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DEVELOPING AN AUTO-PILOT SYSTEM DESIGN USING SINGULAR PERTURBATION

A.TAWFIK*, A.M.WAHDAN**

ABSTRACT

This paper presents developing a suboptimal design of an autopilot system, using singular perturbation approach, to replace a classical system existing in some Russian (MIG-21) aircrafts.

The existing conventional design is discussed. Study of system dynamics leads to two-time scale simplified piece-wise linear model. Applying singular perturbation algorithm to the obtained model; a new suboptimal controller was developed.

Numerical simulation is used to compare between the performances of the existing design, and that of the proposed suboptimal design. The proposed design shows better response than the existing one, especially in pitch attitude.

The suboptimal controller can be implemented using modern digital electronics, which might help to save larger room in the (MIG-21) aircraft for other running modifications. The digital controller would also ensure higher reliability and better accuracy.

* Col. Eng., M.Sc., Main A/C Workshop in Helwan.
** Associate Professor, Dpt. of Electronics and Computers, Ain Shams University, Cairo, Egypt.

1. INTRODUCTION

The levelling control mode of an existing autopilot system, operating with MIG-21 fighter aircrafts is investigated. The purpose of this levelling control mode, is to bring the aircraft back from any deviated flight attitude to level flight conditions. The problem formulation is detailed in section 2.

In section 3, study of the existing conventional autopilot, shows that it consists of two completely decoupled controllers, one for the fast roll channel and the other for the slow pitch channel. The principle of design of each controller is to satisfy certain performance characteristics.

State variable model reformulation, and time scale identification are presented in section 4. A mathematical treatment and an approximation approach for the nonlinear model have been performed, in order to apply singular perturbation technique.

Applying singular perturbation algorithm to the linearized corrected model; a new suboptimal controller was developed in section 5. This new design does not neglect coupling between roll and pitch motions.

Numerical simulation results are depicted to show the performances of both designs.

Conclusions are given in section 6.

2. PROBLEM FORMULATION

We refer to references of aircraft dynamics and mechanics of flight ([1] , [2] , [3]), to get a simplified model of the aircraft system reasonable for control application, and display the effectiveness of such control concepts. The system contains two distinguished dynamics, namely, the roll and pitch motions.

2.1. Roll Motion Dynamics:

The lateral dynamics of an aircraft,(rolling motion around longitudinal axis of aircraft), can be described by the following transfer function

$$\frac{\gamma(s)}{\delta_a(s)} = \frac{K_r}{s\,(1+\tau_r\,s)} \qquad (1)$$

Where:

γ : Roll angle of aircraft.

δ_a: deflection angle of aircraft aileron's surface,

K_r : gain of roll motion of aircraft,

τ_r : time constant of roll motion of aircraft.

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The numerical values for K_r and τ_r are evaluated by, 50 and 0.75 respectively, at specified nominal flight conditions, (forward speed V = 700 - 800 Km/hr, altitude H = 4 - 5 Km).

2.2. Pitch Motion Dynamics:

For pitch control of the considered levelling mode, we are interested only in the long period dynamics (phugoid ossillations). The two main states affecting the long period mode are pitch attitude, and pitch rate. Similarly, the open loop transfer function between aircraft pitch angle (θ) and elevator surface deflection (δ_e), can be formulated by the following general form:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{K_p}{s\cdot(1+\tau_p s)} \qquad (2)$$

Where:

K_p is the gain of pitch motion of aircraft,
τ_p is the time constant of pitch motion of aircraft.

Numerical values, estimated at the previously mentioned nominal flight conditions are given by:

$K_p = 15, \qquad \tau_p = 7.5$ seconds.

Note: the numerical values of model parameters of each channel, were obtained from the aircraft technical data, through experimental and flying tests at certain normal flight conditions.

3. EXISTING CONVENTIONAL CONTROLLER:

The existing conventional autopilot system consists of two completely decoupled subsystems, the roll channel "fast subsystem", and the pitch channel "slow subsystem". See Ref. [4].

3.1. Roll Channel Control:

As shown in Fig.1, the existing conventional controller is designed as (a proportional plus output rate) negative feedback controller. The controller satisfies:

a) Settling time(t_s) about 1.5 seconds.
b) Maximum overshooting about 5 % (This implies damping ratio about 0.7).

The numerical values of controller parameters are:

$K_\gamma = 0.12, \qquad K_{\dot\gamma} = 0.04$

3.2. Pitch Channel Control:

As shown in Fig. 2, the controller is designed as (a proportional plus output rate) negative feedback controller

4. MODEL REFORMULATION USING SINGULAR PERTURBATION APPROACH:

Rearranging both roll and pitch dynamics described in (1) and (2) into one state-space representation for the controlled aircraft, in the levelling control mode, can lead to the following fourth order equation:

$$\dot{\underline{X}} = A\,\underline{X} + B\,\underline{U} \qquad (3)$$

Where $\underline{X}$ is the state vector composed of the following 4-elements $(\theta, \dot{\theta}, \gamma, \dot{\gamma})$

θ = pitch attitude, $\dot{\theta}$ = pitch rate

γ = roll attitude, $\dot{\gamma}$ = roll rate,

and $\underline{U}$ is the control input two-element vector (δ_e, δ_a)

δ_e= elevator deflection, and δ_a= aileron deflection

From aircraft dynamics presented so far, it is clear that the lateral motion (around roll axis) is quite fast compared to the longitudinal motion (around pitch axis). The time constant (τ_p) of longitudinal motion is about ten times greater than that of lateral motion (τ_r).

So, considering a time scale factor

$$\lambda = \frac{\tau_r}{\tau_p} = 0.1 \qquad (4)$$

The state equation (3) can be described as a two-time scale system in the form, (Refer to [5], [6]).

$$\dot{\underline{X}} = A_1\,\underline{X} + A_2\,|\underline{Z}| + B_1\,\underline{U} \qquad (5a)$$

$$\lambda\,\dot{\underline{Z}} = A_3\,\underline{X} + A_4\,\underline{Z} + B_2\,\underline{U} \qquad (5b)$$

Where $\underline{X} = \begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix}$, $\underline{Z} = \begin{bmatrix}\gamma \\ \dot{\gamma}\end{bmatrix}$, $\lambda = 0.1$, $|\underline{Z}|$= ABS $\underline{Z}$

$$A_1 = \begin{bmatrix}0 & 1 \\ 0 & -0.133\end{bmatrix}, \quad A_2 = \begin{bmatrix}0 & 0 \\ 0.06 & 0.04\end{bmatrix}, \quad A_3 = \begin{bmatrix}0\end{bmatrix}$$

$$A_4 = \begin{bmatrix}0 & 0.1 \\ 0 & -0.133\end{bmatrix}, \quad B_1 = \begin{bmatrix}0 & 0 \\ 2 & 0\end{bmatrix}, \quad B_2 = \begin{bmatrix}0 & 0 \\ 0 & 6.67\end{bmatrix}$$

Referring to general mathematical model of the aircraft described by equations (5a,b); it is clear that the coupling effect of each channel on the other is represented by the matrices A_2 and A_3. Concerning matrix A_2, the pitch attitude is affected by value of roll motion regardless its direction. In other words, the aircraft noze tends to pitch down as the aircraft turns left or right, through an angle of roll, due to the resulting decrease of lift force on the aricraft. Thus, it is convenient to replace the fast state vector $\underline{Z}$ in Eq. (5a) by ABSOLUTE vector $|\underline{Z}|$. For the matrix A_3, it is properly approximated by a null matrix. This means that nearly no effect of pitch variations on roll motion dynamics is considered. It is worthy to note that the obtained matrix A_4 appearing in system Eq. (5b), is a singular matrix.

To prepare the aircraft model for application of singular perturbation algorithm, we have to overcome two difficulties, namely, singularity of matrix (A_4), and nonlinearity of Eq. (5a) due to the ABSOLUTE term $|\underline{Z}|$. These two problems are treated as follows:

i) Singularity of Matrix (A_4)

Refferring to roll dynamics described by the transfer function (1), the singularity of matrix A_4 is caused by the pure integration term (1/s). To avoid this singularity; the pure integration term can be replaced by a first-order lag, with sufficiently large time constant compared to the dominant time constant of the considered mode. Thus, the roll dynamics of the aircraft may be reasonably approximated by the following transfer function,

$$\frac{\gamma(s)}{\delta_a(s)} = \frac{66.67}{(s + 0.05)(s + 1.33)} \tag{6}$$

Rearranging the system model in the state-space form, using the corrected transfer function (6), instead of former one (1), would give a nonsingular matrix A_4 given by

$$A_4 = \begin{bmatrix} 0 & 0.1 \\ -0.007 & -0.138 \end{bmatrix} \tag{7}$$

ii) Nonlinearity due to the Absolute term $|\underline{Z}|$

As the pitch attitude is affected by the value of roll motion regardless of its direction, the Eq. (5a) contains $|\underline{Z}|$, not $\underline{Z}$. The absolute term causes Eq. (5a) to be nonlinear. This nonlinearity is treated using piece-wise linear segmentation [7], which results in a linear model for each segment easy to be solved. In our case, we have two segments. One is the positive values of control input (U), and the other is the negative values, (as shown in section 5).

5. PROPOSED SUBOPTIMAL CONTROLLER:

For the near optimal control algorithm, we look for the minimization of a quadratic performance index J in control and state vectors,

$$J = \tfrac{1}{2}\int_0^\infty (\underline{X}^T Q \underline{X} + \underline{U}^T R \underline{U})\, dt \tag{8}$$

Considering Q and R are unity matrices (arbitrary choice based on physical considerations).
Let $\lambda = 0$, then Eqs. (5a,b) will be reduced to:

$$\dot{\underline{X}} = A_1 \underline{X} + A_2 |\underline{Z}| + B_1 \underline{U} \tag{9a}$$

$$0 = A_3 \underline{X} + A_4 \underline{Z} + B_2 \underline{U} \tag{9b}$$

Substituting for $\underline{Z}$ from (9b), Eq. (9a) becomes

$$\dot{\underline{X}} = F \underline{X} + G \underline{U} \tag{10}$$

Where $F = A_1$, G has two cases,
for positive U-segment: $G = B_1 - A_2 A_4^{-1} B_2$,
and for negative U-segment: $G = B_1 + A_2 A_4^{-1} B_2$.
The model in Eq.(10) is called [8] the low order model. The problem can be solved through Reccati Equation to obtain the low order optimal control of the form,

$$\underline{U} = -K_1 (0) \underline{X} \tag{11}$$

This low order design is modified by correcting terms depending on λ for $\lambda > 0$, using singular perturbation theory. Considering only the first two terms of the Taylor' series,

$$K(\lambda) = K(0) + \lambda\, dk(0)/d\lambda \tag{12}$$

$$\text{As } K(\lambda) = \left[k_1(\lambda) \quad , \quad k_2(\lambda)\right]; \tag{13}$$

$$K_1(\lambda) = K_1(0) + dk_1(0)/d\lambda , \; K_2(\lambda) = K_2(0) + dk_2(0)/d\lambda \tag{14}$$

Kokotovic [8] derived the algebra involved in computing the derivatives of Eqs.(14). The suboptimal controller using 0.1 value for λ will be,

$$\underline{U} = -(K_1(0.1)\underline{X} + K_2(0.1)\, \underline{Z}) \tag{15}$$

Where $K_1(0.1) = \begin{bmatrix} 38.489 & 57.854 \\ -0.346 & 14.191 \end{bmatrix}$ (for +ve z_1), $K_1(0.1) = \begin{bmatrix} 38.489 & 57.854 \\ -2.347 & 12.167 \end{bmatrix}$ (for -ve z_1)

$$K_2(0.1) = \begin{bmatrix} 0.041 & 0.0303 \\ 1.153 & 0.8652 \end{bmatrix} \quad \text{for both cases}$$

Applying the control parameters obtained in (section 3, section 5) to the system, and using (Rung-Kutta-4) to simulate the resulting system solution, we get the corresponding system performance for different initial conditions.
A sample of results concerning pitch channel is shown in Fig. 3.

6. CONCLUSIONS:

Referring to Fig.3, the following comments can be pointed out
6.1 Pitch attitude response "$X_1(t)$" is much better for the suboptimal design (dashed line) than the conventional

design (solid line), as it has:
- Shorter settling time.
- No oscillations and no overshoots.

6.2 Pitch rate "$X_2(t)$" is much less in suboptimal design than the conventional one, which is much preferable in aircraft control (higher values of $X_2(t)$ means higher possibility of dangerous STALL situation for an aeroplane).

6.3 Referring to the presented treatment of nonlinearity and matrix singularity problems arising in system model description; gives great confidence in application of singular perturbation algorithm for control of such physical systems having multi-time scales.

6.4 The suboptimal controller can be implemented using modern digital electronics, to replace some of the traditional components of the old existing autopilot system. Thus, we may offer a solution to the problem of "lack of spare parts" of the MIG-21 Russian aeroplanes of the Egyptian Air Force.

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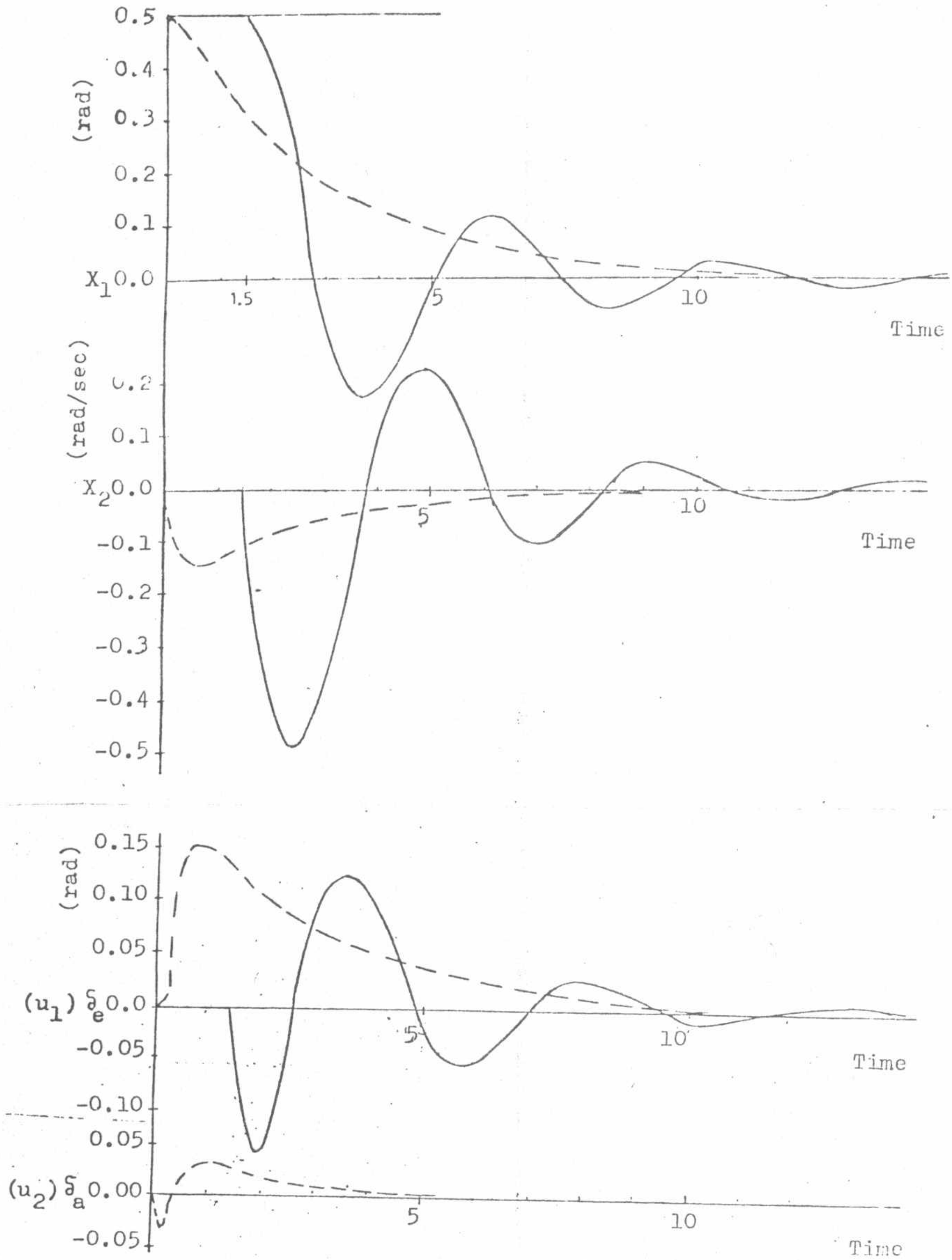

Fig.3 Time response of pitch channel for the initial conditions: ($X_1(0) = Z_1(0) = 0.5$ Rad., $X_2(0) = Z_2(0)=0$)