ASAT
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OPTIMUM POLICIES OF REPAIR RATES FOR AIRCRAFTS AND ITS COMPONENTS

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ABSTRACT

Aircrafts and or its components are subject to failure during its normal operation. These failures may be rectified at different repair rates A_1, A_2, $A_3 \ldots A_k$ with corresponding repair costs r_1, r_2, $r_3 \ldots r_k$. If a unit fails, (A/C or component), it costs C_n per unit time, where n is the state of the system. The problem is to choose repair policy; as a function of the state of the system; which minimizes the long run expected total costs of the system repair.

A model is constructed to deal the problem in two cases:

- Case of poission arrivals and exponential repair time.
- Case of general arrivals and exponential repair time.

An optimization procedure is deduced for both cases, then the problem is computerized through a FORTRAN IV program, the model is then applied through a case study on EGYPTAIR fleet and the optimum policy is deduced through the model's computer program.

MATERIAL PLANNING MANAGER, EGYPTAIR, CAIRO, EGYPT.

NOTATIONS

Consider a system consisting of M units (Aircraft or component) from which R units at least should be operating properly. In case of no failures, the R units are in actual operating conditions and the (M-R) are stand by. A failed unit is repaired in the repair facility at a repair rate A_n which depends on the state of the system.
Let also:

$Q(t)$ = the state of the system at time (t)
m = the capacity at the repair queue
S = the set of all possible states (state space) (0, 1, 2, ...m)
B_n = the mean failure rate when the system is in state n
A_n = the service rate when the system is in state n
K = the number of the available repair rates
r_k = the cost per unit time incurred if the repair rate K is used. It is assumed that $r_1 < r_2 < r_3 \ldots\ldots < r_k$.
$C_n(t)$ = a cost rate incurred by the system owing to loosing production when the state of the system is n at time t.
Z = decision space (Action space), a set of all non-randomized actions.
= 1, 2, ... K
$f_n(t)$ = a function mapping from S to Z with clear meaning that:
$f_n(t) = K$ means that whenever the state of the system is n, the service rate A_K is used.
F = the set of all functions (f)

THE PROBLEM

If the system is in state n at time t and a repair rate A_k is used, the costs of the system at time t will be:

$$C_n(t) + r_{f_n(t)}$$

The average cost of the system during a period (0, T) will be:

$$\frac{1}{T} \int_0^T (C_n(t) + r_{f_n(t)})\, dt$$

and the expected cost of the system will be

$$g_f = \lim_{T \to \infty} E(1/T) \int_0^T (C_n(t) + r_{f_n(t)})\, dt$$

The problem will be so: minimize g_f over all possible $f \in F$

THE MODEL

Two cases at the repair queue are studied:
- case of poission arrivals and exponential repair time.
- case of general arrivals and exponential repair time.

The Model With Poisson Arrivals And Exponential Repair Time:

A system of m unit is considered. Given the number of failed unite n, the failure distribution is negative exponential with a parameter depending only on that number n. Let the repair service has the same property. In particular when the state of the system is n, let B_n be the hazard rate for the unit and A_n be the service rate for the same unit.

One can deduce from the first principales that the stationary probability of being in state n, P_n is given by:

$$P_n = \frac{B_0 B_1 B_2 \cdots\cdots B_{n-1}}{A_1 A_2 \cdots\cdots A_n} P_o$$

where P_o is calculated from the very known relation:

$$\sum_{n=0}^{m} P_n = 1$$

Now; if a policy $f : s \longrightarrow Z$ defines a certain repair rate A_k corresponding to every state of the system, then the long run expected average costs of the system corresponding to this policy will be:

$$g_f = \sum_{n=0}^{n=m} (C_n + r_k) P_n$$

The objective is to find the policy (f) which minimizes g_f

The Model With General Input And Exponential Service Distribution:

Consider the case of exponential service and general input distribution where one examines the system at its regeneration points. The regeneration point occures when a new arrival is joined the repair queue. Let us denote:

n = the number of units in the repair queue in front of a new arrival at time T. That is the state of the system just before a regeneration point.

n^1 = the number of units in the repair queue in front of a new arrival at time t + h.

j = the number of units being serviced during the interval h including the unit arriving at time T if its service is completed during T + h, thus:

$$n^1 = N + 1 - j \qquad \text{with } 0 \leq n^1 \leq n + 1 \tag{1}$$

Assuming that the service distribution is exponential with mean A_n (depends on the state of the system), then:

P (J being serviced / interval h) =

$$\frac{h \; A_{n+1} \; e^{-hA_{n+1}}}{J!} \tag{2}$$

The unconditional probability

$$P\ (J \text{ being serviced}) = \int_0^{\infty} \frac{(h\ A_{n+1})^J e^{-hA_{n+1}}}{J\,!}\ dF\ (h)$$

$$= K_J \quad \text{for} \quad 0 \leqslant J \leqslant n+1 \qquad (3)$$

Where F(h) is the distribution function for interarrival times.

One may represent the system in terms of an imbeded Markovian chain. The Markovian chain matrix is

$$\begin{bmatrix} 0 & 1 & 2 & 3 & \cdots & m-1 \\ 1-K_0 & K_0 & 0 & 0 & & 0 \\ 1-K_0-K_1 & K_1 & K_0 & 0 & & 0 \\ 1-K_0-K_1-K_2 & K_2 & K_1 & K_0 & & 0 \\ \cdot & \cdot & K_2 & K_1 & & 0 \\ \cdot & \cdot & \cdot & K_2 & & 0 \\ \cdot & \cdot & \cdot & \cdot & & 0 \\ \cdot & \cdot & \cdot & \cdot & & \cdot \end{bmatrix} \qquad (4)$$

Define P_n as the stationary probability of n units in front of a unit arriving at regeneration point. Using the matrix (4), one can calculate the values of P_n which satisfies the steady state equations:

$$P_n = \sum_{i=0}^{m-1} P_{i+n-1}\ K_i \quad \text{for } n \geqslant 1 \qquad (5)$$

$$P_0 = \sum_{i=0}^{m-1} P_i\ (1 - \Sigma_i\ ) \qquad (6)$$

Where

$$\Sigma_i = \sum_{J=0}^{i} K_J \qquad (7)$$

The solution of the stationary distribution given in eqmation (5) may be written in the form:

$$P_n = C\ \ S^n \qquad \text{for } C > 0 \qquad (8)$$

Where C & S satisfy both equations (5) & (6)

It can be shown [8] that

$$P_n = (\sum_{n=0}^{m-1} S_o^n\ )\ S_o^n \qquad (9)$$

With $S = F^{*}\left[A_{n+1}(1-S)\right]$ (10)

Where F^{*} is the Laplace-Stieltjes transformation of the interarrival distribution F(h) and S_0 is the unique solution for equation (10) with $0 < S_0 < 1$

P_n is the steady state stationary probability of n units in front of a unit arriving at a regeneration point.

If a policy $f : S \rightarrow Z$ defines certain repair rate A_k corresponding to every state of the system, the expected average costs of the system corresponding to this policy will be:

$$g_f = \sum_{n=0}^{n=m-1} (C_{n+1} + r_k) \; P_n$$

The objective is to find the policy f that minimizes g_f over all possible function mappings

OPTIMIZATION PROCEDURE

The following is a summary for the steps of optimization:

i - Determine all possible decisions giving the values of(A_1, A_2,.......A_k)and (r_1, r_2,.......r_k).

ii - Use the rule of elemenating a decision given in reference [10] to determine the eleminated decisions and so the set of extreme admissable decisions.

iii - Determine the parameters of the system B_n, C(m), M, R, m and H_n.

iv - Determine the set of stationary probabilities P_n for all states of the system.

v - Examin the existence of simple optimal policy as per reference [11]. If there exists a simple optimal policy, calculate g_f for possible simple policies only. If the policy is not simple, g_f is calculated for all possible policies.

vi - The optimum policy in all cases is the one which corresponds minimum value of g_f.

CASE STUDY

A semi-artificial case study is considered due to lag of accurate data.

A fleet of 12 A/C is assumed to be operated properly without stand by (i.e. R = 0). Two service rates in the system:

First rate is of 20 HR mean service time i.e.

A_1 = 0.05 unit / HR

The cost corresponding to this rate is found to be L.E. 30 per HR.

Second rate is of 12 HR mean service time i.e.

$A_2 = 0.083$

The cost corresponding to this rate is found to be L.E. 50 per HR.
The failure rate is 0.01 per HR
The cost due to lost production of an A/C is

$$C_n = \begin{cases} 500\,n & \text{for } 0 \leq n \leq 6 \\ \infty & \text{for } n > 6 \end{cases}$$

Where n is the no. of failed aircrafts.
We shall so have the states 0,1,2,3,4,5 and 6.

Using the model with Poisson input and exponential service time, one has:

$$P_n = \frac{B_0 B_1 \ldots\ldots\ldots\ldots B_{n-1}}{A_1 A_2 \ldots\ldots\ldots\ldots A_n} P_0 \quad \text{and}$$

$$P_0 = \left[1 + \frac{B_0}{A_1} + \frac{B_0 B_1}{A_1 A_2} + \ldots\ldots \right]^{-1}$$

The possible function mapping f from the state S to the action space Z will contain 64 function mapping with S = 0,1,2,......6 & Z = 1, 2

We call these function mappings f_1, f_2, f_3, f_4,......f_{64}.

It is clear that the set of admissible decisions is the set of both available decisions i.e. service rates (A_1, A_2) with service costs (r_1, r_2)

For the existence of a simple optimal policy, one has to study H_i, C(i):

C(i) = C_n is obviously nondecreasing

$C(i+1) - C(i) = 500$ for $i = 1, 2, \ldots\ldots 6$

$$H_1 = -\infty \qquad \& \; H_2 = \frac{r_2 - r_1}{A_2 - A_1} = \frac{20}{0.033} = 600$$

$BH_2 = 0.01 \times 600 = 6$

So $C(i+1) - C(i) > BH_2$

And there exists a simple optimal policy where f(i) is a nondecreasing function with $f(i) \geq f(i-1)$ for all possible states of the system. The set of all simple optimal policies will be:
f_1, f_7, f_{12}, f_{41}, f_{55}, f_{58}, and f_{64} where f_g is defined as per table I.

Table 1: Set Of All Simple Optimal Policies

State	Function mapping (action taken either policy 1 or policy 2)						
	f_1	f_7	f_{12}	f_{41}	f_{55}	f_{58}	f_{64}
1	1	1	1	1	1	1	2
2	1	1	1	1	1	2	2
3	1	1	1	1	2	2	2
4	1	1	1	2	2	2	2
5	1	1	2	2	2	2	2
6	1	2	2	2	2	2	2

The corresponding expected long run average costs for these policies will be:

Simple optimal policy	f_1	f_7	f_{12}	f_{41}	f_{55}	f_{58}	f_{64}
g_f	130.95	130.84	129.38	129.42	125.13	121.24	74.15

The optimal policy is so, f_{64} (2 2 2 2 2 2) which means that the second repair rate (A_2 = 0.0833) should be used for all states of the system.

COMPUTER PROGRAM

A computer package is specially designed in FORTRAN IV for the model. The program is designed for as large no of system states as possible also for as large no of available service rates as possible.

The computer program is shown in appendix 1. The expected long run costs computer output is shown in appendix 2.

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Appendix 1 Computer Program

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 A[illegible] I  Computer Output
LEVEL 2.1  ( JAN 75 )                OS/360  FORTRAN H EXTENDED          DATE 78.137/12[illegible].42        PAGE  1

REQUESTED OPTIONS:

OPTIONS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(60) SIZE 0320K  AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK OBJECT NOMAP [illegible]

             C        PROGRAM TO DETRMINE OPTIMAL REPAIR POLICY
                      DIMENSION A(64,6),C(6),R(2) [illegible]
  ISN 0002         10 FORMAT (4(2X,F8.4))
  ISN 0003         20 FORMAT(4I4)
  ISN 0004         30 FORMAT (6(2X,F8.2))
  ISN 0005         40 FORMAT (4(2X,F8.4))
  ISN 0006         50 FORMAT(F7.3)
  ISN 0007            READ (1,20) M,N,K
  ISN 0008            READ(1,50) S
  ISN 0009            DO 1 I=1,M
  ISN 0010          1 READ (1,10) (A(I,J), J=1,N)
  ISN 0011            READ (1,30) (C(J), J=1,N)
  ISN 0012            READ (1,30) (R(J),J=1,K)
  ISN 0013            DO 2 I=1,M
  ISN 0014          2 WRITE (2,10) (A(I,J),J=1,N)
  ISN 0015            WRITE (2,10)
  ISN 0016            WRITE (2,30) (C(J) ,J=1,N)
  ISN 0017            WRITE(2,10)
  ISN 0018            WRITE(2,30) (R(J),J=1,K)
  ISN 0019            WRITE(2,10)
  ISN 0020            DO 9 I=1,M
  ISN 0021            G=0.0
  ISN 0022            DO3 J=2,N
  ISN 0023            PA(1)=A(I,1)
  ISN 0024          3 PA(J) =PA(J-1)*A(I,J)
  ISN 0025            PO1=0.0
  ISN 0026            P1=0.0
  ISN 0027            DO 4 J=1,N
  ISN 0028          4 P1=P1+(S**J)/PA(J)
  ISN 0029            PO1=1./(1+P1)
  ISN 0030            DO 5 J=1,N
  ISN 0031          5 P(I,J) =(PO1*S**J)/PA(J)
  ISN 0032            DO 8 J=1,N
  ISN 0033            RJ = 0
  ISN 0034            IF(A(I,J).EQ.(.05)) GO TO 6
  ISN 0035            RJ= 50
  ISN 0037            GO TO 8
  ISN 0038          6 RJ=30
  ISN 0039          8 G=G+ (P(I,J))*(C(J)+RJ)
  ISN 0040          9 WRITE(2,50)(G)
  ISN 0041            STOP
  ISN 0042            END
  ISN 0043
```

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Appendix 2

EXPECTED LONG RUN COSTS

POLICY NO.	EXPECTED COSTS
1	130.958
2	87.797
3	114.816
4	126.187
5	129.700
6	130.668
7	130.696
8	76.840
9	111.869
10	125.427
11	129.519
12	130.620
13	87.759
14	84.591
16	86.958
16	87.597
17	114.041
18	114.631
19	114.780
20	126.005
21	126.152
22	129.665
23	74.876
24	76.326
25	76.718
26	76.816
27	84.083
28	84.470
29	84.567
30	86.838
31	86.935
32	87.574
33	111.401
34	111.757
35	111.847
36	113.910
37	114.020
38	114.609
39	125.318
40	125.989
41	129.998
42	125.406
43	74.566
44	74.802
45	74.861
46	76.253
47	76.312
48	76.703
49	84.011
50	84.069
51	86.824
52	111.334
53	111.388
54	113.917
55	125.305
56	113.917
57	84.456
58	111.326
59	84.002
60	76.244
61	74.793
62	74.557
63	74.521
64	74.516