ABSTRACT

Prediction of trajectories of dropped weapon/store from modern hybrid fighter bomber is still of great significance. Computational aerodynamics plays an important role in this domain. The reason is that experimental investigation is very expensive and requires specially equipped wind tunnels and testing facilities. Such laboratories are only available in few countries.

This paper presents a computational method for predicting weapon/store trajectories at the vicinity of the parent aircraft which is a fighter bomber and at different flying conditions.

Three different computer programs are prepared to fulfill this task. The first two programs are used in the calculation of the singularity distribution modeling the aircraft components and the weapon/store as well. The third program is a six degrees of freedom trajectory program.

Computed trajectories are compared with measured ones with good accuracy. The results show that the computational technique could give a good potential in this domain and could be considered as a cost effective one.

NOMENCLATURE

- $a$ Local body radius.
- $a_n$ Polar harmonic singularity coefficient.
- $C_{ac}$ Section drag coefficient of a circular cylinder normal to air stream.
- $C_A$ Axial force coefficient.
- $C_l$ Rolling moment coefficient.
- $C_m$ Pitching moment coefficient.
- $C_n$ Yawing moment coefficient.
- $C_N$ Normal force coefficient.
- $C_Y$ Side force coefficient.
- $V_\infty$ Free stream velocity.

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INTRODUCTION

Modern fighter bombers are laid out to perform multiple mission carrying as many external stores as possible in single, double, and multiple stations and racks at the wing and at the fuselage. Consequently, extensive wind tunnel tests are necessary and vital to check weapon/store trajectories to ensure the safe jettison, and to determine the critical conditions. But such category of testing is too expensive and time consuming. Computational aerodynamics could have a good contribution in this area. The critical cases could be distinguished according to the computed trajectories. Such critical trajectories could be checked by a very small number of wind tunnel tests. On the other hand, such technique could be utilized before flight tests in the absence of the experimental facility.

The physical problem can be summarized briefly, imagining a rigid body representing the weapon/store immersed in a non-uniform flow field. To analyse the weapon/store separation behaviour from an aircraft the store loads must be evaluated at all times after release. The resulting forces and moments are highly unsteady and depend upon the nonuniform flow field around the parent aircraft and on the store position and configuration.

Different approaches are used to solve this problem. Mainly, the empirical approach [1], the theoretical/computational approach [2], the experimental/operational approach [3], and the pure theoretical PAN-AIR technique [4].

Empirical approach needs to work in conjunction with a wind tunnel which is neither the most effective nor the most productive. Theoretical/computational technique success depends on the accuracy of the computational method, the bulk of iteration procedures, and the assumptions used to solve the problem. It does not solve the interference problem completely. On the other hand these techniques can provide quick and reliable indication of the store loads and safe store launch locations. The experimental/operational technique has demonstrated the ability to predict accurately store forces and moments in complex aircraft flow field. However, the requirement that, before predictions for a store could be made the store would have to be calibrated experimentally was an obvious disadvantage in applying this method, particularly when considering nonstandard or noninventory store/weapon. PAN-AIR technique has a good ability to predict store behaviour without the need of any experimental data. The only problem is that the necessity to have a very large computer storage.

In this paper a theoretical/computational separation trajectory program is introduced using Dellenius method [2], the flow models are based on linear potential flow theory in subsonic flow for the components of the aircraft and store. Forces and moments acting on the store in the presence of the parent aircraft was determined by an iterative procedure and comparisons were made between theory and results obtained in a wind tunnel test program to assess the accuracy of computational method.
MATHEMATICAL MODEL

The accurate calculation of weapon/store trajectory requires that the forces and moments acting on the store be known at every point in the trajectory. For this purpose the velocity field in the vicinity of the ejected store relative to that store must first be determined. This field depends not only on the store free stream velocity, but also on the perturbation velocities induced by all the aircraft components and any store still attached to the aircraft. The determination of this velocity field requires that models for the aircraft components be developed and that the important interference between components be accounted for. For noncircular fuselage, fuselage volume is modeled based on an equivalent body of revolution, with the same cross-sectional area as the actual body. The condition for noncircularity will be discussed next.

Basic Assumptions

1- This method is based on linear potential flow theory governed by Prandtl-Glauert equation for linearized compressible flow.
2- All the models to be described are for the equivalent incompressible configuration, this is made through an affine transformation.
3- The effect of the wing contribution is neglected as it has a small effect in the case of releasing a store from under the fuselage centerline.
4- The store is simplified by an axisymmetric body with a cruciform empennage, the triform fore-body empennage is neglected for the first approximation.
5- The aircraft is moving in a uniform rectilinear velocity.
6- The motion of the store is considered quasi-steady.
7- The interference between the store and the aircraft is neglected.

The overall method can be summarized in the following three steps:

1- Determining of mathematical models of various aircraft components and store.
2- Determining the forces and moments acting on the store including damping.
3- Determining the store trajectory by integrating the equations of motion, having the initial flying conditions.

Fuselage and Store Flow Models

The method used to model a fuselage with noncircular cross-section is based on the equivalence rule given in Ref. [5]. The solution for the velocity potential is given by:

\[ \phi_c(r, \theta) = \phi_e(r) + \phi_2(r, \theta) - \frac{U(x)}{2 \pi V} \ln r \]  

and,

\[ \phi_2(r, \theta) = \sum_{n=1}^{M} \frac{\sigma_n \cos n\theta}{r^n} + \frac{U(x)}{2 \pi V} \ln r \]  

where:

- \( \phi_e \) is the outer potential of the equivalent body of revolution, and is modeled by introducing a three dimen-
The perturbation velocity components due to $\Phi_2$ is given by:

$$\mathbf{u}_r(x^*, r^*) = 1 + \sum_{k=1}^{N} \frac{Q_k}{Q_2} \frac{\left( x^* - x_k^* \right)}{[\left( x^* - x_k^* \right)^2 + r^2]^{3/2}}$$

(3)

$$\mathbf{v}_r(x^*, r^*) = \sum_{k=1}^{N} \frac{Q_k}{Q_2} \frac{\left( x^* - x_k^* \right)}{[\left( x^* - x_k^* \right)^2 + r^2]^{3/2}}$$

(4)

The perturbation velocity related to the inner potential $\Phi_2$ is given by:

$$\mathbf{u}_r(r, \theta) = - \sum_{n=1}^{N} r \cos n\theta$$

(5)

$$\mathbf{v}_r(r, \theta) = - \sum_{n=1}^{N} r \sin n\theta$$

(6)

The number of polar harmonics are chosen through a least square estimate of solution of a system of simultaneous equations representing the boundary conditions at a number of control points on the contour surface, in condition that:

$$MN < MC$$

(7)

The fuselage of the fighter bomber Fig(1) is divided into 23 stations along its length as shown in Fig(2), where the relation between the dimensionless radius and the dimensionless coordinate shows the area distribution of the fuselage configuration Fig(3). Similar area distribution is made for the store Fig(4). The shape for the equivalent body of revolution for both fuselage and store is then approximated by a segmented polynomials describing its shapes analytically. This is done through a least square technique program.

**Forces and Moments Calculations**

For calculation of forces and moments acting on the store at each point in its trajectory, the nonuniform velocity field in which the store is immersed, as seen by the store, must be determined at each point in time. The field includes the free stream velocity, the perturbation velocity induced by the parent aircraft and the angular velocities due to the store pitch, yaw, and roll motions. The distribution along the body axis is required in order to calculate the forces and moments acting on the body, and the distribution over the tail fin surfaces is required for the empennage force and moment calculations.

Defining a coordinate system $(X_0, Y_0, Z_0)$ fixed in the store with the origin at the store nose and $X_0$ in the backward direction, and the coordinate system $(X, Y, Z)$ fixed in the aircraft nose center, and with the $X$ in the forward direction, together with the inertial system $(\xi, \eta, \zeta)$ fixed in the aircraft nose, with $\zeta$ in the flight direction. The three velocity components of store are given by:
Slender body theory is the basis for calculating the body normal force and side force distribution. When large values of the combined angle resulting from upwash and sidewash effects occur such that the boundary layer separates, the slender body theory calculation is not continued, simple viscous cross flow theory is used downstream of the separation location. Due to the variation of static pressure field acting in the nonuniform flow, buoyancy forces and moments are also taken into consideration. The expression for calculating buoyancy normal force and side force coefficients are:

\[
\begin{align*}
(C_N)_{BY} &= \frac{2\pi}{Sr} \int_0^{\infty} \left( x_{s,m} - x_s \right)^2 \frac{dW_s^*}{dx_s} \, dx_s \\
(C_Y)_{BY} &= \frac{2\pi}{Sr} \int_0^{\infty} \left( x_{s,m} - x_s \right)^2 \frac{dV_s^*}{dx_s} \, dx_s
\end{align*}
\]

Pitching and yawing moments coefficients due to buoyancy about the center of mass are given by:

\[
\begin{align*}
(C_m)_{BY} &= \frac{2\pi}{Sr} \int_0^{\infty} \left( x_{s,m} - x_s \right)^2 \frac{dW_s^*}{dx_s} \, dx_s \\
(C_n)_{BY} &= \frac{2\pi}{Sr} \int_0^{\infty} \left( x_{s,m} - x_s \right)^2 \frac{dV_s^*}{dx_s} \, dx_s
\end{align*}
\]

Slender body forces and moments coefficients are given by:

\[
\begin{align*}
(C_N)_{SB} &= \frac{2\pi}{Sr} \int_0^{\infty} \frac{d}{dx_s} \left( \sigma^2 w_s^* \right) \, dx_s \\
(C_Y)_{SB} &= \frac{2\pi}{Sr} \int_0^{\infty} \frac{d}{dx_s} \left( \sigma^2 v_s^* \right) \, dx_s \\
(C_m)_{SB} &= \frac{2\pi}{Sr} \int_0^{\infty} \left( x_{s,m} - x_s \right) \frac{d}{dx_s} \left( \sigma^2 w_s^* \right) \, dx_s \\
(C_n)_{SB} &= \frac{2\pi}{Sr} \int_0^{\infty} \left( x_{s,m} - x_s \right) \frac{d}{dx_s} \left( \sigma^2 v_s^* \right) \, dx_s
\end{align*}
\]

The upper limit of the above integral is the assumed separation location, that is, the point at which viscous forces are important. Viscous crossflow forces and moments are given by:

\[
\begin{align*}
(C_N)_{CF} &= \frac{2C_{dc}}{S} \int_{x_{s,0}}^{\infty} w_s^* \, dx_s \\
(C_Y)_{CF} &= \frac{2C_{dc}}{S} \int_{x_{s,0}}^{\infty} v_s^* \, dx_s \\
(C_m)_{CF} &= \frac{2C_{dc}}{S} \int_{x_{s,0}}^{\infty} \left( x_{s,m} - x_s \right) w_s^* \, dx_s \\
(C_n)_{CF} &= \frac{2C_{dc}}{S} \int_{x_{s,0}}^{\infty} \left( x_{s,m} - x_s \right) v_s^* \, dx_s
\end{align*}
\]

where:

\[
\begin{align*}
\sigma^2 &= \left( v_s^{*2} + w_s^{*2} \right)^{1/2} \\
C_{dc} &= \frac{\text{Drag per unit length}}{q_{	ext{b,s}} (2\alpha)}
\end{align*}
\]

Empennage forces and moments are given by:

\[
(C_N)_E = \cos \phi_f \left[ (C_x)_E - (C_x)_f \right] - \sin \phi_f \left[ (C_y)_E - (C_y)_f \right]
\]

\[
(C_m)_E = \cos \phi_f \left[ (C_x)_E - (C_x)_f \right] + \sin \phi_f \left[ (C_y)_E - (C_y)_f \right]
\]

\[
(C_n)_E = \cos \phi_f \left[ (C_x)_E - (C_x)_f \right] - \sin \phi_f \left[ (C_y)_E - (C_y)_f \right]
\]
The rolling moment coefficient is given by:

\[
(C_\theta)_E = \sin \phi \left[ (C_2 - C_\theta)_B \right] + \cos \phi \left[ (C_\gamma - C_\theta)_B \right]
\]

Where:

\[
(C_\theta)_B = \cos \phi \left[ (C_{m_B} - C_{m_B})_B \right] + \sin \phi \left[ (C_{n_B} - C_{n_B})_B \right]
\]

The rolling moment coefficient is given by:

\[
(C_\theta)_E = (C_\theta)_{HV}
\]

Where:

\[
(C_\theta)_{HV} = -\frac{1}{\pi} \int \frac{d\zeta}{(s-\zeta)^2} \left( \frac{d\zeta}{(s-\zeta)^2} \right) (C_\zeta)_E dy
\]

The method employed for calculating store trajectory is given in Ref [5]. The equation of motion is integrated given the initial conditions by the fourth order Runge-Kutta Method, and Adams corrector-predictor Method.

**COMPUTER PROGRAMS**

1- Least Square Program

It has been made to approximate the various shapes of fuselage/store by a series of segmented polynomials specifying these shapes in a least-square sense. The program input is points on the longitudinal axis of the body and their corresponding radii. The output is the segmented polynomials coefficients. The program consists of a main program and two subroutines.

2- Singularity Distribution Program

The purpose of this program is to calculate the sourcesink distribution which is required to represent the volume distribution of the circular fuselage/equivalent body of revolution and axial symmetric bodies. This program consist of the main program and two subroutines. The input to the program is the flying conditions, the location of the first source, and the polynomials coefficients. The output of the program is the source-sink strengths which satisfy the shape of the body under consideration at the given flying conditions.

3- Six Degrees of Freedom Trajectory Program

This program uses the source distribution described in the preceding program as input data. In addition other input data are: the aircraft various components, and flying conditions. The main purpose of the program is firstly to determine the velocity field at which the store is immersed.
secondly, to calculate the aerodynamic forces and moments acting on the store, and finally to integrate the equations of motion to obtain the trajectory.

ANALYSIS OF RESULTS

Input data and configuration are chosen to be consistent with the only available report of measurements in wind tunnel. Obtained results are illustrated in Fig(5) through Fig(8). It could be analysed as follows:

a- Comparison between computed and measured trajectories shows that the computational technique gives trajectories having a good agreement with the experimental ones.

b- Computed trajectories are very close to the measured ones in the vicinity of the parent aircraft up to one-store diameter, in all cases under consideration.

c- The farther the store from the parent aircraft the bigger is the deviation between measured and computed trajectories. The order of magnitude of deviation is less than one-store diameter at a distance greater than one-store length under the airplane.

d- The deviation between measured and computed trajectories have a nonlinear nature.

The reason for this deviation could be attributed to the following reasons:

a- The fore-body empennage is not taken into consideration in the empennage force calculation due to the absence of the necessary and important geometric and aerodynamic data of it. But it could be seen from the comparisons that the fore-body empennage affects to a great extent the pitching moment and angle of attack of the store. Due to its complicated configuration, the drag force on the store is increased leading to the observed change of the slope of the trajectory.

b- The interference effect between the store and the mother airplane is neglected in this stage of research as it represents the first step in the iteration procedure.

c- The effect of the wing is neglected, which may have a small contribution.

CONCLUSION

The computational method used in this paper manifested itself to be comparable to experimental measurements in wind tunnels with good accuracy.

Large capacity and fast digital computers are required and necessary for fulfilling such researches due to the complicated nature of the system of computer programs to be matched.

Fair to good results of predicted trajectories compared to measured ones are obtained. This shows that the effect of the fore-body empennage has to be considered.

As a global conclusion, the computational aerodynamics of predicting weapon/store trajectories has an order of magnitude of time saving and cost effectiveness over the experimental measurements in wind tunnels with comparable accuracy.
REFERENCES


Fig. (4.b): ECM pod geometry.

Fig. (4.a): Mirage 5E geometry.
Fig. 2 Cross-sectional fuselage contours of the airplane

Fig. 3 Area distribution of fuselage
Fig. 5: Comparison of trajectories at (W=0.7, θ=2.7°)

Fig. 6: Comparison of trajectories at (M=0.5, θ=5.4°)
Fig. 8: Trajectories at \( M=0.9, \alpha=1.5^\circ \)

Fig. 7: Trajectories at \( M=0.8, \alpha=12.2^\circ \)
Fig. 4 Area distribution of ECM -POP

Fig. 9 Comparison between store deviations