MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

COMPUTER AIDED DESIGN - COMPUTER AIDED TESTING

MODELLING TECHNIQUE

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ABSTRACT

The dynamic behaviour of many physical systems such as mechanical manipulators are highly nonlinear and complex. Furthermore, the inertia, stiffness and dissipation characteristics of most systems may depend on the environment which is often unknown. Because of these reasons, there is an increasing interest in combining computer aided design with computer aided testing into a unified CADCAT technique to obtain satisfactory modelling for physical systems.

In this paper an integrated CADCAT technique is proposed and investigated. This approach utilizes computer aided finite element modelling "CAD" and computer aided modal testing "CAT" into a system analysis routine to predict the dynamic performance of new systems as well as to select the design modifications for existing systems. Some applications of this technique on some design problems are described.

INTRODUCTION

There are several ways to obtain the mathematical model which describes adequately the dynamic characteristics of a physical system especially when it is complex, nonlinear or has time variant parameters. The most practical analytical method for predicting the dynamic behaviour of such system before it exists, is through the application of linear, nonlinear, or time variant finite element "FE" modelling techniques [1-3]. From an experimental standpoint, the most convenient method of determining the dynamic characteristic of a pseudo-linear model for the system, when it exist, is to use the transient modal testing technique. With the presently available instrumentation, modal parameters of the system can be identified by recording its response at some selected points after it is subjected to an artificial excitation force [4].

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On the other hand, the motion of complex systems such as mechanical manipulators and robotsis usually defined by trajectories, which consist of a sequence of desired positions, velocities, and accelerations of some point such as the manipulator tip, and the forces to be exerted at the end effector . It is then the task of the designer to determine the torques to be exerted at the manipulator joints so as to move in the desired manner. To obtain these torques, the equations of motion for mechanical manipulators have been derived by various authors using Lagrangian approach [5,6]. In the face of the complexity of the Lagrangian formulation, several alternative approaches have been proposed [7,8]. In the same time, the Newton-Euler formulation has been developed and applied to mechanical manipulators [9,10]. Both analytical approaches usually yield poor system modelling due to ignoring many interaction forces such as those due to static friction and gear backlash. Unfortunately, such forces in mechanical manipulators and robots may be comparable to dynamic, interaction and gravity forces. Therefore, straightforward application of analytical methods would almost certainly fail in practice to achieve the desired behaviour.

This paper introduces a hybrid method which combines modal testing and finite element modelling techniques into a system analysis approach to study the dynamic behaviour of complex systems such as mechanical manipulators and robots. It present some theoretical background and illustrates, by examples, use of the method to determine the mathematical model which predicts accurately their performance for the manipulation, development or trouble-shooting redesign of such systems. This approach combines the theoretical and experimental techniques to minimize the amount of assumptions and idealizations and proves superior for defining the dynamic behaviour of those cases.

MATHEMATICAL MODELLING

To model properly the dynamics of a mechanical system, which assumed to have N degrees of freedom, it is required to introduce at least an N-dimensional vector X(t) representing the displacement of N generalized coordinates. Generally, the equation describing the motion of a mechanical system of N-joints can be written as [11]

$$J(X)\,\ddot{X}(t) + C\,\dot{X}(t) + H(\dot{x}_i,\dot{x}_j,X;\ i,j = 1,2,..,N) + G(X) = F\,U(t) \qquad (1)$$

The inherent geometric nonlinearities in the model given by the the set of coupled ordinary differential equations (1) makes the system dynamics characterized by time varying unknown parameters which make it difficult to develop or solve such models. Assuming that some how the instantaneous values of the system parameters J, C, H, G, and F were found and the rate of change of these parameters was relatively slow. Then the system equation of small oscillation about the nominal position can be obtained by the following linearized perturbation equations of motion

$$\bar{J}\,\delta\ddot{X} + \bar{C}\,\delta\dot{X} + \bar{K}\,\delta X = \bar{F}\,\delta U \qquad (2)$$

or simply

$$\bar{J}\,\ddot{x} + \bar{C}\,\dot{x} + \bar{K}\,x = \bar{F}\,u \tag{3}$$

where

$$\bar{J} = \left[\frac{\delta J(X)\,\ddot{X}}{\delta X}\right]_{\bar{X}} ; \quad \bar{C} = \left[C + \frac{\delta H}{\delta X}\right]_{\bar{X}} \tag{4}$$

and

$$\bar{K} = \left[\frac{\delta G}{\delta X} + \frac{\delta H}{\delta X} - \frac{\delta FU}{\delta X}\right]_{\bar{X}} \tag{5}$$

are constant matrices derived from the derivation of the force terms with respect to X and evaluated at nominal conditions.

Equations of similar form in terms of a set of local generalized coordinates y's applied to each separate manipulator element or group of elements (subsystems). For example, the following equation applies to the r-th subsystem

$$[\bar{J}]_r\,[\ddot{y}]_r + [\bar{C}]_r\,[\dot{y}]_r + [\bar{K}]_r\,[y]_r = [\bar{F}_r]\,[u_r] \tag{6}$$

New or modified subsystems may be defined analyticaly using FE modelling technique. Other subsystems, perhaps those from previous design and known to be well behaved or those did not move and still have same configuration may be modelled through modal test data available. Therefore, equations similar to eq. 6 are deduced for all elements or subsystems of the complex system of the mechanical manipulator for example. Then all of these equations can be combined in the following matrix equation:

$$[J]\,[\ddot{Y}] + [C]\,[\dot{Y}] + [K]\,[Y] = [F]\,[U] \tag{7}$$

Equation (7) can be considered as a set of equations of motion for the group of subsystems before assembly. Assembly is a physical process of connecting these subsystems which gives rise to equations of constraint among the components of the vector $[Y]$. Now if there are M components in the vector $[Y]$ and k equations of constraint relating them, then there will exist a subset of these components containing N = M - k displacements that are independent This subset may be related to that set by a linear transformation

$$[Y] = [D]\,[x] \tag{8}$$

It should be noted here that the present analysis is limited to small displacements about the nominal conditions, so that the the linear assumption is not violated. The construction of the $[D]$ matrix requires knowledge of the constraints imposed on all elements by the system of connections. For example, at certain connection a constraint may exist which requires that the translations of a common point on the r-th and s-th subsystems be equal

i.e. $$y_{js} = y_{ir} \tag{9}$$

Rotation constraint can be in the form

$$\theta_s = \theta_r \tag{10}$$

In those cases, the treatment of all displacement constraints at the points of connections leads to a set of linear constraint equations among y's which can be expressed in matrix form

$$[T]\,[Y] = [0] \tag{11}$$

where $[T]$ is a kXM rectangular matrix of constant coefficients. This matrix may be partitioned in the following form:

$$[T] = [A \;\; B] \tag{12}$$

where $[A]$ is a square nonsingular matrix of the order k. Thus, eq. 11 can be expanded to

$$[A]\ [Y]_d + [B]\ [Y]_n = [0] \tag{13}$$

Therefore, the dependent set may be written in terms of the independent set as

$$[Y]_d = -\ [A]^{-1}\ [B]\ [Y]_n \tag{14}$$

From eqs. 8,14 an equation can be derived that relates the complete vector $[Y]$ to the independent subset $[x]$

$$[Y] = \begin{bmatrix} Y_n \\ Y_d \end{bmatrix} = \begin{bmatrix} I \\ -[A]^{-1}\ [B] \end{bmatrix} [E]\ [x] = [D]\ [x] \tag{15}$$

where $[E]$ is another linear transformation matrix introduced for convenience to relate the vector of $[Y]$ to the generalized displacement vector $[x]$ of the overall system. Substitution of eq. 8 into eq. 7 , followed by premultiplication by $[D]^T$ yields

$$[D]^T\ [J]\ [D]\ [\ddot{Y}] + [D]^T\ [C]\ [D]\ [\dot{Y}] + [D]^T\ [K]\ [D]\ [Y] = [D]^T\ [U(t)] \tag{16}$$

METHOD AND IMPLEMENTATION

Now that the basic concepts and principles of the present approach have been developed, this section will deal with its practical applications and illustrates the steps in the procedure. According to the proposed approach the development of a new or existing system can be broadly summerized into six main steps; they are: (1) Deduce analytical pseudo-linear models for all new or modified subsystems as they expected to move in space using FE modelling.approach (CAD). (2) Obtain an experimental model for any existing well behaved subsystem in all its expected configurations with a reasonable spatial step changes using modal testing (CAT). (3) Build a model for the overall system based on steps 1 and 2 above by enforcing constraint equations (ASSEMBLY). If more than one model is required due to the large displacements of the system and the inherent geometric nonlinearities, these models can be obtained sequentially or in recursive form rather than using one-shot approach. (4) Develop system performance and diagnose troubleshooting problems if there is any (ANALYSIS). (5) Simulate suggested modifications to enhance system performance and to overcome any troubleshooting uptil you obtain the best or satisfactory performance. (6) Implement the recommendations and design modifications suggested on the physical system and verify the estimated performance of the entire system using modal testing. To accomplish the above objectives the first four steps in the procedure are described below in detail with more emphasis on the important points required to increase the efficiency of the method and to obtain satisfactory results. The procedure of the proposed technique can be used to refine manipulator design, for example, as shown in Fig.1.

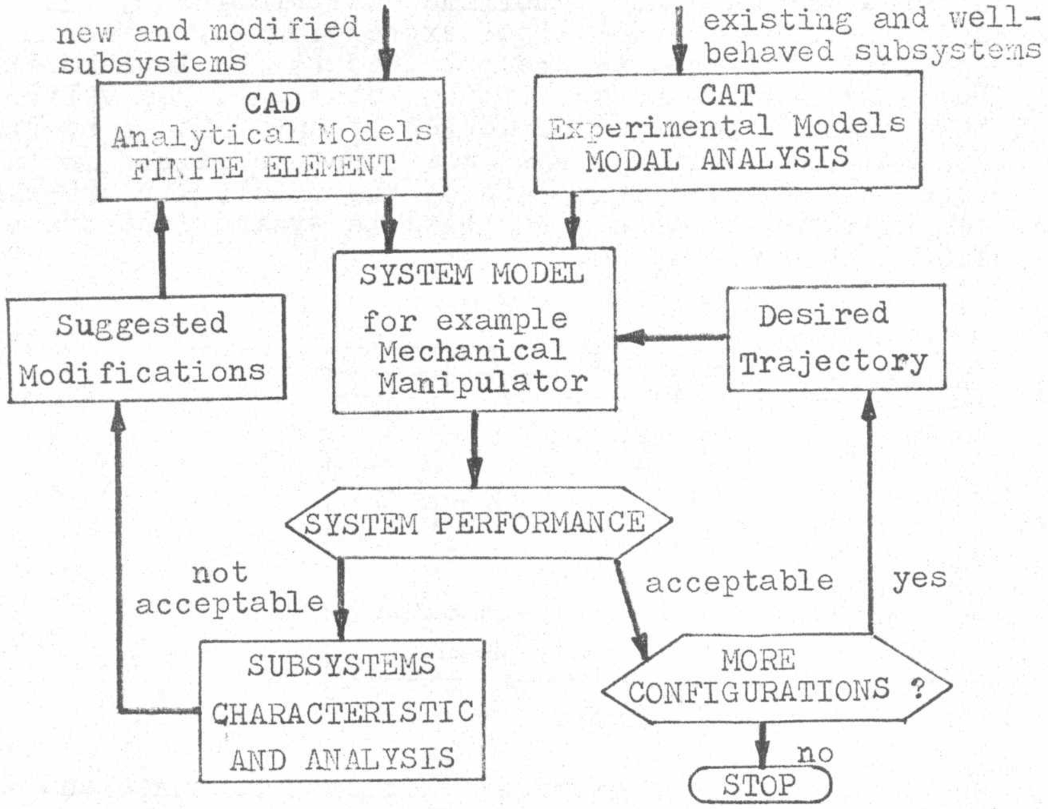

Fig. 1 Proposed system analysis scheme to enhance the performance of complex systems such as mechanical manipulators.

CAD

The finite element technique is used to describe the dynamic characteristics of the new and modified subsystems before they ever exist. The pseudo-linear and time variant approaches are used when it is necessary to obtain adequate modelling. The matrix equation can be generated and then reduced to the proper number of degrees of freedom using Guyan reduction technique. Main emphasis should be given in this stage to the sufficient matrices size to ensure that the dynamic range accurately spans the frequency range of interest in the problem.

CAT

A suitable number of test stations are selected for response measurements and excitation. This number depends on the complexity of the tested subsystem and the end use of the analysis. Modal test is then carried out as it is described in details in [12]. When relatively high noise to signal ratio data sets are employed the results can be greatly enhanced by tacking 10-20 averages. Recycling the data can be used when the frequency window is too small to obtain enough averages during a single run.

Modal parameters calculation from digital data are automated with an FFT program. Inertance response plots can be developed for the system under test and significant resonant frequencies are noted and selected for automatic amplitude computation by the modal analysis program. The analytical expression for transfer functions and modal coefficients are obtained by the implementation of a weighted least squares curve fitting technique. The validity of any model is checked before considering it any further by the reconstruction of some frequency response function exists in the modal data base and not used in obtaining the model. For example, Fig.2 compares measured frequency response to synthesized one on Bode plot of amplitude and phase.

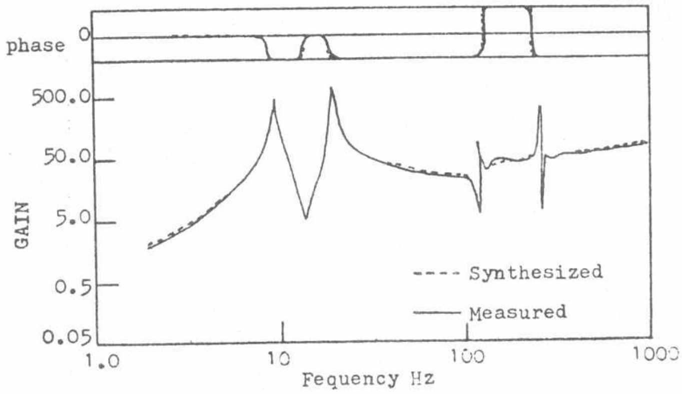

Fig. 2 Comparison between measured frequency response and synthesized one from system model.

Assembly

The proposed method allows the dynamic representation of the overall system to be obtained from subsystems information. After determining the property matrices for each subsystem, the analysis program assembles them in the system dynamic stiffness, mass, and damping matrices by considering constraint and transformation matrices [T] and [S]. Also, subsystems' forces are transformed into system forces by the same transformation. Thus, a set of equations of motion related to the overall system is formulated.

Analysis

Once the system model has been developed, it can be used to study the behaviour of the system in the vicinity of the nominal conditions. The eigenvalues and eigenvectors can be obtained and used to form a data base for the characteristics of the system. Alternatively, forced response can be developed by applying external loads to any degree of freedom in the model. The data base can be modified to predict the effect of altering system configuration constraint equations, stiffness, mass, or damping properties. The analysis procedure is standardized so as to facilitate programming for solution by digital computers. Computer program developed allows an interactive mode in predicting best solution and guides the user through various input and output options.

EXPERIMENTAL WORK AND RESULTS

The applicability and effectiveness of the proposed method have been verified by its application to some simple systems similar in shape to mechanical manipulators and robotic arms such as shown in Fig.3. The modal survey is performed on those systems using the following hardware: (1) PCB piezotronics instrumented 1.1 kg hammer kit model K291A; (2) Brüel and Kjaer triaxial accelerometer model 4321; (3) BK charge amplifiers type 2635; (4) BK four channel FM tape recorder model 7003; (5) GenRad model 2507 minicomputer and data acquisition. The system shown in Fig.3 is measured at 12 stations and the obtained model has 36 D.O.F., and multiple impacting in the x and -z directions on a rubber pad at station number 1 is used to excite the system. The use of triaxial accelerometer and a 4-channel recorder allowes us to simultaneosly record the data required to estimate the transfer function of the 3 D.O.F. at each point, thus reducing the number of tests to be carried in the field by a factor of 3. Data processing is performed at a later time in the laboratory using the GenRad data acquisition system. Typically, this is done by first qualifying all system parameters, i. e. sampling rate, triggering level, channel scale, bandwidth,..etc.. The digitizing rate necessary to properly define the waveform is taken 2-3 times Nyquist sampling rate required to reconstruct the highest frequency of interest [13]. Typical mode shape for the system at 11.4 Hz is shown in Fig. 3.

Finite element modelling is carried out for the same systems using beam elements and a very good agreement is obtained. Finally, the hybrid approach is implemented to the system and the results give even better estimates for modal analysis of the overall system, see Fig. 4 for example.

CONCLUSIONS

This paper presents a hybrid method for analysing the dynamic behaviour of complex systems with nonlinear or time varying parameters. It considers pseudo-linear models and combines finite element modelling and modal analysis experimental modelling into a system analysis routine to provide an important data base for designing superiour mechanical systems. In the proposed method the modelling of each subsystem is determined either analytically or experimentally. Then, the requirement of the system continuity gives rise to equations of displacement compitability at the points of connection. Constraint equations are used to construct a transformation relating local coordinates to the system global coordinates. This transformation is then used to obtain the overall system model.

The proposed method can be used by designer at any step for correlation and verfication of deduced analytical models. The information gained during system simulation can identify not only a number of modifications that would improve system performance, but also those would have no effect or actually worsen the performance.

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Fig.3 System used for experimental investigation.

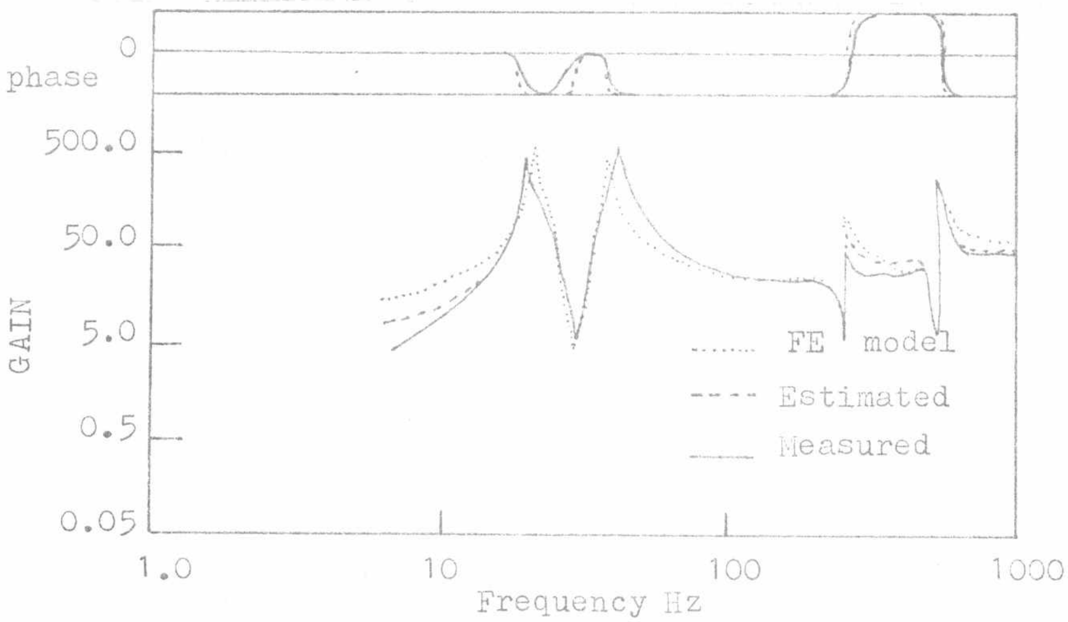

Fig. 4 Comparison between estimated behaviour using proposed scheme and the measured one.