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1 # A NEW SWITCHING CRITERION

2 # FOR THE DUAL-MODE ORBIT PREDICTOR

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4 ## ABSTRACT

The estimation of the orbital states of a satellite is complicated by the fact that a realistic model of satellite dynamics is nonlinear and so is the observation model. Among the various schemes for nonlinear state estimation, it was found in previous work that a combination of invariant imbedding with stochastic approximation provides accurate estimates. The method consists of starting estimation with invariant imbedding and switching over to stochastic approximation at a suitable linstant of time.

A eriterion that governs the switching instant was devised. It was developed by using the statistical properties of the innovations sequence. The "degree of whiteness" of the innovations sequence was used as an indicator of the overall performance of the estimator. However, when the number of observations is small, this switching criterion fails to give acceptable results.

In this paper, a new switching criterion for the dual - mode orbit predictor is presented. This criterion is based on a modification to the measure of whiteness of the innovations sequence. Results of simulation using data supplied by the Communications Research Centre of Canada are included and demonstrate the effectiveness of the proposed criterion when moderate sample sizes are available.

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INTRODUCTION

In order that corrective action may be taken to maintain a satellite in the desired orbit, it is necessary to estimate accurately the orbital states of the satellite from ground-based measurements. The problem is complicated by the fact that a realistic model for the orbital dynamics is nonlinear and so is the observation model[1,2,6].

Several different methods for the recursive estimation of the states of nonlinear systems have been considered for this purpose [7,10,15]. Most of these algorithms employ a Taylor series expansion and use the linearized equations to compute the error covariance matrix and the filter gains. Detchmendy and Sridhar [9], and Kagiwada and others [11] have derived filtering algorithms similar to the first-order filter for nonlinear estimation problems, using the least squares errors criterion and the invariant imbedding technique. These algorithms suffer from a number of major drawbacks, such as the need for prior knowledge of the noise covariance matrices, as well as the covariance matrix of the initial estimation error. Also, the major problem usually encountered in nonlinear state estimation is that of divergence.

These problems were the primary motivation for the development of alternative orbit determination schemes. The use of second-order filters was examined [3], but again, these second-order filters are not easily derived for highly nonlinear systems and most of them cannot be adapted for on-line estimation purposes.

An examination of these methods led to the development of an algorithm combining invariant imbedding with stochastic approximation[3] which does not require a priori knowledge of input and measurement noise statistics and gives convergence to the correct states with much less computation than required for the other methods. The invariant imbedding concept is utilized to obtain a recursive estimator which does not require the statistical information, but requires a considerable amount of computation. The switching over to stochastic approximation at a suitable instant of time gives further estimates with very little computation. The overall algorithm is, therefore, very efficient. The only problem is the difficulty in determining the instant at which one should switch over to stochastic approximation.

In this paper, a new switching criterion for the dual-mode orbit predictor is presented. This criterion is based on a modification to the measure of whiteness of the innovations sequence.

ALGORITHM COMBINING INVARIANT IMBEDDING AND STOCHASTIC APPROXIMATION

Consider a dynamic system described by the nonlinear vector differential equation

$$\dot{x} = f(x,t) \tag{1}$$

where x and f are n-dimensional vectors.

The measurement model is given by the m-dimensional nonlinear vector equation

$$y(t) = h(x,t) + v(t) \tag{2}$$

where y is the observed output vector and v is an m-dimensional random observation noise vector.

The estimation problem is the determination of the optimal estimate, in the least square sense, of the state vector $x(kT)$ from the observations $y(rT)$ over the period $r = 0,1,2,\ldots,k$, where T is the sampling interval.

The approximate nonlinear filter, based on the invariant imbedding concept, is described by the following equations [3,9,11].

Filter Algorithm

$$\hat{x}(k+1) = \hat{x}(k+1|k) + K(k+1)\{y(k+1)-h[\hat{x}(k+1|k),k+1]\} \tag{3}$$

One-Stage Prediction Algorithm

$$\dot{x}(k+1|k) \triangleq x(kT+T) \tag{4}$$

which is the solution of

$$\dot{x}(t) = f[x(t),t], \; kT < t < (k+1)T \tag{5}$$

Filter Gain Algorithm

$$K(k+1) = P(k+1) \frac{\partial h^T[\hat{x}(k+1|k),\; k+1]}{\partial \hat{x}(k+1|k)} R^{-1}(k+1) \tag{6}$$

where $K(k) \triangleq$ the invariant imbedding gain matrix at the k'th sampling instant.
$P(k) \triangleq$ the error covariance matrix at the k'th sampling instant.
$R(k) \triangleq$ the observation noise coveriance matrix at the k'th sampling instant.
and the superscript T indicates transposition.

Prior Error-Covariance Matrix

$$P(k + 1|k) = P(kT + T) \tag{7}$$

where $P(kT)$ is the solution at $kT < t < (k+1)\; T$ of

$$\dot{P}(t) = \left.\frac{\partial f(x,t)}{\partial x(t)}\right|_{\hat{x}(t)} P(t) + P(t) \left.\frac{\partial f^T(x,t)}{\partial x(t)}\right|_{\hat{x}(t)} \tag{8}$$

Error Covariance Algorithm

$$P(k+1) = [P^{-1}(k+1|k) - \frac{\partial}{\partial \hat{x}(k+1|k)} \frac{\partial h^T}{\partial \hat{x}(k+1|k)} R^{-1}(k+1).\zeta(k+1)]^{-1} \tag{9}$$

where

$$\zeta(k) \triangleq y(k) - h[\hat{x}(k|k-1),k] \tag{10}$$

is called the innovations sequence.

The invariant imbedding algorithm described above has the following advantages:

(a) No statistical assumptions are required concerning the nature of the input disturbances or observation errors. In the absence of any prior information, we may start with $R(0) = I$.
(b) A sequential estimation scheme is obtained, which makes it possible to implement in real time.
(c) Convergence of the algorithm is theoretically guaranteed.

The disadvantages of this algorithm are:

(a) More computation is needed as compared to quasilinear or first-order filters.
(b) When the dimension of the state vector is large, the computation time may become larger than the interval between successive observations.
(c) The initial values $x(0)$ and $P(0)$ affect the rate of convergence.

These disadvantages are partly overcome by using the stochastic approximation algorithm,where the filtering equation is the same as Eq. (3), but the gain matrix is given by (Scharf and Alspach, 1972),

$$K(k+1) = K(k) + \frac{a}{b+k} \frac{\hat{x}(k|k-1)\ \zeta^T(k)}{\|\hat{x}(k|k-1)\zeta^T(k)\|^2} \tag{11}$$

where a and b are constants to be determined experimentally.

The stochastic approximation algorithm has the following advantages:

(a) Only a small amount of data needs processing.
(b) Only simple computations are required. These are the computation of the innovations sequence and simple vector multiplications.
(c) A priori knowledge of the process statistics is not necessary. The only requirements are that the regression function satisfies certain regularity conditions and that the regression problem has a unique solution.

The disadvantages of the stochastic approximation algorithm are:

(a) The rate of convergence is rather slow.
(b) The convergence properties depend on the starting values.

From the above, it would appear logical to combine the two algorithms in such a manner as to retain their relative advantages, while disposing with their basic drawbacks. With this point in view, it was proposed [3] to use the invariant imbedding algorithm initially, until the value of the filter gain matrix has reasonably stabilized and then apply stochastic approximation to track any changes in the gain matrix which may bring further improvement. The results of simulation reported by Azim [3] demonstrate the convergence and the efficiency of this combined algorithm in comparison with other approaches in the use of satellite orbit determination.

We shall, therefore, focus our attention to the development of a suitable criterion for determining when one should change over from invariant imbedding to stochastic approximation.

THE INNOVATIONS SEQUENCE AND ITS USE AS A SWITCHING INDICATOR

The idea of innovations was first introduced by Kailath (1968), and since then it has proved a very successful approach to the problem of filtering and estimation [5].

The innovations process can be considered as the new information in the observations, and is given by the sequence

$$\zeta(k) = y(k) - h[\hat{x}(k|k-1),k] \tag{12}$$

In the ideal case, this new information is independent of all the past information, i.e.,

$$E[\zeta(k)\ \zeta^T(s)] = \sigma^2\ I\ \delta(k-s)\quad 0 \leq s < k \tag{13}$$

Thus, the "degree of whiteness" of the innovations sequence can be used as an indicator of the overall performance of the estimator [17].

This idea was used earlier by Mehra [13]. Not only did Mehra use the whiteness of innovations to check the filter performance, but he also used the autocorrelation function of the innovations process to adaptively improve the filter gain matrix. We shall now add another dimension to the usefulness of the innovations process by using it to determine the switching instant for the proposed dual-mode estimation algorithm.

The whiteness of the innovations sequence can be tested statistically by number of different methods [4]. Here we consider a particular method given by Box and Pierce [4]. In this method a quantitative measure of the degree of whiteness is obtained by evaluating a certain quantlty, Q', as follows,

$$Q_i' = N \sum_{k=1}^{p} \hat{r}^2_{\zeta_i}(k) \tag{14}$$

where

$\hat{r}_{\zeta_i}(k) \triangleq$ estimated autocorrelation coefficient of the i'th innovations sequence at lag k.

$N \triangleq$ number of observations.

and p is not larger than say N/4.

If the i'th innovations sequence is white with α percent confidence, then Q_i' is approximately distributed as the Chi-square distribution $X^2_{(1-\alpha)}(p)$. This degree of whiteness may be used as the criterion for switching over from invariant imbedding to stochastic approximation.

It has been shown by Davies, Triggs and Newbold [8] that, for moderate sample sizes, the actual significance level of Q' can be considerably lower than those predicted by asymptotic theory (so that the chance of rejecting the null hypothesis of whiteness is overestimated). However, a simple modification, studied in detail by Ljung and Box [14],

$$Q_i = N(N+2) \sum_{k=1}^{p} (N-k)^{-1} \hat{r}^2_{\zeta_i}(k), \tag{15}$$

appears to have a distribution very much closer to the asymptotic X^2. It would seem preferable, then, to base tests of whiteness in this paper on (15) rather than (14).

RESULTS OF SIMULATION

The algorithm combining invariant imbedding with stochastic approximation was simulated on a CDC 6400 computer for different switching instants. The sampling interval was one second, and the total period of simulation was 300 seconds.

Since we have three observable outputs (azimuth, elevation and range) the number of innovations sequences is three. Therefore, Q_1, Q_2 and Q_3 have been calculated using Eq.(15). A plot of these is shown in Fig. 1 as functions of the switching instant. The acceptable level of whiteness is given by $Q_i = 43.8$ with 95 percent confidence, as given by tables of Chi-squared distribution.

Also, as the switching instant occurs later, more iterations of invariant imbedding are carried out. This causes the total computation time to increase, as shown in Fig. 2.

Hence, we must choose the switching instant in such a manner that an acceptable level of whiteness of the three innovations sequences is obtained without increasing considerably the total computation time.

A study of Figs. 1 and 2 indicates that switching at 70 seconds is a quite reasonable choice. For this particular case, Fig. 3 illustrates the estimation error for the position of the satellite. Figs. 4 to 8 show the mominal trajectories for five of the seven states of the unified state model, together with their estimated values using the algorithm which combines invariant imbedding with stochastic approximation.

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CONCLUSIONS

It may be pointed out that the switching criterion described above can be easily implemented on-line. This is achieved simply by selecting an acceptable level of whiteness, and then proceeding with the invariant imbedding scheme. The three Q-criteria are evaluated after each iteration, and as their values become lower than the selected level, switching over to stochastic approximation can be carried out.

It is also to be noted that for higher confidence levels on the whiteness of the innovations sequence, the number of iterations carried out with invariant imbedding will be higher. Consequently, the total computation time will also be increased.

In practical application with synchronous-orbit satellites the proposed scheme can be used with great advantage. Although the invariant imbedding algorithm may not be usable on-line, after switching over to stochastic approximation, the tracking of the filter-gain matrix and the estimation of the orbital states can be carried-out on-line. Thus, the initial period when the invariant-imbedding algorithm is used off-line, may be regarded as the "learning" period, which is followed by on-line operation.

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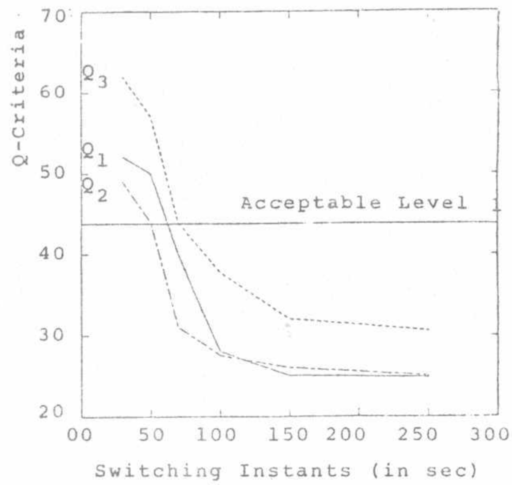

Fig. 1

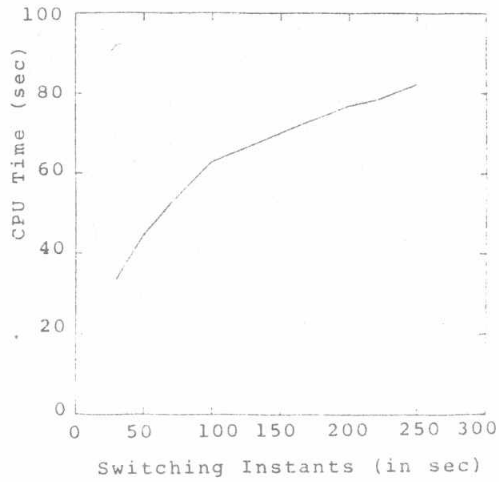

Fig. 2

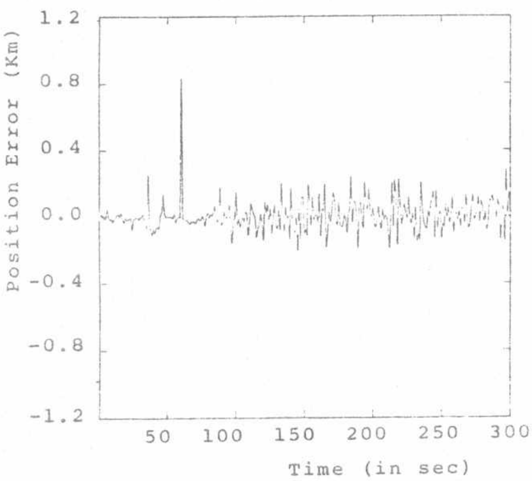

Fig. 3

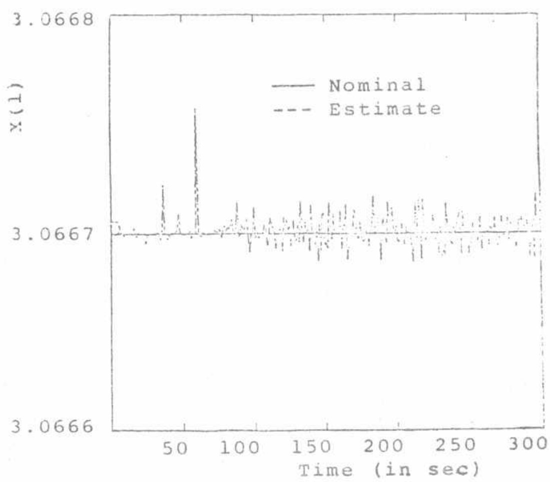

Fig. 4