



DESIGN OF DECENTRALIZED ROBUSTLY STABLE OUTPUT FEEDBACK

CONTROLLERS FOR POWER SYSTEM NETWORKS

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ABSTRACT

In designing controllers for large-scale systems such as spread power systems, three issues need be considered. Firstly, because there may be lack of global information shared by all substations, large-scale power systems are modelled as interconnections of low order subsystems. Hence, decentralized control becomes a feasible approach. Secondly, because system models are usually approximations to the actual process and because of the uncertainties associated with environmental conditions, robust controllers are desired. Finally, the control strategies must be based solely upon output information, i.e., in many cases, the entire static information is not known. Hence output feedback structures are required in the control strategy.

This paper presents a method for designing decentralized robustly stable output feedback controllers for power system. The approach is based upon a state-space formulation of each subsystem and the associated interconnections. An example of a four-generator, four-load system is presented to illustrate the approach.

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I. INTRODUCTION

A variety of procedures for designing feedback control systems and analyzing such strategies have been developed. For large-scale systems such as spread power systems, one can not always assume centrality. Therefore, large-scale power systems are modelled as interconnections of low-order subsystems. The decomposition approach is a natural modelling scheme for large-scale power systems since one can usually identify the subsystems.

When using a control theory to stabilize large-scale power systems, decentralization is one of the important tools that can be used [1,2]. Stabilizing the large-scale system may not be enough unless it is robust, i.e., the system retains its stability in the face of certain uncertainties. These uncertainties may come from many sources. For example, in developing a model for the power network, one may assume a certain range of frequencies or bounded inputs under which the model is constructed. If the system goes out of this range, the behavior of the actual system may be drastically different from that predicted by the model.

The robustness issue is not new in control system design. In single input-output systems, robustness can be specified in terms of gain and phase margins. For multivariable systems, similar measures for robustness are not an easy task and their interpretation must be handled carefully [5].

Many existing decentralized control schemes for large-scale interconnected power systems are designed using static controllers with local state feedback [3,4]. Usually, the entire states of the subsystem are not accessible for control; thus, the control objective has to be achieved by using local outputs. Therefore, it is reasonable to use dynamic output feedback in designing the decentralized controllers for large-scale power systems. The feedback variables which are used in designing the local decentralized controllers are measurable and need not be transmitted from distance away from the subsystem being controlled.

In this paper, then, a decentralized robustly stable output feedback control approach is developed. The system model is briefly discussed in Section II. Once the model is formulated, the decentralized robustly stable controller can be designed. The robustness measure is based upon a singular value decomposition test on a function of the system matrices. This is presented in Section III.

An example of a four-generator, four-load power system is presented in Section IV to illustrate typical results when applying this methodology.

II. THE DECENTRALIZED POWER SYSTEM MODEL

An interconnected power system can be described by the linearized model equations [11]:

$$\begin{aligned} \dot{x} &= Ax + Bu + Fd \\ y &= Cx + Du \end{aligned} \tag{1}$$

where  $x(t) \in R^n$ ,  $u(t) \in R^m$ ,  $d(t) \in R^l$ , and  $y(t) \in R^r$  are state, input, disturbance, and output vectors respectively; A, B, C, F and D are constant

matrices of appropriate dimensions, and  $\underline{x}(t_0) = \underline{x}(0)$  is the initial state at  $t_0$ . The model is composed of  $q$  area models and also associated tie-lines models that can be given in the following decomposed form:

$$\begin{aligned} \dot{\underline{x}}_i &= A_{i-i} \underline{x}_i + a_{-ti} \Delta P_{ei} + B_{i-i} \underline{u}_i + F_{i-i} \underline{d}_i \\ \Delta P_{ei} &= \alpha_{li} \sum_{\substack{j=1 \\ j \neq i}}^q (m_{-ij}^T \underline{x}_i - m_{-ji}^T \underline{x}_j), \quad i = 1, 2, \dots, q \end{aligned} \quad (2)$$

where  $\underline{x}_i(t) \in \mathbb{R}^n$ ,  $\underline{u}_i(t) \in \mathbb{R}^m$ ,  $\underline{d}_i(t) \in \mathbb{R}^r$ ,  $\underline{y}_i(t) \in \mathbb{R}^r$  and  $\Delta P_{ei}$  are the state, the input, the disturbance, the output and the variation respectively, of the total power exchange corresponding to the  $i^{\text{th}}$  area.

$$\Delta P_{ei} = \alpha_{li} \sum_{\substack{j=1 \\ j \neq i}}^q \Delta P_{ij}, \quad i = 1, 2, \dots, q \quad (3)$$

$\Delta P_{ij}$  is the power exchange between area  $i$  and area  $j$ ,  $a_{ti}$  and  $m_{ij}$  are constant coupling vectors and  $\alpha_{li} = (P_{i0}/P_{i0})$  is the normalization factor. Assume that  $\underline{x}_i(t_0) = 0$  and  $\Delta P_{ei}(t_0) = 0$ . The properties of the above model implies the following :

$$n = \sum_{i=1}^q n_i + q - 1, \quad m = \sum_{i=1}^q m_i, \quad r = \sum_{i=1}^q r_i \quad (4)$$

and

$$\sum_{i=1}^q \frac{\Delta P_{ei}}{\alpha_{ij}} = 0 \quad (5)$$

where

$$\alpha_{ij} = \frac{P_{i0}}{P_{j0}} \text{ is the ratio coefficient.}$$

Equation (2) represents the decentralized power system model used here. It is desired to design a robustly stable output feedback controller  $\underline{u}_i(t)$  for each subsystem  $i$ .

One can further describe (2) to include dynamic behavior of the damping winding, the exciter or governor actions or to define the disturbance  $\underline{d}(t)$  more explicitly in terms of input and output interactions. That is, let the model (1) be composed of  $q$  area models ( $q$  is arbitrary) and associated tie-line models which can be given in the following decomposed form :

$$\begin{aligned} \dot{\underline{x}}_i &= A_{i-i} \underline{x}_i + B_{i-i} \underline{u}_i + a_{-ti} \Delta P_{ei} + P_{i-i} \underline{v}_i \\ \underline{y}_i &= C_{i-i} \underline{x}_i + D_{i-i} \underline{u}_i \\ \underline{w}_i &= Q_{i-i} \underline{x}_i, \quad i = 1, 2, \dots, q \end{aligned} \quad (6)$$

where  $P_i$  and  $Q_i$  are constant matrices of appropriate dimensions.

Here  $\underline{v}_i(t)$  and  $\underline{w}_i(t)$  are the interaction inputs and outputs associated with the  $i^{\text{th}}$  subsystem, respectively, which represent the effect of other subsystems on the  $i^{\text{th}}$  subsystem and the effect of the  $i^{\text{th}}$  subsystem and the effect of the  $i^{\text{th}}$  subsystem on the other subsystems. Note  $\underline{v}_i(t)$  and  $\underline{w}_i(t)$  are related to each other by :

$$\underline{v}_i(t) = \underline{f}_i(\underline{w}) \quad i = 1, 2, \dots, q \quad (7)$$

where  $\underline{f}_i(\underline{w})$  is a nonlinear continuous function vector in  $\underline{w}$  satisfying the following condition :

$$\|\underline{f}_i(\underline{w})\| \leq a_i \|\underline{w}\|, \quad i = 1, 2, \dots, q \quad (8)$$

where  $a_i$  is a positive number.

### III. A ROBUSTLY DECENTRALIZED CONTROLLER FOR THE LARGE-SCALE POWER SYSTEM MODEL USING OUTPUT FEEDBACK

In this section, it is desired to find a control strategy to stabilize each decoupled subsystem in (6) using local dynamic output feedback taking the interconnections into consideration. Making the gains of the loops of the interconnected system sufficiently small is useful in retaining the stability in the presence of the interconnections. Now, consider the decoupled form of (6) as :

$$\begin{aligned} \dot{\underline{x}}_i &= A_{i-i} \underline{x}_{i-i} + B_{i-i} u_{i-i} + \frac{a_{-ti} \alpha_{li}}{j=1} \sum_{j=1}^q \Delta P_{ij} + P_{i-i} v_{i-i} \\ \underline{y}_i &= C_{i-i} \underline{x}_{i-i} \quad (D_i \text{ assumed zero here}) \\ \underline{w}_i &= Q_{i-i} \underline{x}_{i-i}, \quad i = 1, 2, \dots, q \end{aligned} \quad (9)$$

Apply the following decentralized control to the interconnected system under investigation:

$$\dot{\underline{z}}_i = G_{i-i} \underline{z}_{i-i} + K_{i-i} \underline{y}_i; \quad \underline{u}_i = -R_{i-i} \underline{z}_{i-i} - E_{i-i} \underline{y}_i, \quad i = 1, 2, \dots, q \quad (10)$$

where  $\underline{z}_i \in \mathbb{R}^{l_i}$  is the state of the decentralized controller and  $G_i, K_i, R_i$  and  $E_i$  are constants of appropriate dimensions. The compensated system becomes:

$$\begin{aligned} \dot{\underline{x}}_i &= A_{i-i} \underline{x}_{i-i} + B_{i-i} (-R_{i-i} \underline{z}_{i-i} - E_{i-i} \underline{y}_i) + \frac{a_{-ti} \alpha_{li}}{j=1} \sum_{j=1}^q \Delta P_{ij} + P_{i-i} v_{i-i} \\ \underline{y}_i &= C_{i-i} \underline{x}_{i-i}; \quad \underline{w}_i = Q_{i-i} \underline{x}_{i-i}, \quad i=1, 2, \dots, q \end{aligned} \quad (11)$$

Simplifying (11) gives :

$$\dot{\hat{\underline{x}}}_i = \hat{A}_{i-i} \hat{\underline{x}}_{i-i} + \hat{\alpha}_i \sum_{j=1}^q \Delta P_{ij} + \hat{P}_{i-i} v_{i-i}; \quad \underline{w}_i = \hat{Q}_i \hat{\underline{x}}_{i-i}, \quad i=1, 2, \dots, q \quad (12)$$

where :  $\hat{\underline{x}}_i = (\underline{x}_{i-i}^T, \underline{z}_{i-i}^T)^T$ ;  $\hat{\alpha}_i = \frac{a_{-ti} \alpha_{li}}{j=1}$

$$\hat{A}_i = \begin{bmatrix} A_{i-i} - B_{i-i} E_{i-i} C_{i-i} & -B_{i-i} R_{i-i} \\ K_{i-i} C_{i-i} & C_{i-i} \end{bmatrix}; \quad \hat{P}_i = \begin{bmatrix} P_{i-i} \\ 0 \end{bmatrix}; \quad \hat{Q}_i = (Q_{i-i} \quad 0), \quad i=1, 2, \dots, q \quad (13)$$

To increase the practicability value of the designing technique, one can assume that the control structure is restricted in such a way that each subsystem is controlled by its own outputs only. Considering this assumption,

i.e. omitting power exchange, the term  $\frac{a_{-ti} \alpha_{li}}{j=1} \sum_{j=1}^q \Delta P_{ij}$  can be deleted

from (12) and the subsystem model can be rewritten as :

$$\dot{\hat{x}}_i = \hat{A}_i \hat{x}_i + \hat{P}_i v_i ; \quad \hat{w}_i = \hat{Q}_i \hat{x}_i , \quad i = 1, 2, \dots, q \quad (14)$$

Note that each open-loop subsystem, can be written as :

$$\dot{x}_i = A_i x_i + B_i u_i + P_i v_i ; \quad y_i = C_i x_i ; \quad w_i = Q_i x_i \quad (15)$$

From (15) then, one can find the frequency response transfer function, between the output  $y_i$  and the input  $u_i$  in a straight forward manner as:

$$G_1(s) = C_i (sI - A_i)^{-1} B_i \quad (16)$$

and between the output  $y_i$  and the interaction input  $v_i$  as :

$$G_2(s) = C_i^T (sI - A_i)^{-1} P_i \quad (17)$$

Correspondingly, the transfer function between the output  $w_i$  and the input  $u_i$  is :

$$G_3(s) = Q_i^T (sI - A_i)^{-1} B_i \quad (18)$$

and between the output  $w_i$  and the disturbance input  $v_i$  as :

$$G_4(s) = Q_i (sI - A_i)^{-1} P_i \quad (19)$$

From (14), i.e., after applying output feedback control the transfer function between input  $v_i$  and output  $w_i$  can be written as :

$$G_5(s) = \hat{Q}_i (sI - \hat{A}_i)^{-1} \hat{P}_i \quad (20)$$

For the decentralized controller in (10), the transfer function between output  $u_i$  and input  $y_i$  can be written as :

$$G_6(s) = E_i + R_i (sI - F_i)^{-1} K_i \quad (21)$$

Assuming that  $P_i = B_i$  in equations (17), (19) and (20) gives :

$$G_5(s) = G_3(s) [I + G_1(s) G_6(s)]^{-1} \quad (22)$$

Similarly, assuming that  $Q_i = C_i$  in equation (18), (19) and (20) gives:

$$G_5(s) = G_2(s) [I + G_1(s) G_6(s)]^{-1} \quad (23)$$

Note that equations (22) and (23) are similar with the assumption that  $P_i = B_i$  and  $Q_i = C_i$ ; however  $Q_i$  and  $P_i$  have, generally the following relations:

$$P_i = B_i \bar{P}_i^T \text{ for } i \in r ; \quad Q_i = \bar{Q}_i C_i \text{ for } i \in q-r ; \quad i=1, 2, \dots, q \quad (24)$$

where  $0 \leq r \leq q$  and  $\bar{P}_i$  and  $\bar{Q}_i$  are matrices of appropriate dimensions.

Let the transfer function  $G_1(s)$  be written as :  $G_1(s) = \alpha_o [N_1(s)/D_1(s)]$ .  $G_3(s)$  and  $G_6(s)$  can also be written as :

$$G_3(s) = G_1(s) = \alpha_o [N_1(s)/D_1(s)] ; \quad (25)$$

$$G_6(s) = \beta_o [N_6(s)/D_6(s)] \quad (26)$$

From Equations (24), (25) and (26), one can rewrite (23) as :

$$G_5(s) = [\alpha_o N_1(s) D_6(s)] / [D_1(s) D_6(s) + \alpha_o \beta_o N_6(s)] \quad (27)$$

Note that  $G_5(s)$  is the transfer function of the subsystem after applying the output robust decentralized controller.

Hence, the parameters of the controller can be chosen as follows :

*Algorithm :*

- (i) The coefficients of the desired polynomial  $\bar{N}_6(s)$ , for each of the  $q$  subsystems, are chosen such that the zeros of  $\bar{N}_6(s)$  are in the closed left half plane.
- (ii) The coefficients of the desired polynomial  $\bar{D}_6(s)$ , for each of the  $q$  subsystems, are chosen as :

$$\bar{D}_6(s) = s^q + \bar{d}_1 \gamma^1 s^{q-1} + \bar{d}_2 \gamma^2 s^{q-2} + \dots + \bar{d}_q \gamma^q \quad (28)$$

The actual polynomial  $D_6(s)$  can be extracted from (26) in the form of :

$$D_6(s) = s^q + d_1 s^{q-1} + d_2 s^{q-2} + \dots + d_q \quad (29)$$

and  $\alpha_o \beta_o$  is chosen such that  $\alpha_o \beta_o = \bar{d}_{q+1} \gamma^{q+1}$  where  $\gamma > 0$  is a parameter to be specified, and  $d_i, i = 1, \dots, q$  are chosen in such a way that all of zeros  $D_6(s)$  are in the closed left half plane.

Assuming the selection of (i) and (ii), one can form the following lemma.

#### Lemma 1

For any  $\sigma > 0$  there exists  $\bar{\gamma} > 0$  such that the following conditions are satisfied, whenever  $\gamma > \bar{\gamma}$  , :

- a)  $|G_6(s)| < \sigma$  for all  $s$  in the closed right half plane.
- b)  $A_i$  has all of its eigenvalues in the left half plane. The proof is given in [6].

A small gain version of the circle criterion in [7] can be developed and is to be used as a fundamental criterion to achieve robust stability [8]. This is provided in Lemma 2.

Consider the compensated interconnected system made up of  $q$  subsystems [9].

$$\dot{\underline{x}} = \hat{\underline{A}} \underline{x} + \hat{\underline{P}} \underline{f}(\underline{w}) ; \quad \underline{w} = \hat{\underline{Q}} \underline{x} \quad (30)$$

#### Lemma 2

Assume that the matrix  $A_i$  of (14) has all of its eigenvalues in the closed left half plane. Then the system of (30) is robustly stable if :

$$\sup_{\omega} \left| \bar{\sigma}_K(G_6(s)) \right| < \frac{1}{\alpha} \quad (31)$$

where  $\bar{\sigma}_K(\cdot)$  is the maximum singular value of  $(\cdot)$ . Using Lemma 1 and

Lemma 2, the following theorem can be stated.

#### Theorem

For each of the subsystems of (14) there exist a set of controllers such that the overall interconnected system is robustly stable if all of the following conditions hold

- i) All of the eigenvalues of the matrix  $A_i$  are in the left half plane.
- ii) The triple  $(A_i, B_i, C_i)$  is controllable and observable.
- iii) The pair  $(P_i, Q_i)$  satisfies (24).

The proof is given in [9].

Hence, using the decentralized controller algorithm guarantees overall stabilization from this Theorem. The selection of  $N_6(s)$  and  $D_6(s)$  to

achieve robustness can be accomplished by applying performance measures, such as graphical or singular value decomposition methods as in [6].

#### IV. Example

A four-machine four-load configuration used in this example is shown in Figure 1. The four machines are assumed to be thermal machines in steady state. The parameters of the machines are given in Table 1; the transmission lines and load flow results are provided in the plot. To check dynamic stability, one of the methodologies is to calculate the system eigenvalues; if they have negative real parts, then the system is dynamically stable.

The operating point terminal voltage  $V_t^0$ , direct and quadrature component voltages  $V_d^0$  and  $V_q^0$  along with direct and quadrature current components for the four machines are listed in Table 2.

The eigenvalues for the system shown in Figure 1 are also listed in Table 3 for the four machines. Notice that the first set of eigenvalues are associated with rotor oscillation and the second set of the modes that damp rapidly are associated with armature circuit; the last set of modes are associated with the governors and they are damping slowly.

In designing the robust controllers for this example, it is assumed that the control structure is restricted in such a way that each machine is controlled by its output only i.e. equation (14) is applicable.

It is clear from the system eigenvalues that the overall system is stable. Now, apply the decentralized controllers of (10) sequentially (one at a time).

Then, the resultant feedback system with state coefficient matrices of  $A_i$  of (13) is robustly stable; one also can see this by observing the system eigenvalues after applying the decentralized controllers (Table 3).

For example, the coefficient matrix of the third machine is obtained after applying the decentralized controller and is :

$$\tilde{A}_3 = \begin{bmatrix} -0.00362 & 0.0429 & 1.58 & -3.52 & -2.251 & 0 & 0 & -0.0052 \\ -0.127 & -0.0798 & 1.342 & 0.982 & 0.871 & 0 & 0 & 0.0081 \\ 0.261 & 0.0052 & -0.089 & 2.63 & -1.77 & 0 & 0 & 0 \\ 4.52 & 3.621 & -0.0471 & 0.092 & 2.42 & 0 & 0 & 0 \\ -3.87 & -2.876 & 1.0331 & 1.132 & -2.53 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & -0.0041 & -0.00084 & 0.0072 \\ 0.031 & 0.0431 & 0.0541 & 0.0621 & 0.0291 & -2.31 & -0.0058 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.057 & -0.0095 \end{bmatrix} \times 10^2$$

The eigenvalues obtained for this subsystem are

$$\lambda_1 = 3.61 + j 99.75 ; \lambda_2 = -3.61 - j 99.75 , \lambda_3 = -26.51 , \lambda_4 = -5.51 , \lambda_5 = -0.369 , \\ \lambda_6 = -0.149 + j 3.0 , \lambda_7 = -0.149 - j 3.0 , \lambda_8 = -13.11 .$$

Note that the third subsystem is made stable under the constraint that the overall system is kept stable. One can apply the same procedures in designing the controller for the second subsystem.

Suppose one chooses machine 1 to apply the second decentralized controller.

It is recognized that the oscillations of  $\Delta\delta$  should be well-damped for an impulse disturbance applied to the system and the excursions of the terminal voltage  $V_T$  should also be reasonable.

After applying the decentralized controller to  $i$ th area,  $i=1,2,3,4$ ; the response is checked. An examination of the results shown in Figures 2 & 3

Machine No.	$T_R$	$T_A$	$K_R$ pu	$V_{Rmax}$ pu	$T_E$	$K_E$ pu	$S_E$	$T_F$	$K_F$ pu	$J$ pu	$D$ pu	$E$ pu	$\delta$ (deg.)	$x_d$ pu	$x_q$ pu	$x_d'$ pu	$R$ pu
1	0.02	1.0	8/-	0.5	-0.06	0.27	0.5	0.02	3	1.0	1.5	20	1.6	1.4	0.25	0.01	
2	0.06	1.0	8	-1	0.6	-0.08	0.29	0.6	0.03	5	1.0	1.5	-7	1.8	1.5	0.27	0.008
3	0.05	0.9	7	-1	0.5	-0.04	0.31	0.8	0.05	10	1.0	1	0	1.5	1.61	0.3	0.031
4	0.04	0.05	7	-1	0.6	0.04	0.2	0.7	0.08	8	1.0	1	28	1.95	1.7	0.23	0.012

Table 1: Machine data of the system shown in Figure 1. (Time constants are in seconds).

Machine No.	$V_T^0$ pu	$V_d^0$ pu	$V_g^0$ pu	$S_E^0$ pu	$E_{FD}^0$ pu	$i_d^0$ pu	$i_q^0$ pu
1	1.293	0.52	1.184	0.25	2.5	-1.62	0.72
2	1.163	0.34	1.112	0.25	2.6	-1.62	0.72
3	0.805	0.36	0.72	0.25	2.5	-1.62	0.72
4	0.909	0.45	0.79	0.25	2.7	-1.62	0.71

Table 2: Operating point.

System Eigenvalues	
Before applying the decentralized controllers	After applying the decentralized controllers
$\lambda_1 = -23.81$	$\lambda_1 = -43.21$
$\lambda_2 = -13.62$	$\lambda_2 = -24.71$
$\lambda_3 = -18.28$	$\lambda_3 = -28.17$
$\lambda_4 = -19.23$	$\lambda_4 = -34.18$
$\lambda_5 = -16.21$	$\lambda_5 = -24.23$
$\lambda_6 = -90.11 + j 9.22$	$\lambda_6 = -90.11 + j 9.22$
$\lambda_7 = -90.11 - j 9.22$	$\lambda_7 = 90.11 - j 9.22$
$\lambda_8 = -0.36 + j 6.98$	$\lambda_8 = -2.11 + j 8.72$
$\lambda_9 = -0.36 - j 6.98$	$\lambda_9 = -2.11 - j 8.72$
$\lambda_{10} = -2.13 + j 65.21$	$\lambda_{10} = -7.14 + j 65.21$
$\lambda_{11} = -2.13 - j 65.21$	$\lambda_{11} = -7.14 - j 65.21$
$\lambda_{12} = -1.51 + j 59.21$	$\lambda_{12} = -1.73 + j 59.92$
$\lambda_{13} = -1.51 - j 59.21$	$\lambda_{13} = -1.73 - j 59.92$
$\lambda_{14} = -275.81 + j 202.12$	$\lambda_{14} = -275.81 + j 202.12$
$\lambda_{15} = -275.81 - j 202.12$	$\lambda_{15} = -275.81 - j 202.12$
$\lambda_{16} = -29.23$	$\lambda_{16} = -29.95$
$\lambda_{17} = -44.62$	$\lambda_{17} = -45.42$
$\lambda_{18} = -29.24 + j 50.31$	$\lambda_{18} = -29.81 + j 50.92$
$\lambda_{19} = -29.24 - j 50.31$	$\lambda_{19} = -29.81 - j 50.92$
$\lambda_{20} = -397.91 + j 870.21$	$\lambda_{20} = -398.21 + j 870.52$
$\lambda_{21} = -397.91 - j 870.21$	$\lambda_{21} = -398.21 - j 870.52$
$\lambda_{22} = -441.31 + j 1690.25$	$\lambda_{22} = -441.31 + j 1690.25$
$\lambda_{23} = -441.31 - j 1690.25$	$\lambda_{23} = -441.31 - j 1690.25$
$\lambda_{24} = -1.32$	$\lambda_{24} = -2.0 + j 0.152$
$\lambda_{25} = -0.35$	$\lambda_{25} = -2.0 - j 0.152$
$\lambda_{26} = -0.31 + j 0.61$	$\lambda_{26} = -1.21 + j 0.74$
$\lambda_{27} = -0.31 - j 0.61$	$\lambda_{27} = -1.21 - j 0.74$
$\lambda_{28} = -0.97 + j 1.27$	$\lambda_{28} = -1.81 + j 1.51$
$\lambda_{29} = -0.97 - j 1.27$	$\lambda_{29} = -1.81 - j 1.51$
$\lambda_{30} = -0.0167$	$\lambda_{30} = -0.0187$
$\lambda_{31} = -0.972$	$\lambda_{31} = -0.972$
$\lambda_{32} = -0.01491$	$\lambda_{32} = -0.079$

Table 3: Four-machine system eigenvalues.



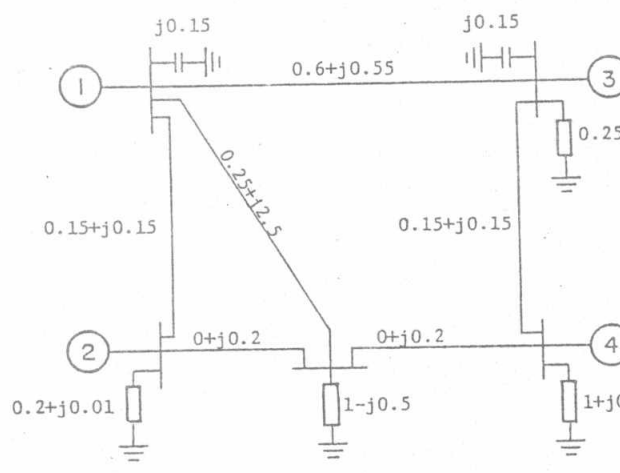


Figure 1: Four-machine four-load power system

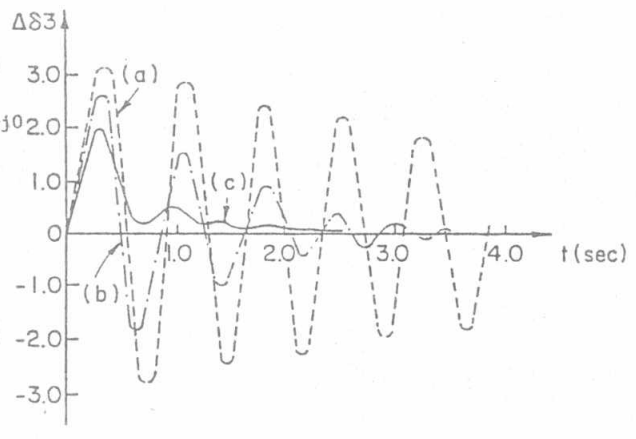


Figure 3: Response of machine No. 3 to a small disturbance

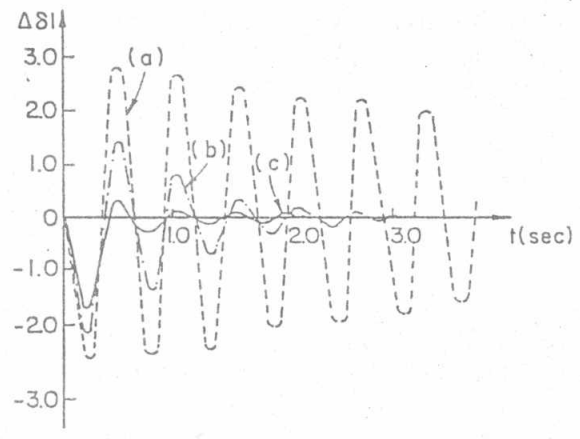


Figure 2: Response of machine No. 1 to a small disturbance

- (a) open loop response
- (b) closed loop response after connecting the first controller
- (c) closed loop response after connecting the second controller

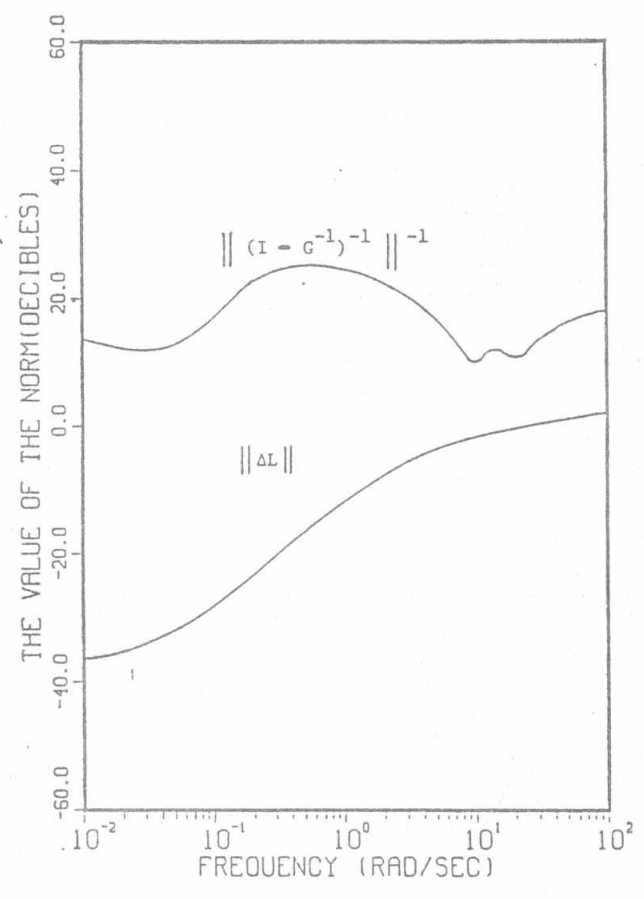


Figure 4: Robustness for Multiplicative Perturbation for the Power System Shown in Figure 1.

indicates the improvement of the response when using the decentralized controllers.

Remark: Figure 4 demonstrates the norm approach to measure the stability margins. It is clear that the system is robust after applying the decentralized controllers. However, Figure 4 illustrates the singular values approach [6] which in this case provides a better measure of robustness for the four-machine four-load power system of Figure 1.

#### V. CONCLUSIONS

A decentralized power system model has been formulated in designing robust controllers. Each subsystem can be independently controlled and the interconnections can be taken into account through the design of the robust controller.

The technique applied here uses a state-space description for each subsystem; the robust controller is designed based upon a singular value decomposition performance measure. The controllers are applied sequentially so as to provide as much robust stability of the overall system as possible.

Currently, this technique is being applied to robotic problems whereby it is desired to have each joint be controlled independently.

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