m,n} = d_{i,j;m,n-1} + f_{m,n}\,a_{m-i,n-j-1;m,n-1} \tag{32}$$

for $o \leqslant i \leqslant m$, and $1 \leqslant j \leqslant n-1$

For calculating the coefficients a_{mn} with $m=n$, Eq's (19), (22), and (23) are used once.

The calculation of the zero-coefficients is performed using the above recursion with (q1,q2) replaces (p1,p2), (M,N) replaces (q1,q2), and rr(m,n) replace ri(m,n). Therefore, the calculation of the pole-, and zero-coefficients is accomplished by employing the 2-D L.A two times. Their calculation requires

$O(p2+p1(1+p2))^2$ and $O(q2+q1(1+q2))^2$ respectively.

The above algorithm is general and could be employed for any type of symmetry.However,for a circularly symmetric filter a 50 % reduction in the complexity of calculation of the original algorithm is attained,since the coefficients of the filter must be symmetric arround the straight line m=n.

IV.PRACTICAL RESULTS

In the following we give an example for designing a low-pass filter to show the performance of the separation of spectra algorithm and the efficiency of the 2-D L.A.for reducing the complexity of calculation of pole- and zero-coefficients.The ideal characteristic is circularly symmetric and specified by (64x64) sampled points.The ideal characteristic of the filter is defined by :

$$RI(e^{j2\pi f1},e^{j2\pi f2})=\begin{cases} 0 \quad dB & ro \leqslant 0.156 \\ -20 \quad dB & ro \leqslant 0.375 \end{cases}$$

where

$$(ro)^2 = (f1/f_{s1})^2 + (f2/f_{s2})^2$$

and f_{s1} and f_{s2} are the sampling frequencies in the f1 and f_2 directions respectively (see Fig.1).The above characteristic is approximated by the recursive filter having p1=p2=2 and q1=q2=1.The resulting characteristic is shown in Fig.2.Increasing p1 and p2 to 3 and 3,the performance of the filter improves while the complexity of calculation increases.The performance(Peak Pass Band Ripples(PPBR),and Minimum Stop Band Attenuation(MSBA)) of the filter and the complexity of calculation in both cases are given in Table 1.From this table,it is clear that using the 2-D L.A.,a big saving in the complexity of calculation of pole-,and zero-coefficients is attained.

Table 1 Performance of the Low-pass Filter Resulting from the Separation of Spectra Algorithm.

Order of Filter	PPBR (dB)	MSBA (dB)	Complexity of calculation	
			G.E	2-D L.A
(2,2,1,1)	1.031	30	O(539)	O(36.5)
(3,3,1,1)	0.89	31	O(3402)	O(117)

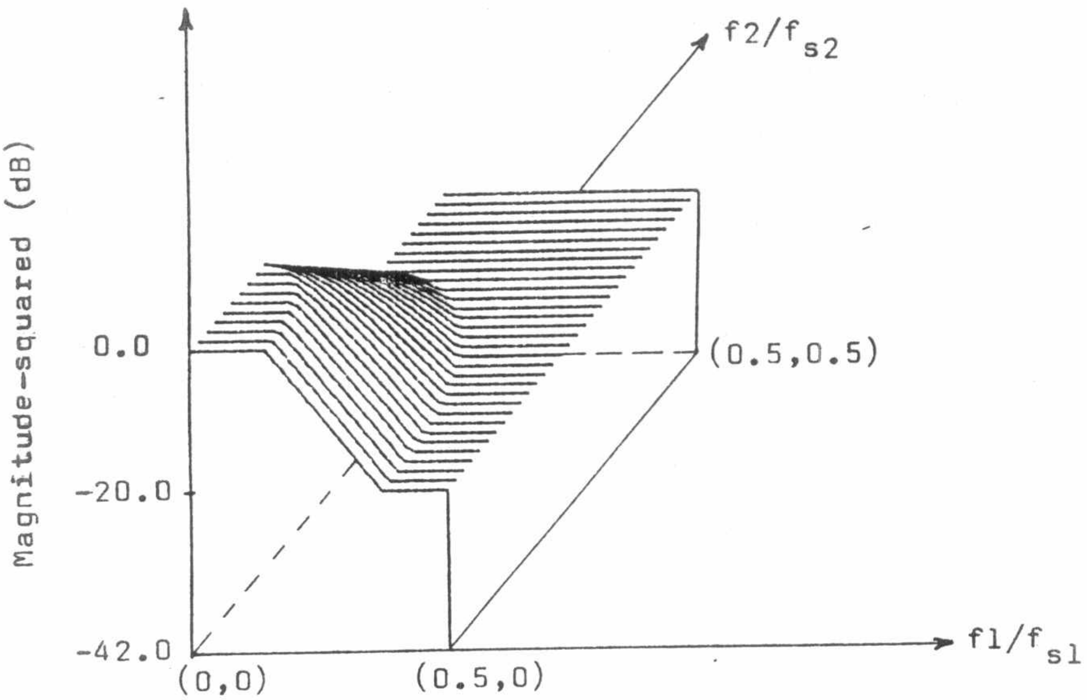

Fig.1 Ideal spectral density characteristic of low-pass filter.

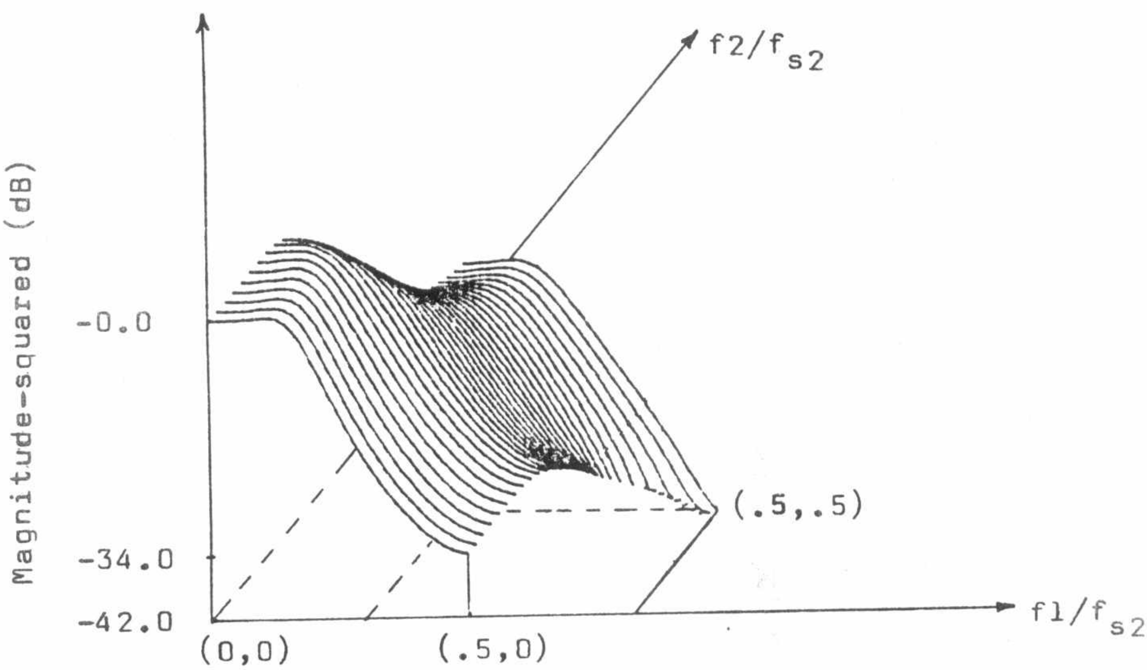

Fig.2 Resulting spectral density characteristic of the (2,2,1,1) low-pass filter.

V.CONCLUSION

The 2-D separation of spectra algorithm is a simple,robust, and straightforward approach for the design of 2-D circularly symmetric causal recursive filters.For speeding up the calculation of pole-,and zero-coefficients of the filter using this approach,a 2-D Levinson algorithm has been deduced.This algorithm could be generally used in the linear prediction of 2-D ARMA filter,and it has been adapted to the problem of designing 2-D causal recursive digital filters.The circularly symmetric constraint put on the correlation function has led to a symmetry in the coefficients of the filter and consequently a rather simplification in the calculation,over the general case,has been attained.The complexity of calculation has been reduced to $O((p2+p1(1+p2))^2 + (q2+q1(1+q2))^2)$ complex operations insead of $O((p2+p1(1+p2))^3 + (q2+q1(1+q2))^3)$ complex operations when using Gauss elimination method.

Using the separation of spectra algorithm together with the 2-D Levinson algorithm we have designed different shapes of 2-D circularly symmetric causal recursive digital filters. Moreover,the algorithm could be adjusted to design 2-D nonrecursive digital filters.

REFERENCES

[1] J.LSHANKS,S.TREITEL,and J.H.JUSTICE," Stability and Synthesis of Two-Dimensional Recursive Filters ",IEEE Trans. Audio and Electroacous.,Vol.AU-20,pp.115-128,june 1972.

[2] J.M.COSTA,and A.N.VENETSANAPAULOS," Design of Circularly Symmetric Two-Dimensional Recursive Filters ",IEEE Trans. Acoust.,Speech,Signal Proces.,Vol.ASSP-22,pp.432-443, December 1974.

[3] P.K.ROJAN,H.C.REDDY,M.N.SWAMY,and V.R.AMACHANRAN," Genration of Two-Dimensional Digital Functions without Non-essential Singularities of the Second Kind ",IEEE.Trans.Acou., Speech,Signal Proces.,Vol.ASSP-28,N° 2,pp.216-223,April 1980.

[4] R.M.MERSEREAU,and D.E.DUDGEON," Two-Dimensional Digital Filtering ",Proc.of the IEEE,Vol.63,N° 4,pp.610-623,April 1975.

[5] S.A.H.ALY,and M.M.FAHMY," 2-D Separable Digital Filters ", ICASSP-81,pp.688-691,Atlanta,1981.

[6] D.E.DUDGEON," The Existance of Cepstra for Two-Dimensional Rational Polynomials ",IEEE Trans.Acous.,Speech,Signal Proces.,Vol.ASSP,pp.242-243,April 1975.

[7] M.P.EKSTROM,and J.W.WOODS," Two-Dimensional Spectral Factorization with Application in Recursive Digital Filtering" ,IEEE Trans.Acous.,Speech,Signal Proces.,Vol.ASSP,N° 2, pp.115-128,April 1976.

[8] A.HANAFY,' Design of 2-D Causal Recursive Filters Using Separation of Spectra ',Third National Radio Symposium, pp.128-139,M.T.C.,Cairo,Egypt,Feb.1985.

[9] A.HANAFY,J.le ROUX,and J.PRADO,' Iterative and Non Iterative Techniques for the Design of Recursive Digital Filters',ICASSP-82,Paris 1982.

[10] A.HANAFY,' Design of Minimal-Phase Nonrecursive Digital Filters ',Third National Radio Sympo.,pp.202-211,M.T.C., Cairo,Egypt,February 1985.

[11] N.LEVINSON,' The Wiener RMS(Root Mean Square)Error Criterion in Filter Design and Prediction ',Journal of Math. and Phsics,Vol.25,N°1,pp.261-278,February 1946.

[12] T.L.MARZETTA,' Two-Dimensional Linear Prediction :Autocorrelation Array,Minimum Phase Prediction Error Filters,and Reflection Coefficient Arrays ',IEEE Trans.Acoustic,Speech ,Signal Processing,Vol.ASSP-28,N° 6,pp.725-733,Oct.1980.