CRACK PROPAGATION LIFE PREDICTION UNDER CREEP-FATIGUE CONDITIONS

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ABSTRACT

Several aircraft structural components operate under cyclic elastic plastic (fatigue) and time dependant (cyclic creep) deformations. Proper estimation of life for these components subjected to such deformation histories is certainly needed for safe operation following safe life philosophy. In addition many of these components are considered defect-free parts and life is estimated based on damage accumulation concepts. However a more rigorous analysis based on fracture mechanics predicts life of such components by crack growth calculations around the preexisted flaws. Following such design technique, tremendous cost savings could be realized if a procedure was developed to provide accurate assessment of inspection and fracture mechanics life prediction techniques.

In this paper cyclic elastic plastic creep fracture mechanics model is developed to predict crack growth in structural components subjected to creep-fatigue conditions typically exists in aircraft structural elements. Based on this model estimates of residual life could be predicted. Such analysis could certainly provide the necessary information needed for establishing preventive maintenance programmes. Results based on the present model and experimental results found in the literature are in close agreement.

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1. INTRODUCTION

The exposure of structural elements to variable mechanical loading coupled with aggressive environment, temperature and loading rates has lead to many failure problems. Often fatigue cracks initiates at stress concentration sites such as structural connections, aircraft engine components, pressure vessels, line pipes, etc. The major complexity in the life prediction analysis for such components is the interaction between fatigue as a result of time independent plasticity and creep as a result of time dependent thermal processes. Crack propagation as a result of such two mechanisms is different and there is no unified method for predicting crack growth under simultaneous action of creep and fatigue.

In this paper an elastic-plastic-creep fracture mechanics model has been developed to predict crack growth rate and hence life estimates of structural components. Based on this model a computer programme has been developed.

2. FATIGUE CREEP FRACTURE MECHANICS MODEL

The success of fracture mechanics concepts in predicting fatigue crack growth behavior at low temperature has been well established [1]. In particular, it has been shown that crack growth can be characterized by the stress intensity factor K in the linear elastic range or by an elastic plastic parameter such as the J integral in the case of plastic crack growth. This success has motivated the use of fracture mechanics at high temperatures and some fracture mechanics parameters have been suggested [1].

Crack growth can occur by cycle dependent and time dependent processes or both. The relative contribution from each process to crack growth depends on temperature, loading frequency, hold time or wave shape, geometry of the structural element, material and environmental effects. Time dependent processes include both creep and environmental effects can introduce a large temperature dependence in the crack growth rate. On the other hand purely cycle dependent processes are rather insensitive to temperature. The effects of temperature and frequency on elevated temperature fatigue crack growth rates for time dependent and cycle dependent processes are shown in figure (1). Low frequencies and high temperature favour the time dependent process, while high frequencies and low temperature favour the cycle dependent process. At intermediate temperatures, and strain rates a combination of both processes can occur, depending on the amplitude and the frequency. In the following sections, fracture mechanics parameters under fatigue, creep and fatigue creep conditions are developed.

For a given strain range and rate the imposed strain range should be subdivided into its three basic components, i.e., elastic, plastic and creep as shown in figure (2). Estimation of each of these strain components is needed in order to estimate the elastic-plastic and creep fracture mechanics parameters necessary for crack growth rate determination and hence life predictions.

2.1 ESTIMATION OF DIFFERENT STRAIN COMPONENTS

Elastic, plastic and creep strain components could be estimated for a given strain range and rate as discussed below.

a- Basic cyclic stress range-strain rate data for a given material and temperature could be experimentally obtained and plotted for different strain ranges as shown in figure (3).
b- At relatively low strain rates \(10^{-6}\) \(\text{Sec}^{-1}\) and high temperature, deformation behaviour is considered as creep. Under these conditions, the stress-strain loops are relatively flat and therefore the stress range \(\Delta \sigma\) can only take the cyclic yield value, independent of the imposed strain range as indicated by the lower line shown in figure (4).

c- At relatively high strain rates \(10^{-1}\) \(\text{Sec}^{-1}\) and even at high temperature, deformation behaviour is considered as elastic and plastic. At strain rates higher than a specific value \(10^{-1}\) \(\text{Sec}^{-1}\) the stress range becomes much less sensitive to the strain rate. The stress-strain response under this condition defines a case of pure fatigue [2] which can be represented by the upper line shown in figure (4).

d- At intermediate strain rates, mixed elastic, plastic and creep mechanisms controls overall deformation behaviour.

As shown in figure (4) the creep strain component \(\Delta \varepsilon_c\) corresponding to a given strain range \(\Delta \varepsilon_r\) and rate \(T\) could be determined. Therefore values for elastic and plastic strain components can also be obtained from the following equation.

\[
\Delta \varepsilon_p + \frac{\Delta \sigma}{E} = \Delta \varepsilon - \Delta \varepsilon_c
\]

In the present work, the relationship between \(\Delta \sigma\) and \(\log \dot{\varepsilon}\) is assumed linear.

2.2 ELASTIC-PLASTIC CREEP FRACATURE MECHANICS SOLUTIONS

Elastic plastic J integral fracture mechanics solutions have been developed previously [3] and employed successfully to analyse fatigue cracks growing in elastic plastic strain field. These solutions are given below and extended to analyse cracks growing under elastic, plastic, and creep conditions.

In making an estimate of the J integral for small crack under uniform fatigue strain, linear elastic and exponential hardening plastic cases may be considered separately and then combined to approximate elastic plastic stress strain behaviour [4]. For the special case of linear elastic material, J reduces to the strain energy release rate, G. Assuming plane stress, Je, for the elastic case is simply related the elastic stress intensity factor, K and youngs modulus, E as follows:

\[
\Delta J_e = G = \frac{\Delta K^2}{E} = \frac{F^2 \Delta \sigma^2}{2 \pi (1+\nu)} (l+L_o)
\]  

or

\[
\Delta J_e = 2\pi F^2 W_e (l+L_o)
\]

where \(W_e\) is the elastic strain energy density, \(F\) is a geometry correction factor accounts for the shape of the crack and \(L_o\) is a material correction constant accounts for the growth of short fatigue cracks [5].

An approximate solution of \(J_p\) for the exponential plastic case may be obtained as follows [3]:

\[
\Delta J_p = \frac{F^2 W_e (l+L_o)}{2\pi}
\]
\[ \Delta J_p = 2\pi F^2 f(s) W_p (L + L_o) \]  

where \( f(s) \) is a function of the strain hardening exponent \( s \), which is defined by the cyclic stress strain relationship given below.

\[ \frac{\Delta \sigma}{2} = R \left( \frac{\Delta \varepsilon_p}{s} \right)^s \]  

where \( \Delta \sigma \) and \( \Delta \varepsilon_p \) are the stress and the plastic strain ranges respectively. 

The quantity \( W_p \) is the plastic strain energy density, which may be expressed in terms of \( s \), stress range, \( \Delta \sigma \) and plastic strain range, \( \Delta \varepsilon_p \). Reducing equation (3) to yield:

\[ \Delta J_p = 2\pi F^2 f(s) \frac{\Delta \sigma \Delta \varepsilon_p}{(s+1)} (L + L_o) \]  

For combined elastic plastic deformation, the total \( J \) corresponding to fatigue condition per cycle may be approximated by adding equations (2) and (5) as follows:

\[ \Delta J = 2\pi F^2 (L + L_o) \left\{ \left( \frac{\Delta \sigma^2}{2\varepsilon_p^2} \right) + \left( \frac{\Delta \sigma \Delta \varepsilon_p}{s+1} \right) f(s) \right\} \]  

Equation (6) gives the \( J \) integral estimate for a cracked uniaxial structural member subjected to uniform fatigue strain. In this equation \( L_o, s, E \) and \( f(s) \) are known constants for a given material. The plastic strain range \( \Delta \varepsilon_p \) can be estimated based on the procedure outlined in the previous section for a given applied total strain range. The stress range \( \Delta \sigma \) can be estimated using equation (4).

Equation (6) can be extended to estimate \( J \), corresponding to creep strain component per cycle provided that material of the cracked structural element is undergoing secondary creep deformation simulated by the following stress strain rate relation:

\[ \sigma = B (\dot{\varepsilon})^n \]
Where $\sigma$ and $\dot{\varepsilon}$ are the applied peak tensile stress and strain rate respectively. $B$ and $n$ are material property constants for a given temperature. It should be noticed that equation (7) is similar to equation (4) and therefore by analogy estimate for $J_c$ similar in form to equation (5) can be obtained as follows:

$$
\Delta J_c = 2 \pi F^2 f(n) \frac{\Delta \sigma \Delta \varepsilon_c}{n+1} (L+L_0)
$$

Equation (8) gives the $J$ integral estimate corresponding to the creep component of strain $\Delta \varepsilon_c$. This component can be estimated for a given total strain range $\Delta \varepsilon$ based on the procedure described in the previous section. $f(n)$ and $n$ are know material constants.

### 2.3 Model for Fatigue-Creep Crack Growth

Fatigue cracks grow under elastic-plastic conditions with time-independent mechanisms. While creep cracks grow with time dependent process. Therefore different crack growth rate models for each case should be employed as follows:

$$
\left[ \frac{dL_c}{dN} \right] = M (\Delta J_c)^c
$$

$$
\left[ \frac{dL_f}{dN} \right] = Z (\Delta J_f)^b
$$

Where $\frac{dL_c}{dN}$: crack growth rate under fatigue conditions; and $\frac{dL_f}{dN}$: crack growth rate under creep conditions. $M, Z, C$ and $b$ are material constants for a given temperature. For a given strain cycle $\Delta J_c$ and $\Delta J_f$ can be estimated using equations (8) and (6) respectively for the same crack size. Crack growth increment per each cycle can be obtained by integrating equations (9) and (10) and added to yield the total crack growth increment corresponding to such loading cycle as follows:

$$
dL = \int M (\Delta J_f)^c dN + \int Z (\Delta J_c)^b dN
$$

Closed form solution for equation (11) is difficult, therefore a computer programme has been developed which numerically integrates such equation cycle by cycle and the total crack growth increment $(dL)$ is continuously added to the crack size obtained at the previous cycle. Based on this computer programme life estimates as a function of crack size versus number of cycles can be obtained for a given applied strain range.

### 3. Prediction Examples

High temperature low cycle fatigue data [6] for type AISI 316 stainless steel at 650°C at strain rates ranges between $4 \times 10^{-4}$ Sec$^{-1}$ and $4 \times 10^{-3}$ Sec$^{-1}$ were used to check the accuracy of the present proposed model. Material
constants employed in the present model are given in Table (I). Figures (5, 6, and 7) show a comparison between experimental and predicted results. It should be noted that the predicted life has been based on a final crack size of 3 mm. A value of $F = 0.71$ is employed assuming half circular surface cracks. A close agreement between experimental and predicted results is clear. According to this model, most of the life is spent in propagating small cracks and the initiation life can be neglected.

Figure (7) also shows the predicted number of cycles required to grow a crack to a specific size as a function of the imposed strain range. Such plots are useful for design to a specific crack size for structural components operating at high temperatures and given strain rates. Such curves are certainly useful in planning maintenance programs for such structural components.

The present computer program can be used also to predict life limits of components subjected to certain local strain history. The lower life limit is achieved by assuming the total local strain range to be all creep, while the upper life limit is attained by considering that the total local strain range is all fatigue. Such predicted limits for the present case are shown in Figure (8). Lives corresponding to different strain rates are located within these lower and upper limits. In cases of uncertain strain rates and slight temperature variations, the lower limit curve should be employed as a conservative criteria for design.

CONCLUSIONS

Cyclic elastic-plastic-creep fracture mechanics model has been developed to predict crack growth in structural components subjected to creep-fatigue conditions at elevated temperatures. This model is simulated in a computer programme. Based on this programme, the residual life of structural elements containing cracks could be predicted. Life predictions based on this model and experimental results are found in close agreement. The present analysis plays an important role in setting maintenance programs.

REFERENCES


| Table (I): Material Properties of 316 Stainless Steel At 650°C |
|-----------------|-----------------|----------------|----------------|----------------|----------------|
| M               | C               | B               | b              | E              | n              | L_o             | S               |
| 2X10⁻⁷          | 1               | 2.2X10⁻⁶        | 1.125          | 151X10⁶ KN/m²  | 0.125          | 0.00002 meter   | 0.38            |
FIG. 1 CRACK GROWTH RATE AS A FUNCTION OF TEMPERATURE AND STRAIN RATE

FIG. 2 STRAIN COMPONENTS AT HIGH TEMPERATURE

FIG. 3 STRESS STRAIN RATE RELATION FOR AISI 316 STAINLESS STEEL AT 650 °C. REF[6]
FIG. 4  CYCLIC STRESS STRAIN CURVES FOR AISI 316 STAINLESS STEEL AT 650 °C [REF.6]

FIG. 5  PREDICTED AND EXPERIMENTAL STRAIN LIFE RELATION FOR AISI 316 STAINLESS STEEL

FIG. 6  PREDICTED AND EXPERIMENTAL STRAIN LIFE RELATION FOR AISI 316 STAINLESS STEEL
FIG. 7  PREDICTED CRACK INITIATION LINES FOR AISI 316 STAINLESS STEEL

FIG. 8  PREDICTED FATIGUE AND CREEP LINE LIMITS