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MILITARY TECHNICAL COLLEGE

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A ROTATING MASS-SPRING-DAMPER SYSTEM UNDER COMBINED HARMONIC AND RANDOM EXCITATION

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ABSTRACT

A vibratory system of mass-spring-damper rotates about an axis perpendicular to its plane of vibration, and is subjected to harmonic forces in that plane. By taking into consideration the various coupling terms that may be present, we can measure the small rate of turn around the axis perpendicular to the plane vibration from the amplitude and phase angle relations. It was found previously that the phase angle is independant of the damping factor (i.e. independant of the transiant response) at the value of the rate of turn which is smaller than the system natural frequencies.

Therefore, we can improve the transient performance without affecting the sensitivity of the devise.

In this work, we study the sensitivity of the system when it is subjected to random noice.

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Introduction

The purpose of this paper is to study the effect of random noise on the measurment of small rate of turn by vibratory system.

This system, described by Linnett (1969), is used to measure a small rate of turn. It is a two degrees of freedom vibratory rate sensor which consists of a single mass mounted at the point of measurment. The mass is deduced from the analysis of the effect of coupling due to inertia; damping; and stiffness.

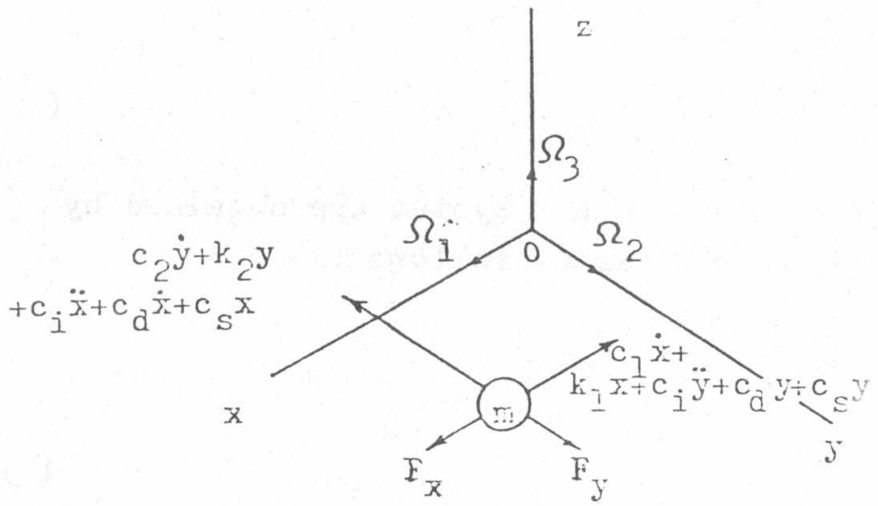
It is possible to measure very small rate of turn about an axis perpendicular to the plane of vibration, by means of the phase relationship between the induced and exited vibrations.

The phase relationship, unlike the amplitude ratio, is independant of the damping present in the system. Hence the sensitivity of the instrument is not affected by the increase in damping.

This system measures small rates of turn by a mass-spring -damper system under some simplified assumptions. One of these, is that the excitation acting on the vibratory system is harmonic. This assumption, however, can lead to the system response directly by assuming a harmonic solution. This paper is devoted to study the capability and the efficiency of the mass-spring-damper system to measure rates of turn when excited by small random forces in addition to the harmonic forces.

Equations of motion and system describtion :-

The system is in the figure. It consists of a point mass m constrained to move in the plane OXY of a rectangular set of axes OXYZ , which is rotating in space at an angular velocity Ω about a non-accelerating origin O.



When the mass is displaced from eqilibrium a distance r where r is measured from the origin O. the mass will be subjected to the forces F_x and F_y in the OX and OY directions respectivily due to the effects of stiffness, damping, inertia and the coupling terms.

The forces in relation to the displacement , velocity and acceleration are expressed as :

$$F_x = C_1 \dot{x} + K_1 x + C_i \ddot{y} + C_d \dot{y} + C_s y \quad ;$$

$$F_y = C_2 \dot{y} + K_2 y + C_i \ddot{x} + C_d \dot{x} + C_s x \quad . \qquad (1)$$

Where

C_1 and C_2 are the viscous damping coefficient in the OX and OY directions respectivily ;

K_1 and K_2 are the spring constants in OX and OY directions respectivily ;

C_i , C_d , and C_s are inertia, damping, and stiffness coupling coefficients; and are assumed equal in both OX and OY directions.

The exciting forces which act on the mass are harmonic forces in the OX and OY directions . The forces are of the same frequency ω , but have a phase difference ψ between them :

$$P_x = P_1 \, e^{j \omega t}$$

$$P_y = P_2 \, e^{j(\omega t + \psi)} \qquad (2)$$

The equations of motion for the system are obtained by applying Newton's second law as follows :

$$m \, a_1 = P_x - F_x$$

$$m \, a_2 = P_y - F_y \qquad (3)$$

Where

a_1 and a_2 are the absolute accelerations of the system in the OX and OY directions

The absolute accelerations of the mass , in general , can be written in a vector form as :

$$\vec{a} = \frac{\partial^2 \vec{r}}{\partial t^2} + \frac{\partial \vec{\Omega}}{\partial t} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + 2\vec{\Omega} \times \frac{\partial \vec{r}}{\partial t} \qquad (4)$$

Where

$$\vec{\Omega} = \Omega_1 \vec{i} + \Omega_2 \vec{j} + \Omega_3 \vec{k} \qquad (5)$$

$$\vec{r} = x \vec{i} + y \vec{j}$$

(Since the mass is constrained to move in the OXY plane , then the displacement in the Z-direction equals zero)

Thus, equations (1) are reformed by the derivation (1) as follows:

$$\ddot{x} + 2 \zeta_1 \omega_{n1} \dot{x} + \omega_{n1}^2 x + u_i \ddot{y} + (\omega_{n1} u_{d1} - 2\Omega) \dot{y}$$

$$+ u_{s1} \omega_{n1}^2 y = \omega_{n1}^2 X_s \, e^{j \omega t} .$$

Similarlly the second equation are :

$$\ddot{y} + 2 \zeta_2 \omega_{n2} \dot{y} + \omega_{n2}^2 y + u_i \ddot{x} + (\omega_{n2} u_{d2} + 2\Omega) \dot{x}$$

$$+ u_{s2} \omega_{n2}^2 y = \omega_{n2}^2 Y_s \, e^{j(\omega t + \psi)} . \qquad (6)$$

Where ω_{n1} and ω_{n2} are undamped natural frequencies in OX and OY directions.

The effect of random forces is studied by adding a small random force term to the harmonic force term. These random forcing functions are expressed as Gaussian stochastic processes.

When small random force terms are added, the equations of motion becomes as follows :-

$$\ddot{x} + 2\,\zeta_1\,\omega_{n1}\,\dot{x} + \omega_{n1}^2\,x + u_i\,\ddot{y} + (\omega_{n1}\,u_{d1} - 2\Omega)\,\dot{y}$$

$$+ u_{s1}\,\omega_{n1}^2\,y = \omega_{n1}^2\,(X_s\,e^{j\omega t} + \epsilon\,W_1(t))$$

$$\ddot{y} + 2\,\zeta_2\,\omega_{n2}\,\dot{y} + \omega_{n2}^2\,y + u_i\,\ddot{x} + (\omega_{n2}\,u_{d2} + 2\Omega)\,\dot{x}$$

$$+ u_{s2}\,\omega_{n2}^2\,x = \omega_{n2}^2\,(Y_s\,e^{j(\omega t + \psi)} + \epsilon\,W_2(t)) \qquad (7)$$

Where ; $W_1(t)$, and $W_2(t)$ are Gaussian or Normal stochastic processes with zero mean and spectral density $\sigma = .2$.

By using the state space method, these two equations are rewritten as follow :

$$\dot{\underset{\sim}{Z}}_R\,(t) = \underline{F}\;\underset{\sim}{Z}_R(t) + \underline{G}\;\underset{\sim}{u}_R(t)$$

$$\dot{\underset{\sim}{Z}}_I\,(t) = \underline{F}\;\underset{\sim}{Z}_I\,(t) + \underline{G}\;\underset{\sim}{u}_I\,(t)$$

Where subscripte R and I refere to the real and the imaginary parts

Where,

$$\underline{F} = \frac{1}{1-u_i^2}\begin{bmatrix} 0 & 1-u_i^2 & 0 & 0 \\ -\omega_{n1}^2 + \omega_{n2}^2 u_{s2} u_i & -2\zeta_2 \omega_{n1} + u_i(\omega_{n2} u_{d2} + 2\Omega) & -\omega_{n1}^2 u_{s1} + \omega_{n2}^2 u_i & -(\omega_{n1} u_{d1} - 2\Omega) + 2\zeta_2 \omega_{n2} u_i \\ 0 & 0 & 0 & 1-u_i^2 \\ \omega_{n1}^2 u_i - \omega_{n2}^2 u_{s2} & 2\zeta_1 \omega_{n1} u_i - u_i(\omega_{n2} u_{d2} + 2\Omega) & \omega_{n1}^2 u_{s1} u_i - \omega_{n2}^2 & u_i(\omega_{n1} u_{d1} - 2\Omega) - 2\zeta_2 \omega_{n2} \end{bmatrix}$$

$$\underline{G} = \frac{1}{1-u_i^2}\begin{bmatrix} 0 & 0 \\ \omega_{n1}^2 & -\omega_{n2}^2 u_i \\ 0 & 0 \\ -\omega_{n1}^2 u_i & \omega_{n2}^2 \end{bmatrix}$$

$$\underset{\sim}{z}_R^T(t) = \begin{bmatrix} x_1 & x_3 & y_1 & y_3 \end{bmatrix}$$

$$\underset{\sim}{z}_I^T(t) = \begin{bmatrix} x_2 & x_4 & y_2 & y_4 \end{bmatrix}$$

$$\underset{\sim}{u}_R^T(t) = \begin{bmatrix} X_s \cos \omega t + \epsilon W_1'(t) & Y_s \cos(\omega t + \psi) + \epsilon W_2'(t) \end{bmatrix}$$

$$\underset{\sim}{u}_I^T(t) = \begin{bmatrix} X_s \sin \omega t + \epsilon W_1'(t) & Y_s \sin(\omega t + \psi) + \epsilon W_2'(t) \end{bmatrix}$$

Simulation::-

The numerical method can be effeciently used in solving the system of equations. A case study must be considered for solving the system equations numerically. The performance of the system at any case will be estimated after we know the performance of this case study.

A case study:-

The parameters of the system are selected as follows :

Frequencies ;-

The natural frequency in OY direction (ω_{n2}) must be equal to the frequency of the exciting forces (ω) which is very important to be at resonance. The amplitude of (Y) is maximum at resonance and the arg (Y/X) not affected by the change of the damper charactristic.

The natural frequancy in OX direction (ω_{n1}) can have any value which will not affect the system solution. The natural frequancy ω_{n1} must not be equal to the frequancy of the exciting forces (ω) .

The assumed values of the frquancies are as follows :-

$$\omega_{n1} = 3 \quad sec^{-1},$$

$$\omega_{n2} = 2 \quad sec^{-1}.$$

$$\omega = 2 \quad sec^{-1}.$$

Damping factors :- (ζ_1 & ζ_2)

The damping factors in OX and OY directions are assumed equal to 0.1 . The change in this values does not change the phase angle.

Coupling terms :- (u_i , u_s , & u_d)

The system responses are obtained at different values of coupling terms.

Results

Using the computer program, the responses of the system are obtained with respect to the rates of turn. Some of the obtained results will be discussed as follows.

The Effect of Inertia coupling :-

The coupling terms $u_i = 0.01$ and the rest equal zero inserted into equations (7) we obtain :-

$$\ddot{x} + 2\zeta_1 \omega_{n1} \dot{x} + \omega_{n1}^2 x + u_i \ddot{y} - 2\Omega \dot{y} = \omega_{n1}^2 \left(X_s e^{j\omega t} + \epsilon W_1(t) \right)$$

$$\ddot{y} + 2\zeta_2 \omega_{n2} \dot{y} + \omega_{n2}^2 y + u_i \ddot{x} + 2\Omega \dot{x} = \omega_{n2}^2 \left(Y_s e^{j\omega t} + \epsilon W_2(t) \right)$$

Figure (1) shows plots of the magnitude $|Y/X|$ versus Ω with some selected values of ϵ . From figure (1) , note the error in measuring rates of turn. The symmetry about the vertical line $\Omega = 0$ is lost. Figure (3) , shows plots of the percentage error in the magnitude. We note that the error percent in measuring the magnitude of the rates of turn is increased while the parameter ϵ increased. The error percent when Ω positive is less than when Ω negative.

Figure (2) shows plots of the arg (Y/X) versus Ω with the same selected values of ϵ . Figure (4) shows plots of the error percent in the phase angle shift between Y and X. this figure shows the percentage error in the direction of the small rates of turn. Note that the error percent is increased while the randomness increased.

The Effect of Stiffness coupling :-

Introducing the coupling terms u_{s1} and u_{s2} not equal zero and the rest equal zeros into equations (7) we obtain :

$$x + 2\zeta_1\omega_{n1}x + \omega_{n1}^2 x - 2\Omega y + \omega_{n1}^2 u_{s1} y = \omega_{n1}^2 (X_s e^{j\omega t} + \epsilon W_1(t))$$

$$y + 2\zeta_2\omega_{n2}y + \omega_{n2}^2 y + 2\Omega x + \omega_{n2}^2 u_{s2} x = \omega_{n2}^2 (Y_s e^{j\omega t} + \epsilon W_2(t))$$

Typical plots of the magnitude of the solution $|Y/X|$ versus Ω for u_{s1} and u_{s2} equal 0.01 and various values of ϵ are shown in figure (5). Note that the symmetry about the vertical line $\Omega = 0$ is lost and there are certain error in measuring the magnitude of the small rates of turn.

The error percent in measuring the magnitude and directions of the small rates of turn are shown by the two figures (7), and (8). Figure (7) shows plots of the two curves of the percentage error related to the parameter equal 0.1, and 0.2 . We note that the error percent are increased while randomness increased. Figure (8) shows plots of the error percent in the direction of the small rates of turn for the same selected values of ϵ as the previous amplitude curves, which permit the observation that the percentage error have negative values up to Ω near to zero and then have positive values.

Conclusions :-

The percentage of error of the measurments of the mass - spring -damper system with harmonic forces and small random forces does not exceed .15 percent, when the random forces are equal 20 percent of the harmonic forces. It has been recommended to be used in measuring both small and very small rates of turn.

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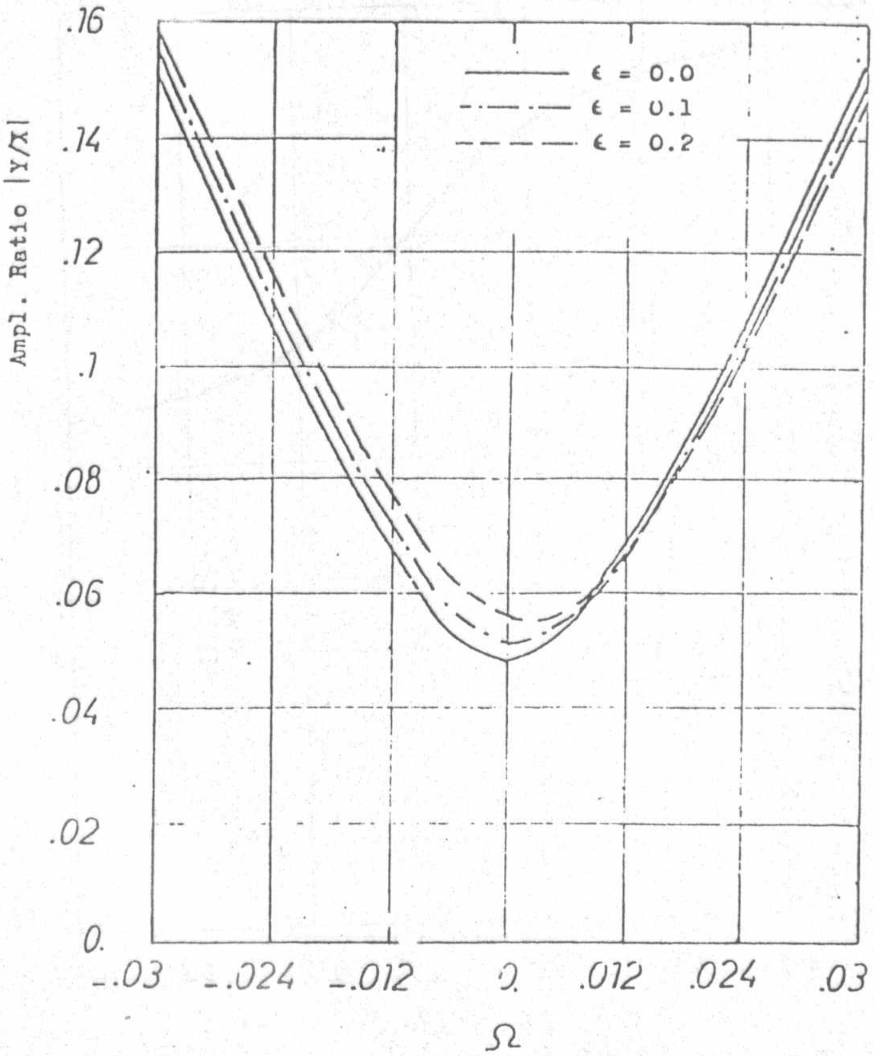

Fig. 1 Variation of $|Y/X|$ versus Ω for $r_2=1$, $\zeta=.1$, $u_i=.01$, $u_{s_{1,2}} = u_{d_{1,2}}=0$, and verious values of ϵ .

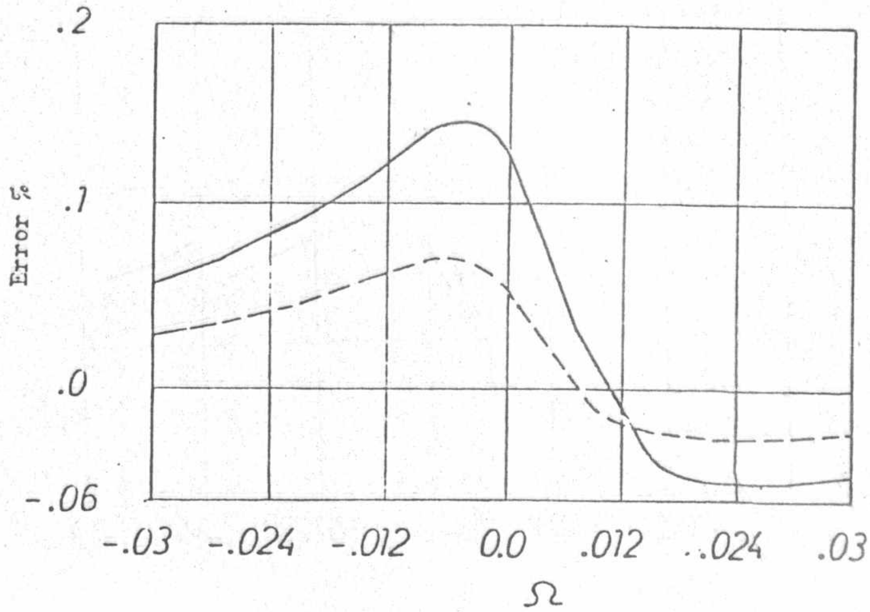

Fig. 3 Variation of the percentage error in magnitude versus Ω for ϵ = 0.1, 0.2 .

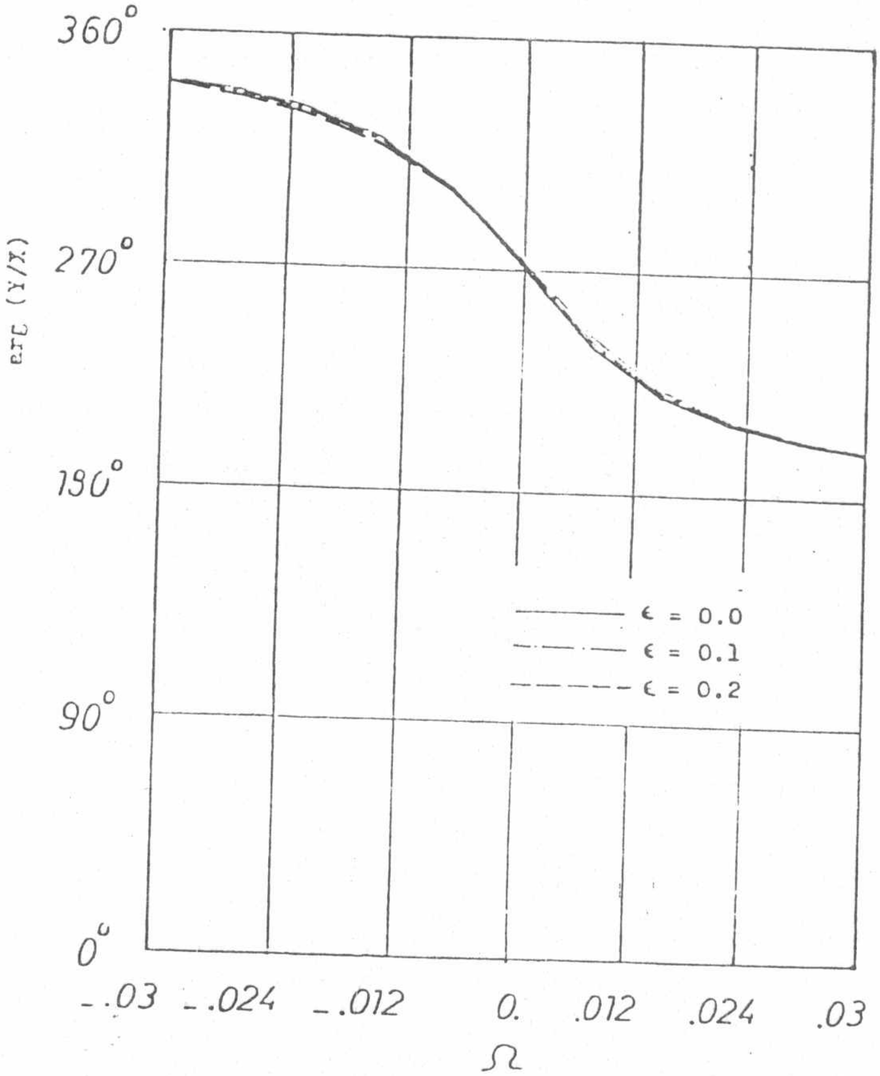

Fig. 2 Variation of arg(Y/X) versus Ω for $r_2=1$, $u_i=.01$, $u_{s_{1,2}}=u_{d_{1,2}}=0$, and verious values of ϵ .

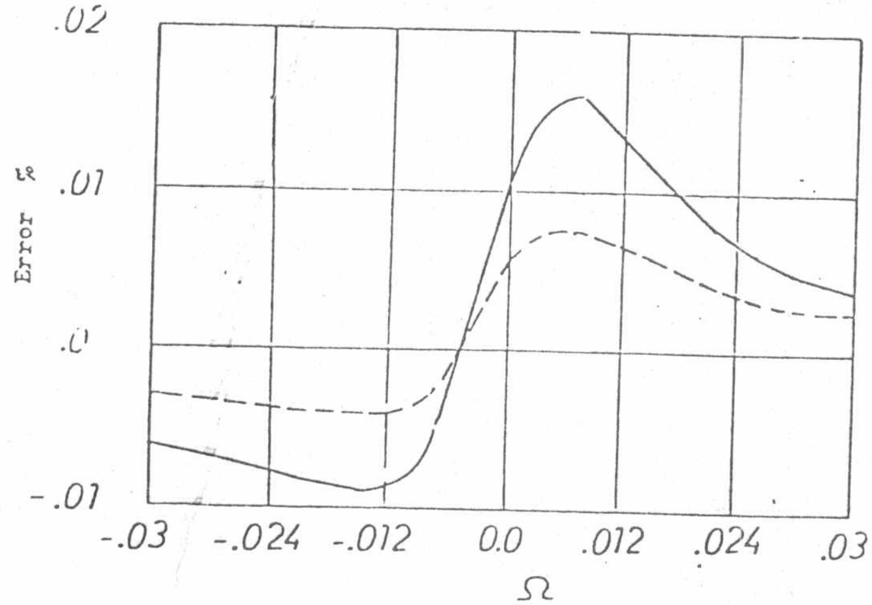

Fig. 4 Variation of the percentage error in direction versus Ω for ϵ = 0.1, 0.2 .

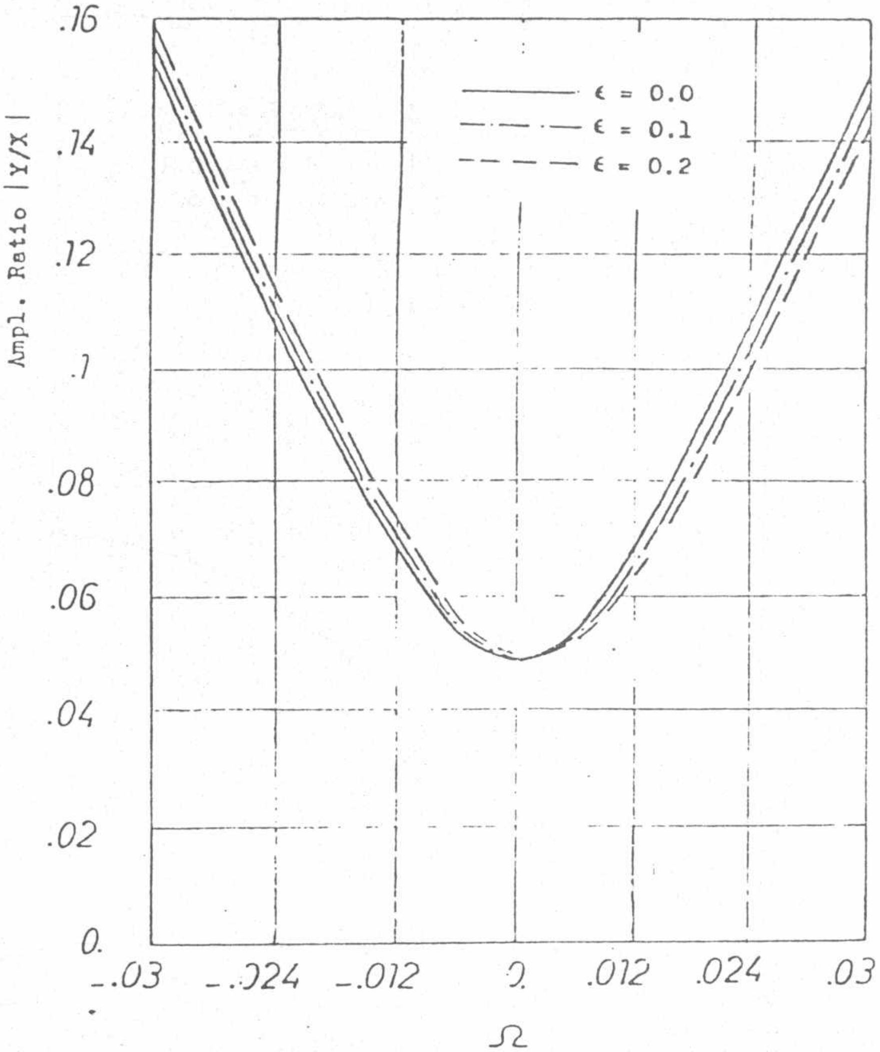

Fig. 5 Variation of $|Y/X|$ versus Ω for $r_2=1$, $\zeta=.1$, $u_{s_{1,2}}=0.01$, $u_i=u_{d_{1,2}}=0$, and various values of ϵ .

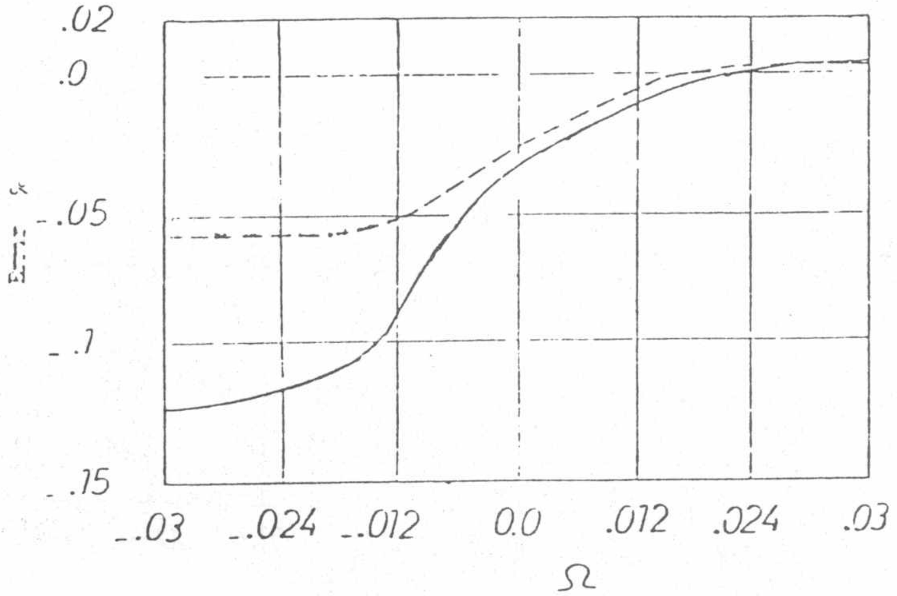

Fig. 7 Variation of percentage error in direction versus Ω for ϵ = 0.1, 0.2 .

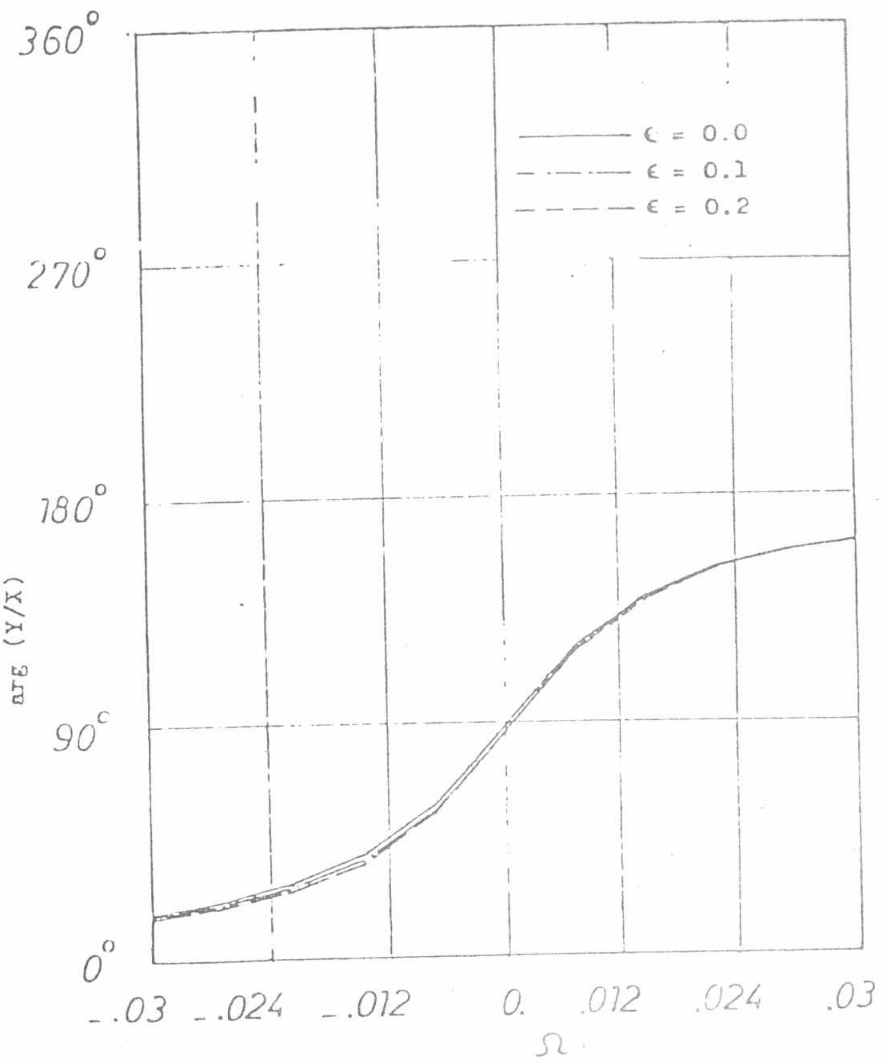

Fig. 6 Variation of arg(Y/X) versus Ω for $r_2=1$, $u_{s_{1,2}} = 0.01$, $u_i = u_{d_{1,2}} = 0$, and various values of ϵ .

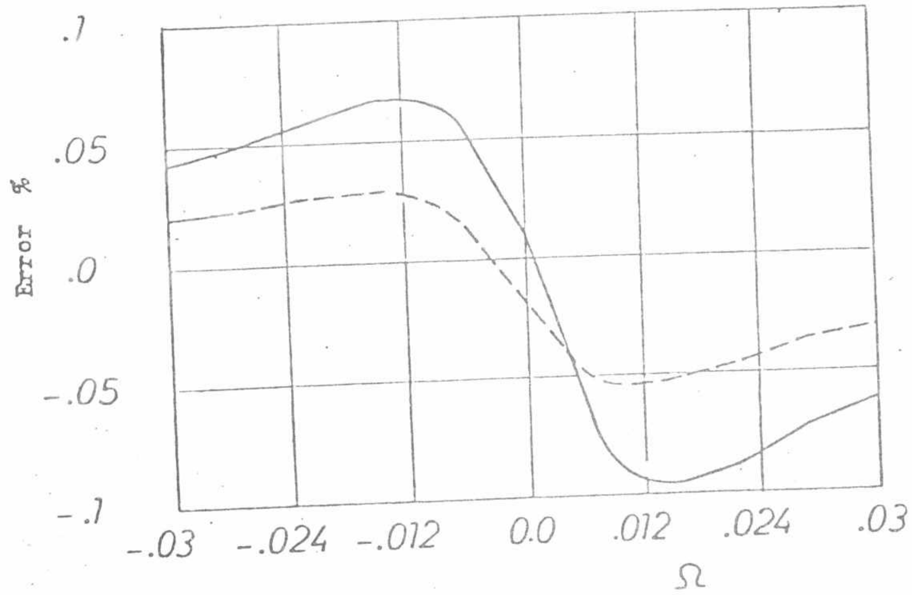

Fig. 8 Variation of percentage error in magnitude versus Ω for $\epsilon = 0.1, 0.2$.