GC-12 | 1025

MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

A DESCRIBING FUNCTION FOR MULTIPLE NONLINEARITY CONTROL SYSTEMS

DR. E.E. ZAKZOK*, MS S. EL-NAHASS**

1. ABSTRACT

This paper deals with the problem of using the Describing Function (DF) technique, in the analysis of practical control systems which contain more than one nonlinear element in the control loop. The DF technique is analysed to investigate its limitation in nonlinear system design. Both single input-single output and multiple input-single output systems are considered. Computer simulation is utilized to check results. It has been shown that this technique can be used in control systems which contain only one nonlinearity and with single input applications.

However, in the case of systems which have more than one nonlinearity in the control loop, it is shown that the DF Is not an accurate method of analysis specially in the multiple input applications.

2. INTRODUCTION

The DF philosophy is simply to replace the system nonlinearity element chosen such that it will render the same response of nonlinearity for some specific input (1).

It is represented by the amplitude and phase relationship between the sinusoidal input and fundamental component of the expected output. For a sinusoidal input signal A sin $(\omega t + \theta)$, the DF, denoted by N (A,) is defined as :

N (A,ω) = phasor representation of the output component of frequency $\omega \div$ phasor representation of the input component of the frequency ω. (1)

The accuracy of the DF is based upon measuring the ignored part of the nonlinearity output (2). There is no general method for evaluation of this accuracy; but it can be obtained for some specific applications.

In this paper, "the DF technique" is assumed to investigate its limitation in the analysis of the control systems. Three nonlinear elements belonging to different classes are chosen and the DF is calculated for each individual element. The multiplicative DF is then calculated for all combinations of the three nonlinearities. The multiplicative DF for two nonlinearities is given as the product of their individual DF's as in the case of linear systems.

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* MTC, Ministry of Defence, Cairo, A.R.E.

** Air Force, Ministry of Defence, Cairo, A.R.E.

However, actual DF's for all these combinations are obtained using the exact definition. Errors between the actual and multiplicative DF in both the amplitude and phase are calculated. For the system which contains these combinations, limit cycle amplitude and frequency errors are also investigated. This is done for the case of single input systems, and when the error is not exceeding 20%, is repeated for multiple input systems.

3. MULTIPLE NONLINEARITY SYSTEMS

The results obtained when analyzing systems having single nonlinearity through the utilization of DF technique have been quite successful (3). There, it is advantageous to extend the technique for multiple nonlinearity systems.

A general two-nonlinearity control system is illustrated in Fig. (1).

The nonlinearities $N1(x,x')$, and $N2(x,x')$ are replaced by their respective DF's $N1(A1,\omega)$ and $N2(A2,\omega)$ respectively, were A1 and A2 are the amplitude of the input signals to the first and second nonlinearities, while ω is the frequency.

$N(A,\omega)$ is defined by :

$$N(A,\omega) = \frac{j}{2\pi A}\int_0^{2\pi} Y\,(A\sin\psi\,,\ A\cos\psi)\, e^{-j\psi}\, d\psi \qquad (2)$$

where $Y\,(A\sin\psi\,,\ A\cos\psi)$ is the output signal, and A is the amplitude of the input signal.

The frequency response of the control system (error ratio) is given by :

$$\frac{X1}{R}\,(j\omega,A) = \frac{1}{1 + N1\,(A1,\omega)\;N2(A2,\omega)L(j\omega)} \qquad (3)$$

Therefore, the limit cycle is obtained from the solution of the characteristic equation :

$$1 + N1(A1,\omega)N2(A2,\omega)L(j\omega) = 0 \qquad (4)$$

4. NONLINEAR ELEMENTS IN COMBINATION

A) A selection of the most important nonlinearities is done by chosing the following :

(1) Ideal relay (N1)

(2) Amplifier with dead zone (N2)

(3) Hysteresis (N3)

The sinuusoidal and multiple input DF's are computed for every chosen nonlinearity, and the results are summerized in Table (1).

B) All possible combination pairs for those nonlinearities are analysed taking into consideration the proper arrangement. This is explained through Fig. (2).

The multiplicative DF = NnNm is calculated, and the actual DF = Nnm is obtained from the analysis of Y(t), which is calculated point by point through graphic representation. Resultas are checked by computer simulation for cases of sinusoidal input.

Asin $(\omega t + \theta)$, and multiple input B + Asin $(\omega t + \theta)$.

Table (2) gives the results for sinusoidal inputs, while Table (3) gives the results for multiple inputs.

C) Relative errors in amplitude, and phase for systems having two nonlinearities are given by :

$$\text{amplitude} = \frac{|Nn.Nm| - |Nnm|}{|Nnm|} \tag{5}$$

$$\text{phase} = \frac{[\text{phase of } Nn) + \text{phase of } (Nm)] - [\text{phase of } (Nnm)]}{[\text{phase of } (Nnm)]} \tag{6}$$

These errors are illustrated in Table (2) for the case of sinusoidal inputs, and in Table (3) for multiple inputs.

5. LIMIT CYCLE EVALUATION

For the nonlinear control system shown in Fig. (3), N(x,x') is replaced by either NnNm or Nnm, which represents the multiplicative and actual DF for the two nonlinearities respectively, while $L(j\omega)$ is a third order linear system with transfer function.

$$L(S) = \frac{k\,\omega^2}{S(S^2 + 2\xi\omega_n S + \omega_n^2)} \tag{7}$$

The percentage errors of limit cycle amplitude δE_o, and the limit cycle frequency $\delta\omega_o$ are given by :

$$\delta E_o = \frac{E_o\,(Nn.Nm) - E_o\,(Nnm)}{E_o(Nnm)} \tag{8}$$

$$\delta\omega_o = \frac{\omega_o\,(Nn.Nm) - \omega_o(Nnm)}{\omega_o\,(Nnm)} \tag{9}$$

As an illustrative example, a computer simulation is done for the case of dead zone followed by ideal relay in multiple input applications. The flowchart for the methodology of calculation is shown in fig. (4)

6. CONCLUSIONS

The DF technique in nonlinear control systems which contains more than one nonlinearity is analyzed. Different types of nonlinearities are used and their arrangements are considered to evaluate the interaction strength. Nonlinearity parameters are changed to show their effect in DF accuracy.

The following concuslsions are obtained :

A) The following factors affect the errors in both the amplitudes of the DF and limit cycle :

- Nonlinearity type
- Multiple value nonlinearities produce higher error than single value ones
- Nonelinearity arrangement

The last nonlinearity controls the error. If it is multiple value, the error increases, and vice versa.

- Nonlinearity parameter with respect to the input signal.

B) DF Phase and limit cycle frequency are controlled by the following factors :

- If the input signal is known in advance, which is very difficult, especially in feedback loops.
- When the accuracy is not an important factor in the system design.

REFERENCES

1. Alistair I. Mess and Auther R.B.: "Describing Function, Revised", IEEE Trans. on Automatic Control, Vol. AC-20, No. 4, August 1975.

2. C.A. Karybaks: "Synthesis of Nonlinear Characteristics from Complex Describing Function Data." IEEE Trans. on Industrial Electronics and Control Instrumentation, Vol. ICEI-21, No. 4 (November, 1974).

3. E.C. Servetas: "A Nonlinear Electronic Compensator for Automatic Control System." IEEE Trans. on Industrial Electronics and Control Instrumentation, Vol. IECI-22, No. 2 (May, 1975).

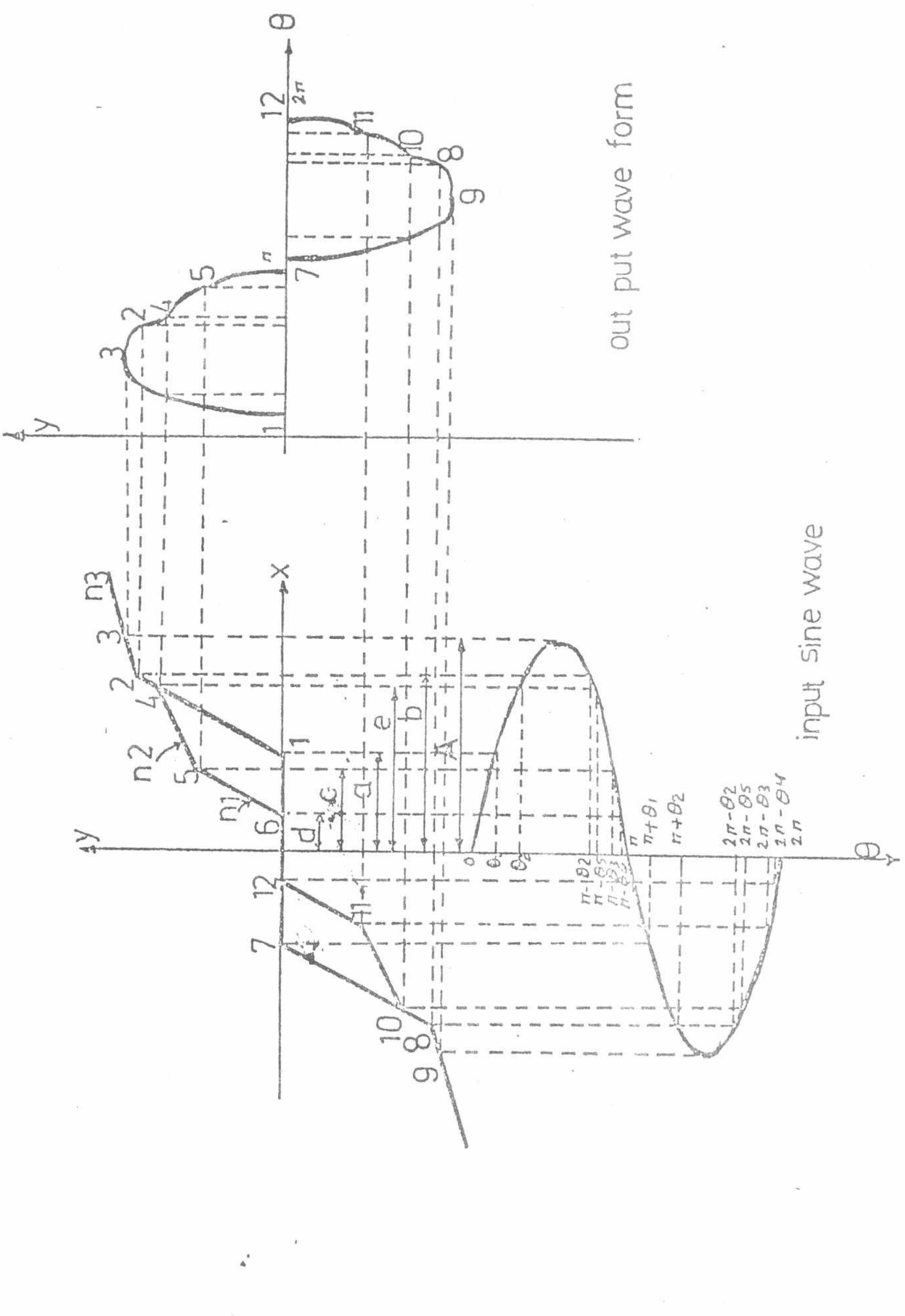


FIG.- 1 General Nonlinearity DF

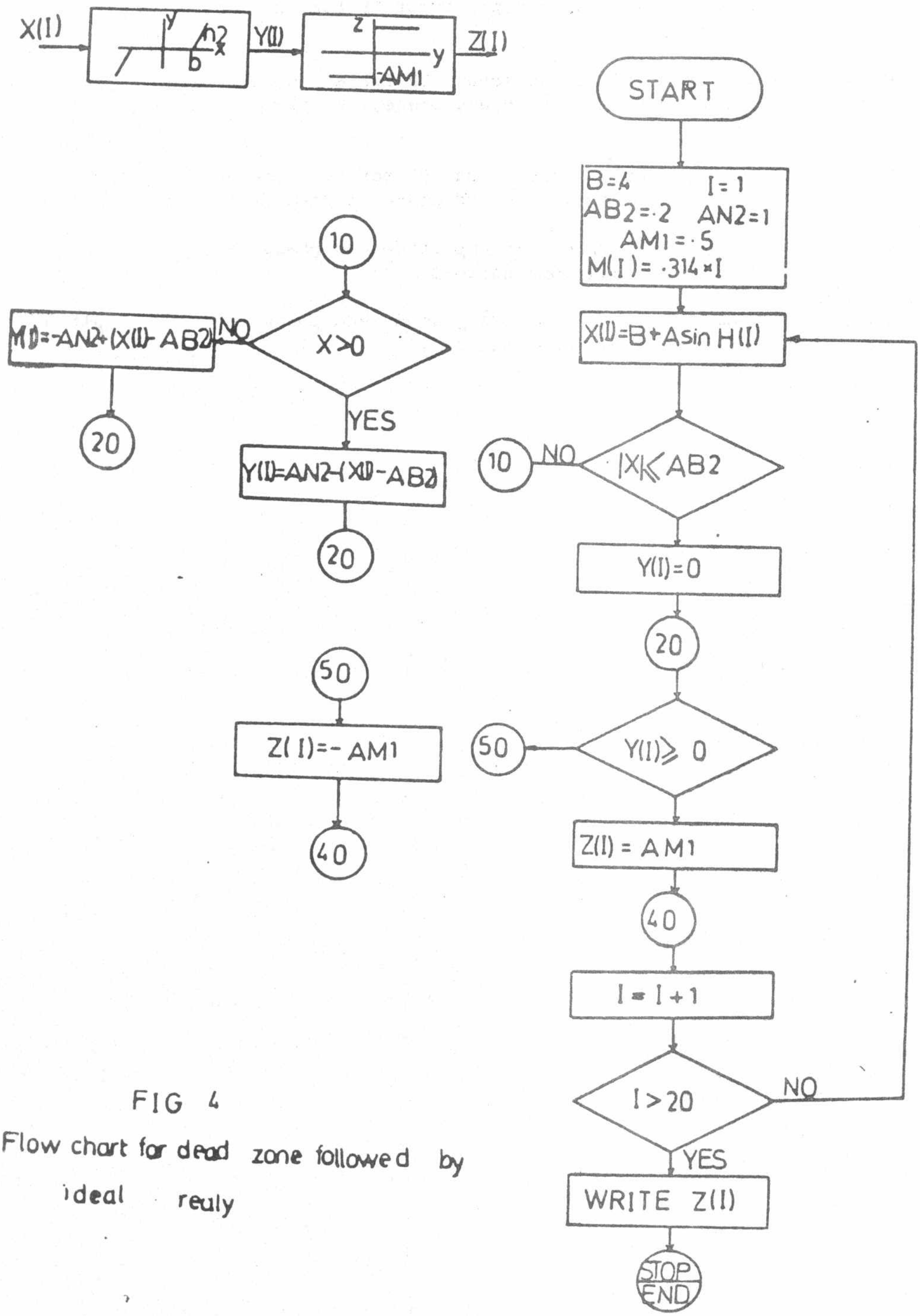


FIG 4

Flow chart for dead zone followed by ideal reuly

LIST OF TABLES

Table (1)

Sinusoidal and multiple input DF for the chosen nonlinearity

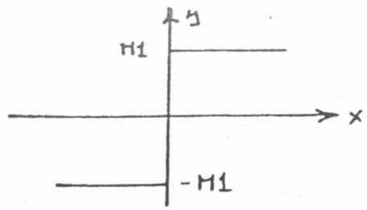
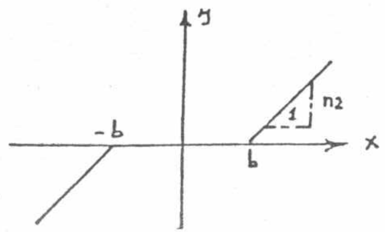
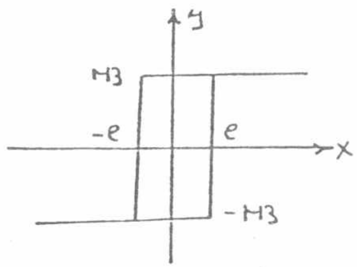
Nonlinearity	Comments	Sinusoidal input DF	Multiple input DF
	$\theta_1 = \sin^{-1} \frac{B}{A}$	$N1 = \frac{4M1}{\pi A}$	$NB = \frac{2M1}{\pi B} \theta 1$ $NA = \frac{4M1}{\pi A} \cos\theta 1$
	$\theta_2 = \sin^{-1} \frac{b-B}{A}$ $\theta_2' = \sin^{-1} \frac{b+B}{A}$	$N2 = \frac{n2}{\pi A}[A(\pi-2\theta 2)$ $+ A\sin 2\theta 2 - 4b\cos\theta 2]$	$NB = n2[1 - \frac{A}{\pi B}(\theta_2' \cdot$ $\sin\theta_2' + \cos\theta_2') -$ $(\theta_2 \sin\theta_2 + \cos\theta_2)]$ $NA = n2[1 - \frac{1}{\pi}(\theta_2' +$ $\sin\theta_2' \cos\theta_2') - (\theta_2 +$ $\sin\theta_2 \cos\theta_2)]$
	$\theta_3 = \sin^{-1} \frac{e-B}{A}$ $\theta_3' = \sin^{-1} \frac{B+e}{A}$ $\theta_3'' = \sin^{-1} \frac{2e}{A}$	$N3 = \frac{4M3}{\pi A}[\cos\theta 3 -$ $j \sin\theta 3]$	$NB = \frac{M4}{\pi B}(\theta_3' - \theta_3)$ $NA = \frac{2MB}{\pi A}[(\cos\theta 3 +$ $\cos\theta_3') - j2\sin\theta_3'']$

Table (2)

Error in DF amplitude, and phase for sinusoidal input

Nonlinearities	Actual Df "N actual"	Multiplicative DF "N multiple"	Error in amplitude "E_{amp}"	Error in DF phase
→ N1 → N2 →	$\frac{4M1n2}{\pi A}$	$\frac{4M1n2}{\pi A}\ [1-\frac{L}{\pi}-\frac{b}{M1}\cos\theta_2]$	$-[\frac{b}{M1}\cos\theta_2+\frac{L}{\pi}]$	ZERO
→ N2 → N1 →	$\frac{4M1\cos\theta_2}{\pi A}$	$\frac{4M1}{\pi A}$	$-1+\frac{1}{\cos\theta_2}$	ZERO
→ N1 → N3 →	$\frac{4M3}{\pi A}$	$\frac{4M3}{\pi A}\ e^{-j\theta 3}$	ZERO	ZERO
→ N3 → N1 →	$\frac{4M1}{\pi A}\ e^{-j\theta 3}$	$\frac{4M1}{\pi A}\ e^{-j\theta 3}$	ZERO	ZERO
→ N2 → N3 →	$\frac{4M3}{\pi A}\ e^{-j\theta'_3}$	$\frac{4M3}{\pi A}\ e^{-j\theta'_3}$	ZERO	$\theta_3-\theta'_3$
→ N3 → N2 →	$\frac{4M3}{\pi A}n2\ e^{-j\theta_3}$	$\frac{4M3n2}{\pi A}\ e^{-j\theta_3}(1-\frac{L}{M3}-\frac{b}{M3}\cos\theta_2$	$\frac{L}{\pi}-\frac{b}{M3}\cos\theta_2$	ZERO

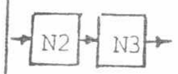
$L = 2\theta_2 - \sin 2\theta_2$

Table (3)

Error in DF amplitude, and phase for multiple input

	Actual DF "N_{act}"	Multiplicative DF N_{mult}	Error in DF amplitude E_{amp}	Error in DF phase E_{phase}
→ N2 → N1 →	$NB = \frac{M1}{\pi B}[\theta_2' - \theta_2]$ $NA = \frac{2M1}{\pi A}[\cos\theta_2 + \cos\theta_2']$	$NB = \frac{2M1}{B}\theta_1$ $NA = \frac{4M1}{\pi A}\cos\theta_1'$	$\varepsilon NB = -1 + \frac{2\theta_1}{\theta_2' - \theta_2}$ $\varepsilon NA = -1 + 2\frac{\cos\theta_1}{\cos\theta_2 + \cos\theta_2'}$	ZERO
→ N1 → N3 →	$NB = \frac{2M3}{\pi B}\theta_1$ $NA = \frac{2M3}{\pi A}\cos\theta_1$	$NB = \frac{M3}{\pi B}(\theta_3 - \theta_3')$ $NA = \frac{2M3}{\pi A}[G^2 + 2\sin\theta_3''^2]^{\frac{1}{2}}$	$\varepsilon NB = -1 + \frac{\theta_3 - \theta_3'}{2\theta_1}$ $\varepsilon NA = -1 + \frac{\sqrt{G^2 + 2\sin\theta_3''^2}}{2\cos\theta_1}$	$\tan^{-1}\frac{\sin\theta_3''}{G}$
→ N3 → N1 →	$NB = \frac{M1}{\pi B}(\theta_3' - \theta_3)$ $NA = \frac{2M1}{\pi A}[G + jG']$	$NB = \frac{2M1}{\pi B}\theta_1$ $NA = \frac{4M1}{\pi A}\cos\theta_1$	$\varepsilon NB = -1 + \frac{2\theta_1}{\theta_3 + \theta_3'}$ $\varepsilon NA = \frac{-1 + 2\cos\theta_1 \div}{\sqrt{G^2 + G'^2}}$	$-\tan^{-1}\frac{G'}{G}$

Table (3) -- (Cont'd)

	Actual DF N_{act}	Multiplicative DF N_{mult}	Error in DF amplitude E_{amp}	Error in DF phase E_{phase}
	$NB = \frac{M3}{\pi B}(\theta_3''' - \theta_3'')$	$NB = \frac{M3}{\pi B}(\theta_3' - \theta_3)$	$\varepsilon NB = -1 + \frac{\theta_3' - \theta_3}{\theta_3''' - \theta_3''}$	$-\tan^{-1} \frac{2\sin\theta_3'''}{G}$
	$NA = \frac{2M3}{\pi A}\sqrt{(G^2 - (G' - \sin\theta_2 + \sin\theta_2)^2}$	$NA = \frac{2M3}{A}\sqrt{G^2 + \sin\theta_3'''^2}$	$\varepsilon NA = -1 + [(G - 4\sin\theta_3'')] \div \sqrt{F(2,4) + (\cos\theta_3'' + \cos\theta_3''')^2}$	$-\tan^{-1} \frac{F(2,4)}{\cos\theta_3'' + \cos\theta_3'}$

$$F(2,4) = (\sin\theta_2 + \sin\theta_2') - (\sin\theta_3' - \sin\theta_3''')$$

$$G = \cos\theta_3 + \cos\theta_3'$$

$$G' = \sin\theta_3 + \sin\theta_3'$$

Table (4) Error in limit cycle amplitude and frequency for sinusoidal input

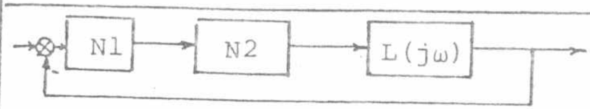
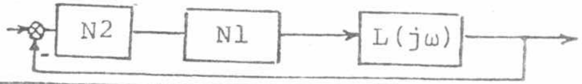
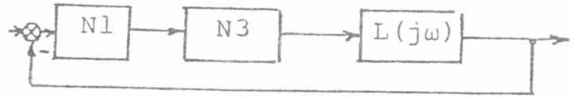
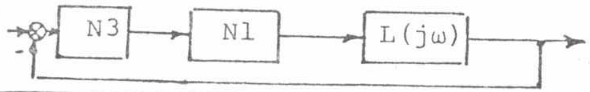
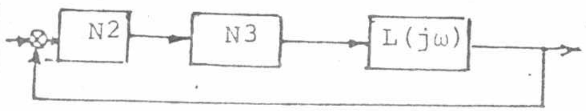
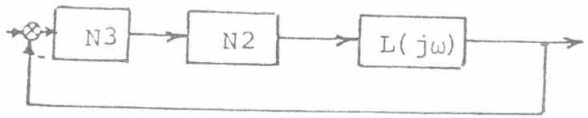
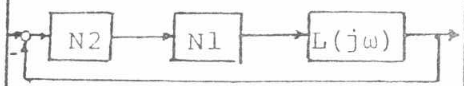
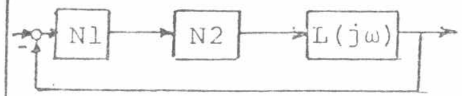
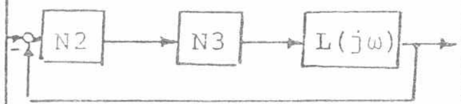
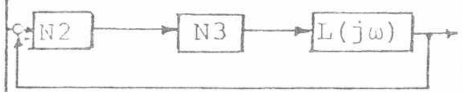
	Error in limit cycle amplitude δE_o	Error in limit cycle frequency $\delta\omega_o$
	$-\frac{1}{\pi}(2\theta_2 - \sin 2\theta_2) + \frac{b}{M1}\cos\theta_2$	ZERO
	$-1 + \frac{1}{\cos\theta_2}$	ZERO
	$-1 + \frac{\cos\theta_3}{[-\xi\tan\theta_3+\sqrt{1+\xi^2\tan^2\theta_3}\ ]^2}$	$-1 +[-\xi\ \tan\theta_3 +\sqrt{1+\xi^2\tan^2\theta_3}]$
	ZERO	ZERO
	$-1 + \frac{\cos\theta_3}{\cos\theta'_3}\ \frac{1}{-\xi\tan\theta_3+\sqrt{\xi^2+\tan^2\theta_3}}$	ZERO $-1+[-\xi\tan\theta'_3+\sqrt{\xi^2\tan^2\theta'_3+1}\ ]$
	$-[\frac{1}{\pi}(2\theta_2 - \sin 2\theta_2) + \frac{b}{M3}\cos\theta_2)\ ]$	ZERO

Table (5)

Error in limit cycle amplitude and frequency for multiple input

	Error limit cycle amplitude δE_o	Error in limit cycle frequency $\delta\omega_o$
	$-1 + 2\,\frac{\cos\theta_1}{\cos\theta_2 + \cos\theta_2'}$	ZERO
	$-1 + \frac{G}{\cos\theta_1\left[-\xi\sin\theta_3'' + \sqrt{1+\left(\frac{\xi\sin\theta_3'''}{G}\right)^2}\right]}$ $\cos\theta_3 + \cos\theta_3' = G$	$-1 + \frac{-\xi\sin\theta_3'''}{G}\sqrt{\left(\frac{\xi\sin\theta_3'''}{G}\right)^2 + 1}$
	$-1 + \frac{\cos\theta_1}{G}\,\frac{1}{[1+\delta\,\omega_o]^2}$ $L = \sin\theta_3'' + \sin\theta_3'''$	$-1 + \frac{1}{-\xi\,\frac{G'}{G} + \sqrt{\left(\xi\,\frac{G'}{G}\right)^2 + 1}}$ $\sin\theta_3 + \sin\theta_3' = G'$
	$-1 + \frac{G}{K}\,\frac{1}{[1+\delta\omega_o]^2}$ $K = \cos\theta_3'' + \cos\theta_3''$	$-1 + \frac{-\xi\sin\theta_3'''}{-\frac{2\xi}{L}(K'-L) + \sqrt{\left[\frac{2\xi}{L}(K'-L)\right]^2 + 1}}$ $K' = \sin\theta_2 + \sin\theta_2'$