MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

MODA CONTROL SYSTEM INCORPORATING DERIVATIVE FEEDBACK

M.S. MADY*

ABSTRACT

In the present paper we investigate the problem of modal-control using proportional-plus-derivative output-feedback controllers (PD-type). With the derivative part of the feedback, we can get additional information about the system state vector, which are necessary for the solution of the pole-assignment problem.
This group of PD-controllers has the advantage of providing additional parameters without increasing the order of the system under control. The additional derivative term of the system output-compared with purely proportional controllers-helps to reduce the overshoots in the system outputs during the transient period, thus it has no effect on the steady-state characteristics of the system, and it also improves the system stability. A sufficient condition for complete pole-assignment of closed-loop system is derived in the form of lemma. A simple and computationally easy design procedure of PD-controllers is developed, and it is applied to the synthesis of stabilizing systems of a helicopter.

1. INTRODUCTION

The problem of modal control; commonly known as pole- assignment problem; in linear-multivariable systems using proportional output feedback was investigated and received considerable attention in many research works. It is known that there are cases where no solution of this problem exists. This fact had motivated the trial to propose the use of proportional-plus-derivative output feedback controllers (PD-type). The derivative part of the feedback gives additional information about the state vector of the system, which is needed for the solution of pole-assignment problem. This class of controllers (PD-type) has the main advantage that it does not change the order of the system under control. Bengetson and Lindhal [1] proposed the use of PD-controllers for the stabilization of systems in the case of incomplete state-feedback. Seraji and Tarokh [9] developed a technique for design of PD-controllers in the frequency domain, having the dyadic structure (of unity rank).
In the present paper we introduce a method for designing PD-controllers in the time domain, where no restrictions are imposed on their structure.

* Dept. of Aircraft Electrical Equipment and Armament , Military Technical College, Kobry El-Koba, Cairo, Egypt.

2. THEORITICAL ANALYSIS

Let us consider the linear time-invariant multivariable system, which is controllable and observable, and governed by state and output equations of the form

$$\dot{x}(t) = A\, x(t) + B\, u(t) \tag{1}$$

$$y(t) = C\, x(t) \tag{2}$$

where x is the n-dimensional state vector, u is the m-dimensional control vector, and y is the ℓ-dimensional output vector.and where A is an n x n constant matrix, and B,C are constant matrices of full rank and appropriate dimensions. The output feedback law is assumed to have the form

$$u(t) = -K_1\, y(t) - K_2\dot{y}(t) + r(t) \tag{3}$$

where K_1 and K_2 are the proportional and derivative output feedback constant matrices (each of dimension m x ℓ), and r(t) is the command vector of dimension m. The state equation can now be written as

$$\dot{x}(t) = A\, x(t) + B\left[-K_1C\, x(t) - K_2C\, \dot{x}(t) + r(t)\right] \tag{4}$$

or

$$\dot{x}(t) = (I_n + BK_2C)^{-1}(A - BK_1C)\, x(t) + (I_n + BK_2C)^{-1}B\, r(t) \tag{5}$$

assuming that (I_n+BK_2C) is invertible.

The closed-loop system plant matrix has the form

$$A_c = (I_n+BK_2C)^{-1}\ (A-BK_1C) \tag{6}$$

and it is of the same order n, as that of the open-loop one, which means that the introduction of the PD-controller does not affect the system order. It is well known that (Wonham[11]) the eigenvalue spectrum of the plant matrix (A-BP) of the closed-loop system governed by eqn.(1) and the state-feedback control-law of the form.

$$u(t) = -P\, x(t) + r(t) \tag{7}$$

can be assigned arbitrarily by appropriate choice of the constant state-feedback matrix P (of dimension m x n). Hence, a method of assigning the closed-loop poles using PD-controller is to solve the equation

$$A_c = A - BP = (I_n + BK_2C)^{-1}\ (A-BK_1C) \tag{8}$$

from which we can find that

$$B\left[K_1C + K_2C\ (A-BP) - P\right] = 0_{nxn} \tag{9}$$

As the matrix B is assumed to be of full rank, this implies that

$$K_1C + K_2C\ (A-BP) = P \tag{10}$$

and thus we can verify eqn.(9).We notice that eqn. (10) can be put in the form

$$\begin{bmatrix} K_1 & \vdots & K_2 \end{bmatrix} \begin{bmatrix} C \\ \hline C(A-BP\ ) \end{bmatrix} = P \tag{11}$$

Introducing the followingn notations

$$\hat{K} = \begin{bmatrix} K_1 & \vdots & K_2 \end{bmatrix} \tag{12}$$

and

$$\hat{C} = \begin{bmatrix} C \\ \hline C(A-BP) \end{bmatrix} \tag{13}$$

where the matrix $\hat{K}$ (of dimension $m \times 2\ell$) represents an equivalent output-feedback controller, and the matrix $\hat{C}$ (of dimension $2\ell \times n$) represents an equivalent output-matrix. We can assume that the matrix $\hat{C}$ is generally of full rank, i.e.

$$\text{rank } \hat{C} = \min\ (\ 2\ell, n) \tag{14}$$

Considering eqns.(7),(11),(12) and (13), the control-law can now be written as

$$u(t) = -\ \hat{K}\ \hat{C}\ x(t) + r(t) \tag{15}$$

From eqns.(13) and (15) we can notice the advantage of the application of PD-controllers, since greater number of state variables are accessible for feedback. Thus we can have greater possibilities of controlling the position of closed-loop system poles using this class of controllers compared with the case of proportional-output feedback controllers. Moreover , we get the following result:

PROPOSITION

If $n \le 2\ell$, for nearly all the pairs (C, A-BP), the matrix $\hat{C}$-eqn.(13)- has the rank n.

We can conclude the following lemma

LEMMA

When the condition $n \le 2\ell$ is satisfied, it is possible-in almost all cases-to have complete pole assignment of closed loop system using the compensation by feedback of the system output vector and its derivative.

Now, eqn.(11) can be put in the form

$$\hat{K}\ \hat{C} = P \tag{16}$$

Two cases may arise

- Case I:the state feedback matrix P is imposed. In this case, the matrix equation (16) can be solved to get $\hat{K}$ (see [3],[6-8] and [10]). When $n > 2\ell$, eqn. (16) can be solved or not according to the satisfaction or not of its consistency conditions;
- case II:the matrix P is not given. In the literatures we can find many algorithms to construct such a matrix P satisfying the modal-control problem requirements (e.g [2],[3],[5]and [12]). Having P, eqn. (16) can be solved as explained in the previous case.

3. PRACTICAL APPLICATION

In this paragraph, appropriate control laws for the stabilization of a helicopter in hovering flight are synthesized using the modal control design procedure developed in this paper.

3.1. MATHEMATICAL MODEL OF UNCONTROLLED SYSTEM

The state equation of the open-loop system has the form [4]

$$\begin{bmatrix} \dot{x}_I(t) \\ \dot{x}_{II}(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_I(t) \\ x_{II}(t) \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} u_I(t) \\ u_{II}(t) \end{bmatrix} \qquad (17)$$

where $x_I(t)$ is the longitudinal state vector, given by

$$x_I(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} u(t)/V_R \\ \omega(t)/V_R \\ q(t) \\ \theta(t) \end{bmatrix} \qquad (18)$$

and $x_{II}(t)$ is the lateral state vector, given by

$$x_{II}(t) = \begin{bmatrix} x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \\ x_9(t) \end{bmatrix} = \begin{bmatrix} v(t)/V_R \\ p(t) \\ r(t) \\ \phi(t) \\ \psi(t) \end{bmatrix} \qquad (19)$$

Also, the longitudinal input vector, $u_I(t)$, is defined as

$$u_I(t) = \begin{bmatrix} U_p(t) \\ U_c(t) \end{bmatrix} \qquad (20)$$

and the lateral input vector, $u_{II}(t)$, is defined as

where

$$u_{II}(t) = \begin{bmatrix} U_R(t) \\ U_T(t) \end{bmatrix} \qquad (21)$$

where:

$u(t)$ = longitudinal velocity;
$\omega(t)$ = verticalv velocity;
$q(t)$ = pitch rate;
$\theta(t)$ = pitch angle;
V_R = rotor-tip velocity;
$v(t)$ = lateral velocity;
$p(t)$ = roll rate;
$r(t)$ = yaw rate;
$\phi(t)$ = roll angle ;
$\psi(t)$ = yaw angle;
$U_p(t)$ = longitudinal cyclic pitch-control angle;
$U_c(t)$ = main-rotor collective pitch-control angle;
$U_R(t)$ = lateral cyclic pitch-control angle;
$U_T(t)$ = tail-rotor collective pitch-control angle.

It is assumed that we can neglect the coupling effect between the longitudinal and lateral motions. Hence, the coupling matrices A_{12}, A_{21}, B_{12}, and B_{21} in equation (17) are all null. The matrices A_{11}, A_{22}, B_{11}, and B_{22} (as given in ref. [4]) are

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$$A_{11} = \begin{bmatrix} -0.016 & 0 & 0.0025 & -0.05 \\ 0 & -0.3242 & 0.0002 & 0 \\ 1.97 & 1 & -0.542 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{22.a}$$

$$A_{11} = \begin{bmatrix} -0.033 & -0.0025 & 0.0009 & 0.05 & 0 \\ -7.25 & -1.96 & 0.01 & 0 & 0 \\ 5.59 & -0.0043 & -0.303 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{22.b}$$

$$B_{11} = \begin{bmatrix} 0.005 & 0.05 \\ -0.424 & 0 \\ 0.69 & -6.15 \\ 0 & 0 \end{bmatrix} \tag{22.c}$$

$$B_{22} = \begin{bmatrix} 0.05 & 0.022 \\ 21.81 & 0.3475 \\ 0.174 & -7.48 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{22.d}$$

The eigenvalues of the matrix A_{11} are

$$\begin{aligned} \lambda_{1,2} &= 0.0887 \pm 0.3552\, i \\ \lambda_3 &= -0.7354 \\ \lambda_4 &= -0.324 \end{aligned} \tag{23}$$

and the eigenvalues of the matrix A_{22} are

$$\begin{aligned} \lambda_{5,6} &= 0.0315 \pm 0.4137\, i \\ \lambda_7 &= -2.0565 \\ \lambda_8 &= -0.3024 \\ \lambda_9 &= 0 \end{aligned} \tag{24}$$

LONGITUDINAL MODE CONTROL

If it is required to find a modal controller such that the eigenvalues of A_{11} will be transfered to the positions

$$\begin{aligned} \rho_{1,2} &= -0.8 \pm 1.2\, i \\ \rho_3 &= -1.0 \\ \rho_4 &= -0.6 \end{aligned} \tag{25}$$

Porter and Crossley [4] found the matrix P_I of state-feedback; eqn.(7); as

$$P_I = \begin{bmatrix} 0.61 \times 10^{-3} & -0.651 & -0.99 \times 10^{-4} & 0.94 \times 10^{-4} \\ 6.441 & 1.592 & -0.27 & -0.586 \end{bmatrix} \quad (26)$$

which solves this pole-assignment problem, under the assumption that all the elements of $x_I(t)$ are accessible.
If the state variables $x_2(t)$ and $x_3(t)$ are not available for feedback, i.e.

$$y_I(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_I(t) \quad (27)$$

then, we search for another type of controllers. Introducing the control-law (3), and following the procedure deseribed in section (2) , we get the next results

$$(K_1)_I = \begin{bmatrix} 2.8832 & 0.1766 \\ -0.6034 & -1.0177 \end{bmatrix} \quad ; \quad (K_2)_I = \begin{bmatrix} 8.5271 & -0.14037 \\ -20.853 & 0.06404 \end{bmatrix} \quad (28)$$

and the closed-loop poles are found to be as specified.

3.3. LATERAL MODE CONTROL

If it is required to find a modal controller such that the eigenvalues of A_{22} will be shifted to the new locations

$$\begin{aligned} \rho_{5,6} &= 0.5 \pm 1.0 \quad i \\ \rho_7 &= \lambda_7 = -2.0565 \\ \rho_8 &= -1.0 \\ \rho_9 &= -0.6 \end{aligned} \quad (29)$$

the matrix P_{II}, found as [4]

$$P_{II} = \begin{bmatrix} 1.987 & 0.0441 & 0.0022 & 0.145 & 0 \\ -1.184 & -0.0111 & -0.177 & -0.0439 & -0.0819 \end{bmatrix} \quad (30)$$

will lead to the required pole-assignment.
If the state variables $x_5(t)$, $x_7(t)$, and $x_9(t)$ are only accessible for feed-back , i.e.

$$y_{II}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_{II}(t) \quad (31)$$

then, applying the above mentioned procedure, we get

$$(K_1)_{II} = \begin{bmatrix} 0.33036 & -0.36374 & -0.27468 \\ -0.75925 & 0.0083 & -0.00901 \end{bmatrix} \quad (32.a)$$

$$(K_2)_{II} = \begin{bmatrix} -0.31737 & -0.44931 & -0.36374 \\ -0.04286 & 0.11885 & 0.00831 \end{bmatrix} \qquad (32.b)$$

and we find that the closed-loop system poles are exactly assigned.

4. CONCLUSION

We have shown that the introduction of a PD-controller in the control loop may help to find a solution for the modal control problem, especially in the case that the proportional-controller fails. In addition, the PD-controller does not change the order of the system under control, which can be considered as a great advantage in comparison with dynamic controller for example. In the case of position control systems and servomechanisms, the derivatives of the output variables are physical quantities and can be measured directly, thus we can avoid the differentiation of the output signals.

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