MEASUREMENTS OF MEAN FLOW VELOCITY PROFILES
OVER ROTATCNG SMOOTH AND ROUGH DISKS

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ABSTRACT

Measurements of the mean flow velocities over a disk rotating in a quiescent air are presented in this work. The measurements were performed on flat and rough disk surfaces such that the effect of surface roughness can be investigated. The surface roughness employed in the present work was 0.03 mm. Measurements were extended from the laminar region to the turbulent region where the maximum value of Reynolds number employed in the present investigation is $3.7 \cdot 10^5$. A hot wire anemometer with a single sensing wire was used in the measurements. The three components of the mean velocity were separated by performing measurements in two distinct directions of the sensing wire. The transition region was detected by employing an oscilloscope jouned to the anemometer circuit.

The results indicate that the boundary layer velocity is composed mainly of radial and tangential components in the laminar region. In the transition and turbulent regions, away from the disk surface by a small distance, the magnitudes of the three components of velocity are of the same order. The effect of surface roughness is to cause the transition region to start at smaller values of Reynolds number and to increase the magnitudes of the three components of velocity in the transition and turbulent regions. Comparisons between this work and previous works are presented and discussed.

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INTRODUCTION

The flow over a finite disk rotating in its own plane in an otherwise quiescent air is a complicated three dimenensional one. Near the disk axis is a region of laminar boundary layer flow. At large radii, there is a region of transition flow. At even greater radii, a fully developed turbulent boundary layer occurs.

The laminar flow over an infinite disk was investigated by Von Kármán [1] and the solution was improved by Cochran [2]. Cham & Head [3] and Cooper [4] solved the turbulent boundary layer equations with empirical correlations based on two dimensional flow. Their solutions were greatly improved over the solutions of Von Kármán [1] and Goldstien [5] whose correlations were not based on experiments. However, these improved solutions were not compatible with the experiments [3]. This was due to the ignoring of the axial velocity component in the theoretical works. The previous measurements of the laminar and turbulent boundary layers on a rotating disk were not comprehensive. Earlier experiments were performed by Theodorsen and Regier [6] who measured the torque produced from the turbulent boundary layer. Kreith and Taylor [7] measured the mass transfer rates and the friction coefficients in the laminar and turbulent regions. Cham and Head [3] measured the frictional torque and the velocity components in the radial and tangential directions of turbulent flow. In their measurements, they ignored the axial velocity component.

The object of the present investigation is to obtain reasonably complete and detailed measurements of the mean components of velocity in the three regions of flow. This includes the measurements of the axial velocity component which has been ignored by previous workers. In addition, the effect of the roughness of the disk surface on the flow is also invest-igated.

VELOCITY VECTOR ANALYSIS
AND MEASURING TECHNIQUE

Boundary layer measurements were made by using a hot wire anemometer with a single sensing wire. Separation of the three components of velocity require the measurements to be performed in distinct positions of the sensing wire. This is discussed in the present section.

The paper of Jorgnesen [8] showed that two factors k_1 and k_2 are important when a single hot wire probe is placed in a three dimensional velocity field and reference to fig. 1 should serve to clarify this point. Two planes of particular concern are illustrated in the figure. Plane (1) is defined by the sensing wire and the probe axis. Plane (2) is perpendicular to plane (1) and passes through the probe axis. The probe is calibrated in the position where $\phi = 0$. Considering V flowing along the probe axis, this velocity will be registered as being of the same magnitude of V_{eff}. Now if ϕ is not equal to zero and V is allowed to rotate about the probe axis as shown in fig.1, the cooling effect is greatly influenced by the rotation of q while W has no influence since it lies in the same direction of calibration. In general V is not equal to V_{eff}. If V lies in plane (1) ($\alpha = 0$), the cooling effect of q will be minimum because q is tangent to the sensing wire. For this reason a yaw factor K_1 has to be included as a correction factor for the tangential

cooling velocity. If V lies in plane (2) ($\alpha = 90^0$), the cooling effect of q will be maximum, since q is normal to the wire, and for this reason a second factor K_2, the pitch factor, is required to correct for the velocity in plane (2). Referring to fig.1 and using the two factors described above, the effective colling velocity V_{eff} acting on the sensor at any angle α can be expressed as

$$V_{eff}^2 = w^2 + K_1^2 q_1^2 + K_2^2 q_2^2 \qquad (1)$$

Where q_1 and q_2 are the components of q in planes (1) and (2) respectively. Eq.(1), known as the characteristic equation, can be written in the form

$$V_{eff}^2 = w^2 + q^2 (K_1^2 \cos^2\alpha + K_2^2 \sin^2\alpha) \qquad (2)$$

In order to apply Eq.(2) at any angle α , the values of V_{eff}, K_1, and K_2 must be known. This was attained by performing the required correlations in a wind tunnel where V was measured by a standard pitot tube. The probe equation is given by

$$e^2 - e_0^2 = BV_{eff}^n \qquad (3)$$

Calibration between V_{eff} and ($e^2 - e_0^2$) was performed by aligning the probe axis in a direction parallel to V. In that position, $V_{eff} = V$. The results are shown in Fig.2 and the values of B and n are 6.11 and 0.46 respectively. The probe axis was then turned such that it became normal to V and its sensor was tangent to V. In this position $V_{eff} = K_1V$. The correlation between K_1 and V was achieved by knowing the values of e_0, e, V,B, n, and employing Eq.(3). To obtain correlation between K_2 and V, the sensor was turned by 90^0 about its axis from the above position ($V_{eff} = K_2 V$) and similar procedure was performed. The results are shown in Fig.3.

To enable the three components of velocity to be separated, measurements were carried out in the two positions shown in Fig.1 (notice that in the measurements, the sensor was rotated about its axis until it lies in plane (1) or plane (2), while V takes a fixed direction). Application of Eq.(2) to the first position where $\alpha = 0$, yields

$$V_{eff}^2 (1) = w^2 + K_1^2 q^2 \qquad (4)$$

Application of Eq.(2) to the second position where $\alpha = 90^0$, yields

$$V_{eff}^2(2) = w^2 + K_2^2 q^2 \qquad (5)$$

The above two positions were identified by the readings of the registered voltages, where they are of minimum and maximum values. $V_{eff}(1)$ and $V_{eff}(2)$ can be obtained from Fig.2. Equations (4) and (5) can be solved for w and q. knowing the yaw angle at any one of the above positions, u and v can be computed. It is to be noticed that the solution to equations (4) and (5) is obtained by iteration, since both K_1 and K_2 are functions of V as shown in Fig. 3.

APPARATUS

The main parts of the apparatus are simply a wooden disk (R = 24.35 cm,
t = 1.3 cm) and a sensing wire placed at a location where the velocity is
measured. The disk is rotating in a horisontal plane and the wire is
parallel to it. The probe axis is normal to the disk surface. The disk
is connected to a vertical shaft which is aligned by two ball-bearings,
30 cm apart. The shaft is driven by a 2 hp motor via pullies and a V-belt.
The rotational speed can be varied by employing pullies of variable
diameters, and it is measured by a stroboscope. The maximum disk rotational
speed attained in the present work is 880 rpm. The probe axis, which carry
the sensing wire, is mounted on the lower end of a traversing mechanism. The
later can move in both radial and vertical directions. The radial motion
of the traversing mechanism is recorded by a metering scale of 0.5 mm
accuracy, and its axial movement is recorded by a dial gage of 0.01 mm
accuracy. The yaw angle is measured by a protractor of 0.5^{o} accuracy, placed
parallel to the disk surface and can rotate with the probe axis. Precautions
were made such that vibration was avoided to a great extent. In addition to
the above mentioned parts of the apparatus,an oscilloscope joined to the
anemometer circuit was used to detect the transition region of flow.

RESULTS AND DISCUSSIONS

Experiments were performed on smooth and rough disk surfaces. For smooth
disk, the surface was flat to 0.15 mm. Rough surface was obtained by placing
sand papers of mean roughness 0.03 mm on the smooth disk surface. The
transition regions were detected at various speeds. The results indicated
that there are approximately constant values of R_e at which transitions from
laminar to transition regions and from transition to turbulent regions occur.For
smooth surface, these values are $1.8*10^5$ and $2.8*10^5$ respectively,and for
rough surface, they are $1.4*10^5$ and $2.2*10^5$ resepectively(see Fig.9).

Fig.4 shows the velocity profiles in the laminar region for two different
values of R_e. It is seen from this figure that the profiles are identical
and hence a single profile at any value of R_e can represent the solution in
the whole region. This supports the laminar theory of Von Kármán [1]. The
theoritical solution of Cochran [2] is also presented in the figure. The
agreement between theory and experiment is good everywhere which indicates
that the calibration process and the experimental accuracy of the present
work is satisfactory. As seen from the figure, the magnitude of $\bar{w}$ is very
small as compared with the other two components. The surface roughness
has no effect on the profiles up to a value of $R_e = 1.4*10^5$ where
transition occurs over rough disk surface.

Figures 5,6, and 7 illustrate the profiles of $\bar{w},\bar{v}$, and $\bar{u}$ in the transition
and turbulent regions respectively. As seen, the effect of surface
roughness is to increase the magnitudes of all velocity components. The
profiles in the transition and turbulent regions are of the same behaviour.
Fig.4 shows that $\bar{w}$ vanishes over the disk surface by an axial distance which
increases by increasing R_e. $\bar{w}$ is increased by increasing $\bar{z}$ until it reaches
a peak value $\bar{w}_p$ at a location $\bar{z}_p$. The magnitudes of $\bar{w}_p$ and $\bar{z}_p$ are shown to
increase by increasing R_e. Fig. 7 shows that, near the disk surface, $\bar{u}$
increases with high rates until it reaches a peak value $\bar{u}_p$ at a small distance

of $\bar{z}$. $\bar{u}_p$ is shown to decrease by increasing R_e until it reaches a constant value of about 0.12 in the turbulent region (Notice that although $\bar{u}_p$ is decreased by increasing R_e, its dimensional value increases). Fig. 6 shows that near the disk surface, the profiles of $\bar{v}$ become more flat as R_e increases. Figures 5 to 7 show that the boundary layer thickness $\bar{\delta}$ increases by increasing R_e. At the edge of the boundary layer, the magnitudes of $\bar{u}$ and $\bar{v}$ approach zero, while that of $\bar{w}$ has small magnitude which does not vanish since there must be an axial inflow to feed the radial outflow at the disk periphery.

A comparison between the velocity profiles of the present work and of previous workers is shown in Fig.8 (for smooth surface). Figure 8-a shows that the agreement between experiments is good. However, there is a disagreement between theory and experiments in the range $3 < \bar{z} < 16$ (notice the disagreement between theory and experiment of the same reference). The agreement shown in Fig. 8-b is good everywhere except at the region of the outer layer. The disagreement between theory and experiment is due to the fact that the axial momentum equation was ignored in the theoretical work. This assumption is only correct at small values of $\bar{z}$ where $\bar{w}$ vanishes or has small magnitudes. At intermediate and high values of $\bar{z}$, the magnitude of $\bar{w}$ is of the same order of $\bar{u}$ and $\bar{v}$, and thus the above mentioned assumption can not be applied. Boundary layer parameters are shown in Figures 9 and 10. The theoritical solutions (for flow over smooth disks) of Von Kármán [1] and Cooper [4] are also presented. The agreement of all works is only seen at large values of R_e. Von Kármán assumed that the flow is fully turbulent allover the disk surface. This indicates the reason of the big departure between his results and the present results at low values of R_e and as seen the departure is decreased by increasing R_e. This means that the solution of Von Kármán is only correct at high values of R_e. The same conclusion were obtained by Cham and Head [3]. The solution of Cooper was based on the assumption that the flow is laminar up to a value of $R_e = 3.04*10^5$ and above this value the flow is fully turbulent. The agreement between the present work and that of Cooper is seen in the laminar and turbulent regions, and the disagreement is shown in the transition region which was neglected by Cooper, who mentioned that his results were not correct in this region. As seen from the figures, the effect of surface roughness is to increase $\bar{\delta}^*$ and H in the turbulent region.

CONCLUSIONS

In the present study, it was shown that a hot wire anemometer with a single sensing wire can be used to measure the mean velocity components in the three cylindrical directions. Analysis of the directional characteristics of the anemometer showed that the three components can be separated by performing measurements in two distinct positions of the sensing wire.

From the measurements performed in the present study, the following important results are concluded: For a disk with a smooth surface, the laminar region is extended from the disk axis to a radius corresponding to a value of Reynolds number of $1.8*10^5$. In this region, the velocity is composed mainly of radial and tangential components; the profiles are identical at any value of Re; and at any speed, the boundary layer thickness is constant allover the region.

In the transition and turbulent regions, the velocity profiles are of the same behaviour; the boundary layer thickness is increased by increasing R_e; and the axial velocity component vanishes over a small axial distance from the disk surface. However, at further axial distance, the three components of velocity have the same order of magnitudes. The surface roughness has no effect on the flow in the laminar region, but this region ends at earlier radial positions when rough disk surface is used. The effect of surface roughness on the other two regions of flow is to increase the magnitudes of all components of velocity.

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NOMENCLATURES

e	output voltage, volts
e_0	Voltage at stagnant conditions
H	Shape factor
K_1	yaw factor
K_2	Pitch factor
R_e	Reynolds number $= \omega r^2 / \nu$
r	radius from axis of rotation
V	magnitude of absolute velocity m/s
V_{eff}	effective cooling velocity, m/s
u,v,w	mean velocity components in radial, tangential, and axial directions, respectively
$\bar{u},\bar{v},\bar{w}$	the values of u,v, and w divided by ωr
z	axial direction
$\bar{z}$	$z\sqrt{\omega/\nu}$
δ^*	displacement thickness $= \int_0^{\bar{\delta}} \bar{v} \, d\bar{z}$
ω	disk rotational speed
ν	kinematic viscosity

α = angle between q and plane(1)

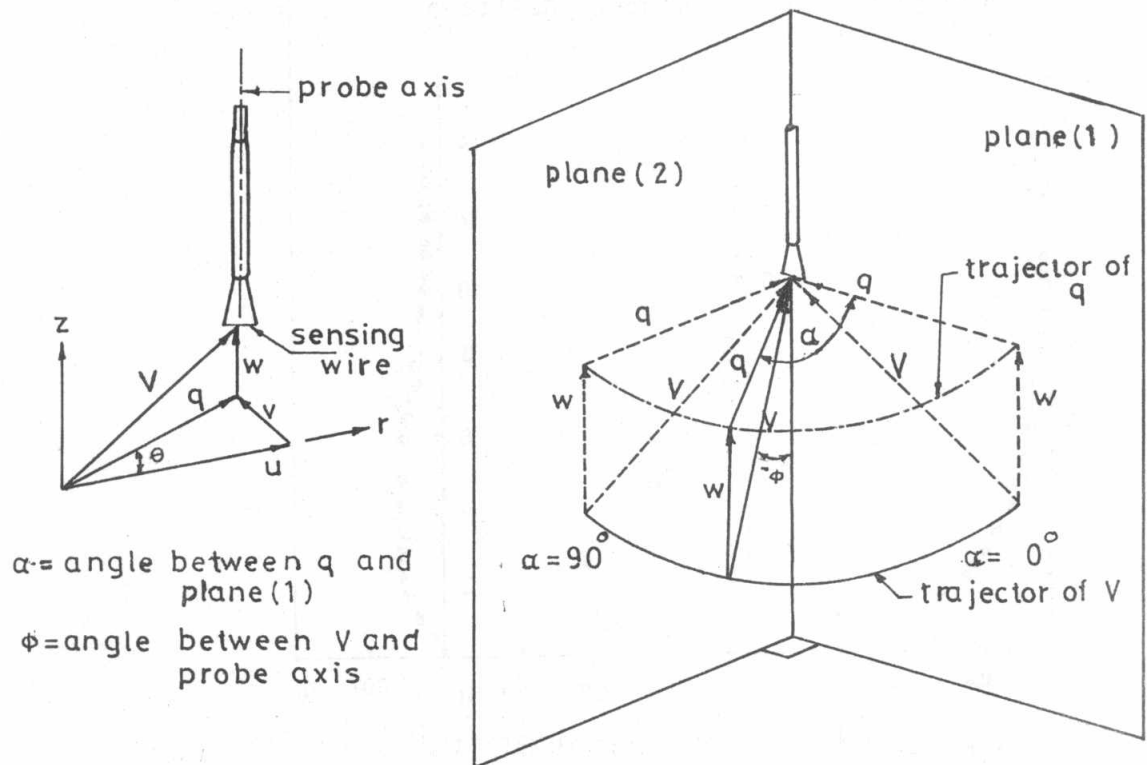Φ = angle between V and probe axis

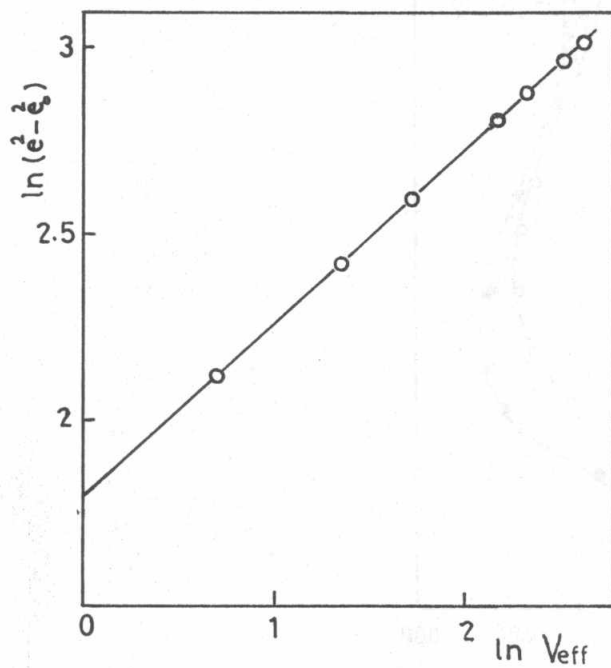Fig.1. Directional characteristics of the hot wire anemometer
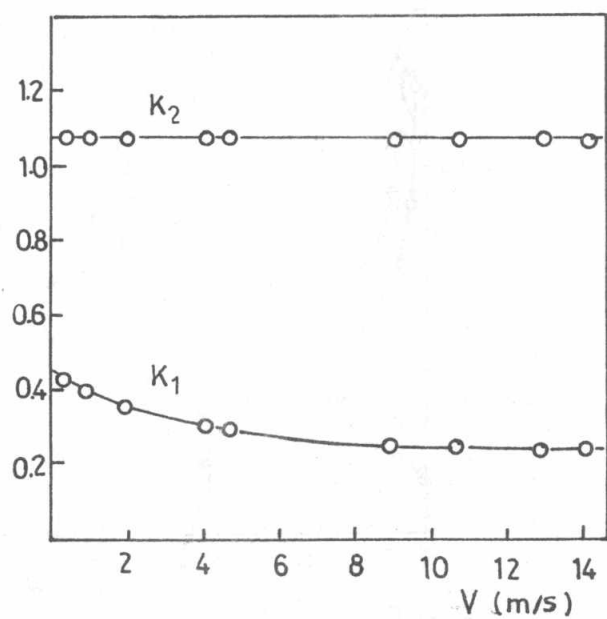

Fig.2. Calibration of hot wire
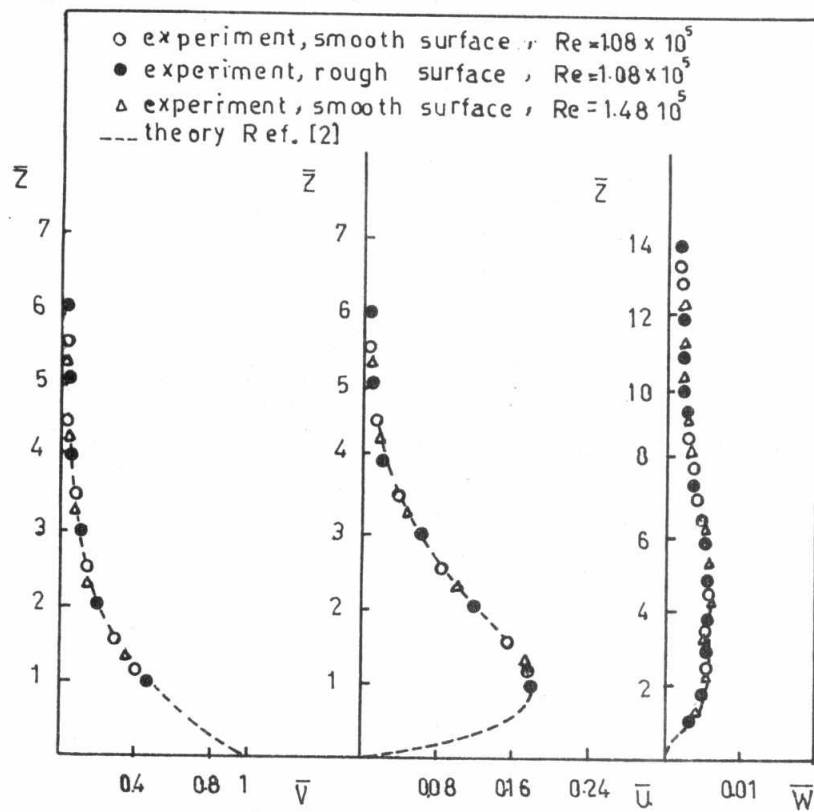

Fig.3. Correction factors, K_1 and K_2

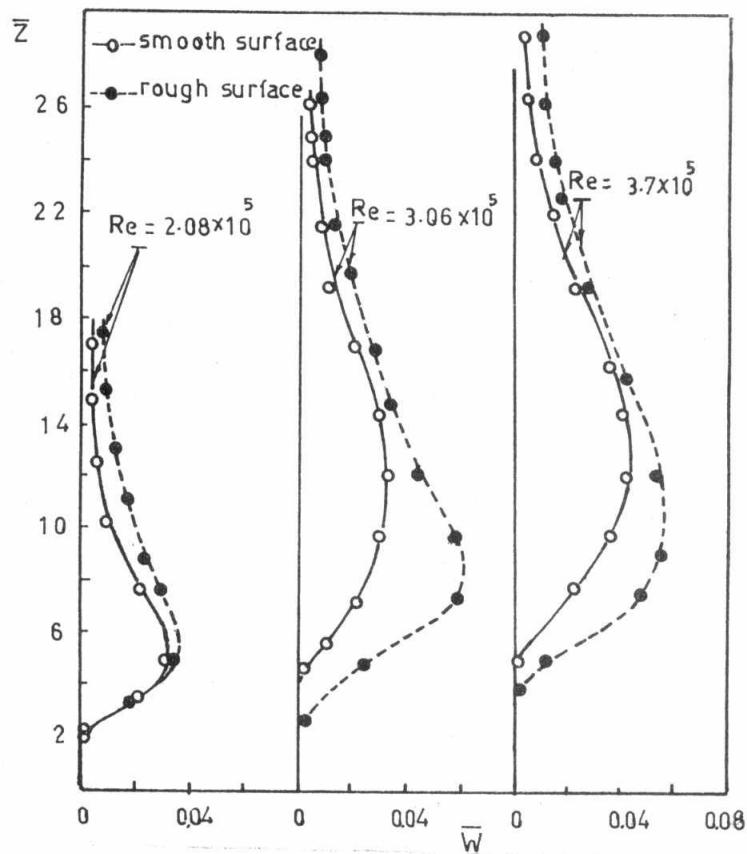Fig. 4. Flow in the laminar region

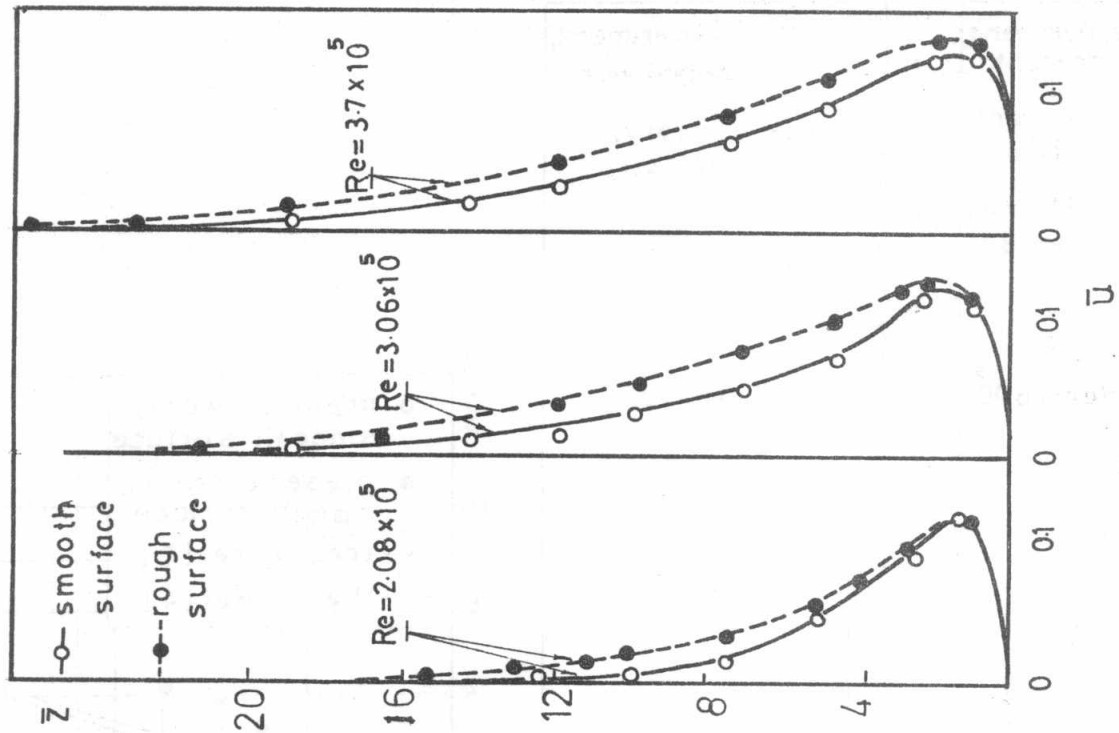Fig. 5. Axial velocity in the transition and turbulent regions

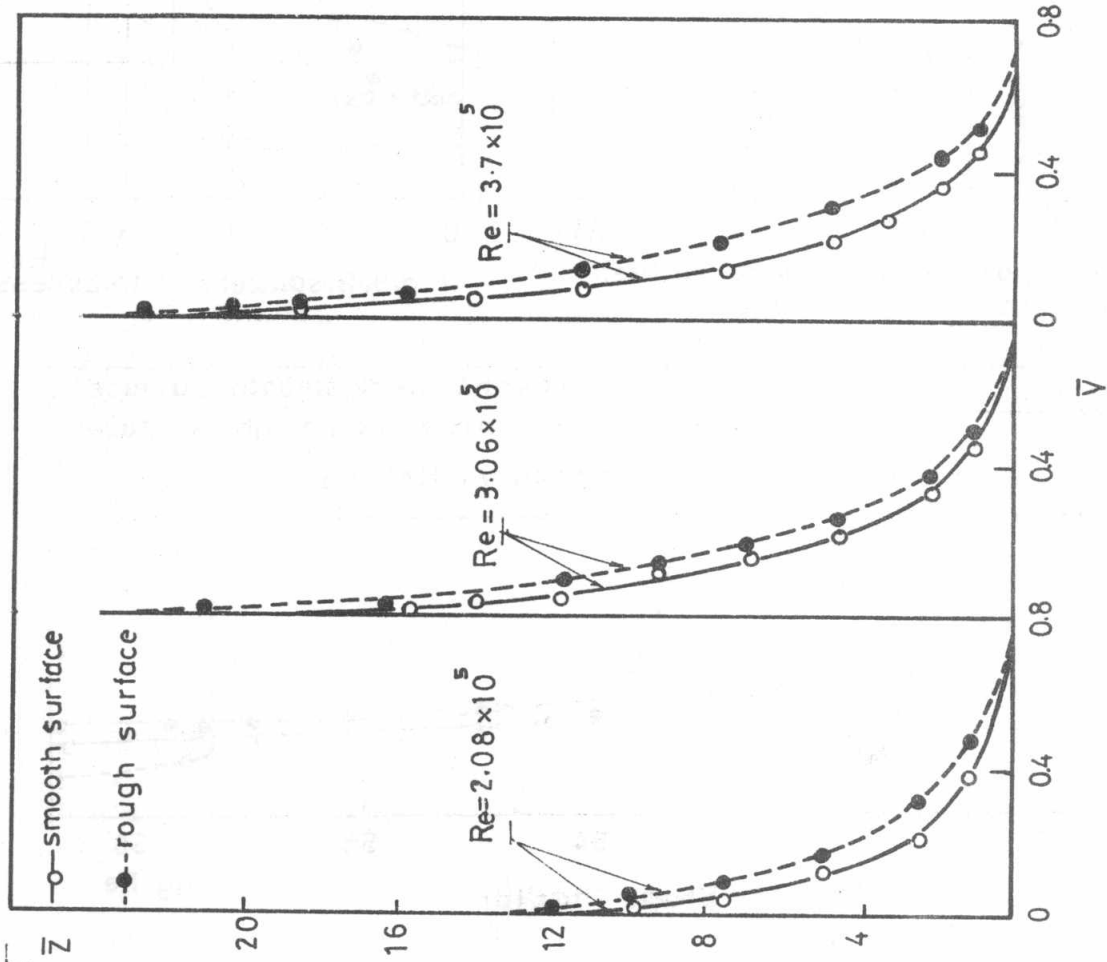Fig. 7. Radial velocity in the transition and turbulent regions

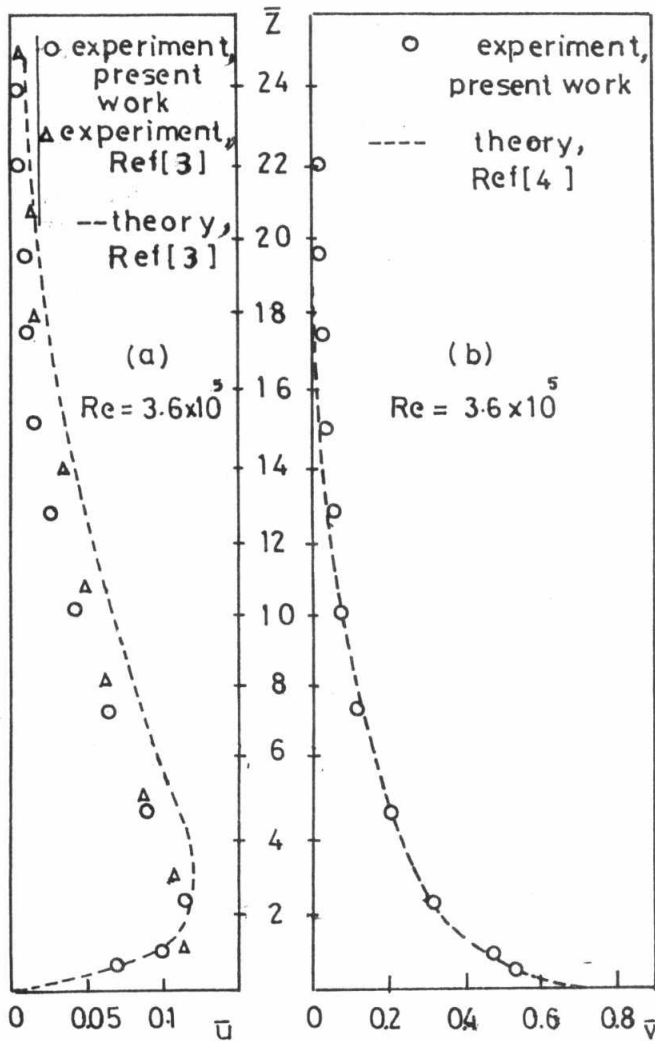Fig. 6. Tangential velocity in the transition and turbulent regions
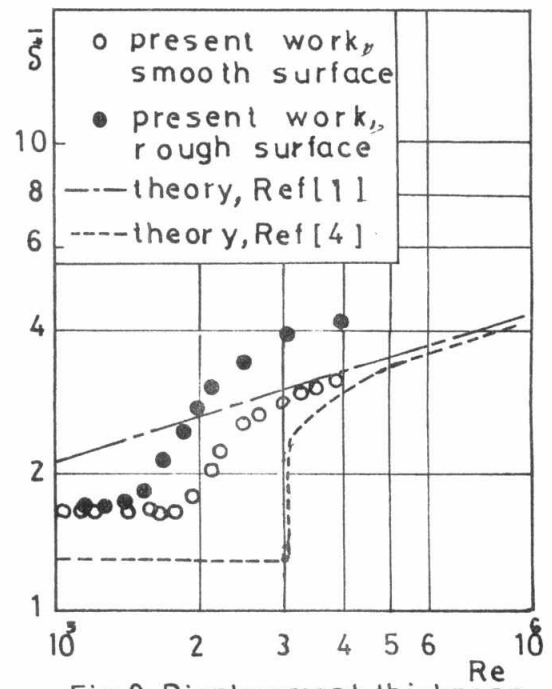
Fig.8, Comparison of velocity profiles with previous works
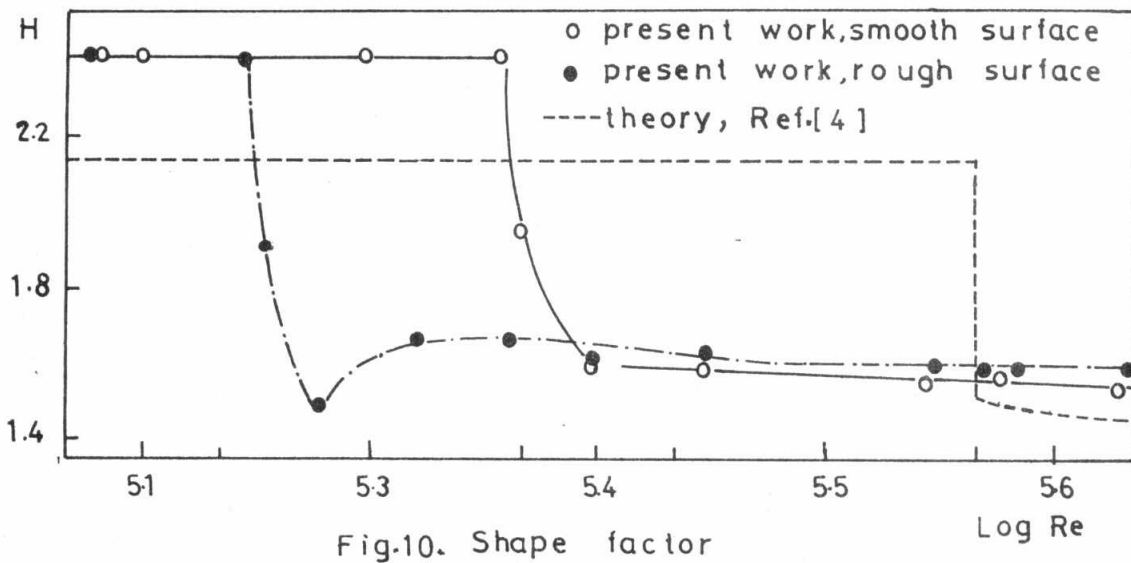

Fig.9. Displacement thickness

Fig.10. Shape factor