1 A PARAMETRIC SOLUTION TO THE MODAL CONTROLLERS

2 DESIGN PROBLEM

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4 ABSTRACT

The main purpose of this paper is to present a method for determining a parametric solution to the problem of modal-control of controllable and observable linear time-invariant multivariable systems via output-feedback. The approach followed is based, at first, on finding a general set of state-feedback modal controllers, then a subset of it is chosen such that it satisfies the necessary and sufficient conditions for the existance of an output-feedback modal-controller . This approach makes use of the controllable canonical form of the given system. A design algorithm is established and it can be implemented with ease. Then, this algorithm is extended to the case of designing modal dynamic controllers with arbitrarily fixed poles. Two numerical examples are provided to illustrate the theoritical aspects.

The paper consists of two main parts:
- the first part describes the design of static modal controllers;
- the second part is related to the design of dynamic modal controllers.

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Part I: STATIC MODAL CONTROLLERS

I.1. Problem Development

We shall start by considering the linear time-invariant multivariable system, which is described by state-space and output-equation of the form

$$\dot{x}(t) = A\,x(t) + B\,u(t) \tag{1.a}$$

$$y(t) = C\,x\,(t) \tag{1.b}$$

where $x(t)$ is an n-dimensional vector of state, $u(t)$ and $y(t)$ represent the (m) inputs and (l) outputs of the system respectively; A,B and C are constant real matrices of appropriate dimensions, B and C are of full rank m and l respectively. It is well known (Wonham [9]) that using constant states-feedback of the form

$$u(t) = P\,x(t) + v(t) \tag{2}$$

where P is mxn state-feedback matrix, $v(t)$ is mxl vector of reference inputs; then if the pair (A,B) is completely controllable, all of the eigenvalues of the closed-loop system

$$\dot{x}(t) = (A+BP)\,x(t)+B\,v(t) \tag{3}$$

can be arbitrarily assigned in the s-plane (subject to complex paring) . For every controllable pair (A,B), and desirable set of closed-loop poles $\{\rho_i\}$, there exists an infinite set of P-matrices achieving this requirement. However, since in most practical situations only the system outputs defined by eqn. (1.b) are available for feedback, we seek an output feedback solution to the modal-control problem, i.e. having an output-feedback control-law of the form

$$u(t) = K\,y(t) + v(t) \tag{4}$$

the closed-loop system

$$\dot{x}(t) = (A+BKC)\,x(t) + B\,v(t) \tag{5}$$

attains the desired eiganvalue pattern, where K is mxl constant output-feedback matrix. The conditions under which this output feedback solution exists are presently unknown in spite of numerious studies. One approach is to derive K when P is given. This suggests the searching for a solution to the matrix equation.

$$K\,C = P \tag{6}$$

i.e. for a prescribed state-feedback matrix P, achieving a desired set of closed-loop system poles, is there a corresponding output-feedback matrix K that solves the same problem?

But it may happen-in many cases-that eqn. (6) can be inconsistent, hence there is no solution to the problem of pole-assignment via output-feedback. In another words, not all P's can yield K, and one faces the difficulty of searching for those P's (if any) which satisfy the problem requirements.

To this end, we can say that this point motivated the searching for a general solution to the modal-control problem using output-feedback. We notice that the matrix P has great number of degrees of freedom, since in order to assign n-poles, we need only to impose n-constraints on P and the remaining n x(m-1) elements can be chosen either arbitrarily or to satisfy other design requirements. Hence, we can utilize this flexibility to find a set of P's which can produce the required K's. Again, a general family of state modal-controllers is obtained by forcing the closed-loop system matrix (A+BP) to be equal to some other matrix H possessing the desired closed-loop eigenvalues with a sufficient number of free elements, thus we can get a sec of P's. A subset of it is chosen to agree with the consistency conditions of eqn. (6). Solving we can derive a general family of K's satisfying the modal-control problem requirements.

I.2. Theoritical Determination of State and Output-Feedback Matrices

I.2.1. Definitions and Fundamental Relations

Let us suppose that the multivariable system (1) is in its controllable canonical form, i.e.

$$A=\begin{bmatrix} A_{11} & A_{12} \cdots & A_{1m} \\ A_{21} & A_{22} \cdots & A_{2m} \\ \vdots & \vdots & \vdots \\ A_{m1} & A_{m2} \cdots & A_{mm} \end{bmatrix} ; \quad B=\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_m \end{bmatrix} \tag{7.a}$$

$$x=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} ; \quad u=\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} \tag{7.b}$$

where

$$A_{ii}=\left[\begin{array}{cccc} 0 & 1 & 0 \ldots & 0 \\ 0 & 0 & 1 \ldots & 0 \\ \vdots & \vdots & \vdots & \\ 0 & 0 & 0 \ldots & 1 \\ \hline -(a_{ii})_o & -(a_{ii})_1 & \ldots & -(a_{ii})_{\sigma_i -1} \end{array}\right] = \left[\begin{array}{c} \bar{A}_{ii} \\ \hline \tilde{A}_{ij} \end{array}\right] \tag{8.a}$$

$$A_{ij} = \left[\begin{array}{c} 0 \\ \hline -(a_{ij})_o - (a_{ij})_1 \ \ldots \ -(a_{ij})_{\sigma_i -1} \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline \tilde{A}_{ij} \end{array}\right] ; \quad i \neq j \tag{8.b}$$

$$B_i = \left[\begin{array}{c} 0 \\ \hline 0 \quad 0 \ \ldots . \ 0 \ \ \underset{\uparrow \, i^{th} \text{ column}}{1} \ \ 0 \ \ldots \ 0 \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline \tilde{B}_i \end{array}\right] \tag{8.c}$$

and σ_i; i=1,2,...,n are the controllability indices of the pair (A,B) such that

$$\sum_{i=1}^{m} \sigma_i = n \tag{9}$$

Now, our problem is to find the set $\hat{P}$ (of matrices P) such that $\forall\, P \in \hat{P}$

$$\det (\rho_i I - A - BP) = 0 \; ; \quad i=1,2,\ldots,n \tag{10,a}$$

or

$$\det (sI-A-BP) = s^n + d_{n-1} s^{n-1} + \ldots + d_1 s + d_o \tag{10.b}$$

where $\{\rho_i\}$ is the set of desired poles, and such that the closed-loop characteristic polynomial

$$\left.\begin{aligned} \phi(s) &= \prod_{i=1}^{n} (s-\rho_i) = s^n + d_{n-1} s^{n-1} + \ldots + d_1 s + d_o \\ &= \det (sI-A-BP) \end{aligned}\right\} \tag{11}$$

Thec closed-loop system matrix has necessarily the same structure as A. Hence, we can write

$$A + BP = H \tag{12}$$

where

$$H_{ii} = \left[\begin{array}{ccccc} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & & & \\ 0 & 0 & 0 & \ldots & 1 \\ \hline -(h_{ii})_o & -(h_{ii})_1 & \ldots & & -(h_{ii})_{\sigma_i -1} \end{array}\right] = \left[\begin{array}{c} \bar{H}_{ii} \\ \hline \tilde{H}_{ii} \end{array}\right] \tag{13.a}$$

$$H_{ij} = \left[\begin{array}{cccc} & & 0 & \\ \hline -(h_{ij})_o & -(h_{ij})_1 & \ldots & -(h_{ij})_{\sigma_j -1} \end{array}\right] = \left[\begin{array}{c} \bar{H}_{ij} \\ \hline \tilde{H}_{ij} \end{array}\right] \; ; \; i \neq j \tag{13.b}$$

In view of this cobservation, the problem of modal-control (pole assignment) can be stated as follows:

Given that $\hat{H}$ is the family of all nxn matrices having the form (13), and satisfying the condition

$$\forall\, H \in \hat{H} \quad \det (sI-H) = s^n + d_{n-1} s^{n-1} + \ldots + d_1 s + d_o \tag{14}$$

find the set of matrices P such that eqn. (12) holds. Let us introduce the selection matrix S_ν (of diemension mxn) such that each column s_j, j=1,2,...,n is defined by

$$s_{\nu_k} = e_k \tag{15}$$

where

$$\nu_k = \sum_{i=1}^{k} \sigma_i \tag{16}$$

and e_k is the k^{th} column of the identity matrix $I_{m,m}$; and $s_j=0$ otherwise. It results that

$$S_\nu B = I_{m,m} \tag{17}$$

and

$$S_\nu(A+BP)=S_\nu A+P = S_\nu H \tag{18}$$

If we define

$$\tilde{A} = \left[\tilde{A}_{ij}\right] \; ; \quad \tilde{H} = \left[\tilde{H}_{ij}\right] \tag{19.a}$$

then

$$\tilde{A} = S_\nu A \quad ; \quad \tilde{H} = S_\nu H \tag{19.b}$$

we now get

$$P = S_\nu(H-A) = \tilde{H} - \tilde{A} \tag{20}$$

We can notice that eqn. (20) gives the set $\hat{P}$ as parameterized in the elements of H, and the constraints (14) imposed on $H \varepsilon \hat{H}$ are translated by the intermediate of eqn. (20) - to those constraints on $P \varepsilon \hat{P}$. We impose other constraints on $P\varepsilon \hat{P}$, such that the conditions of consistence of eqn.(6) will be satisfied. A necessary and sufficient condition for the consistency of (6) is that the relation

$$P \bar{C} = 0_{m,n}$$

will be valid, where $\bar{C}$ is any matrix whose columns generate the kernel of C:

$$\operatorname{Im} \bar{C} = \operatorname{Ker} C$$

We can take a selection of independent columns of $(I-C^+C)$ - where C^+ is the pseudo inverse of C- or more simply we choose the matrix $\bar{C}$ proposed by Seraji [7] :

$$P \bar{C} = 0_{m\mathrm{x}(n-1)} \tag{21}$$

It is clear that equation (21) represents a set of linear constraints on the elements of P, (which we can consider as a great advantage of the method introduced in this paper). Now, the matrix $P \varepsilon \hat{P}$, given by (20), must satisfy the following two equations:

$$\det (sI-A-BP)= \phi(s) \tag{11}$$

and

$$P \bar{C} = 0_{m\mathrm{x}(n-1)} \tag{21}$$

We can notice that, after the assignment of n-poles of the compensated system, we still have N_1-elements of the matrix P which are not specified, where

$$N_1 = nx\ (m-1) \tag{22.a}$$

and the number of equations in the set (21) is

$$N_2 = m\ x(n-1) \tag{22.b}$$

Thus, we can conclude that, in order to have a solution for almost all cases, the following condition must be satisfied

i.e. $N_1 \geq N_2$

$$m\ x\ 1 \geq n \tag{22.c}$$

I.2.2. Determination of the Output Modal Controller

We shall study how to search for the solution of the set of equations (11) and (21).

We can find a solution of (21) parameterized in certain number of elements of matrix P, called " free elements". Let us introduce the vector η which is composed of these free elements , then we write for the solution of (21)

$$P = P_1(\eta)$$

such that

$$\forall \eta : P_1(\eta)\ \bar{C} = 0.$$

The matrix $(A+BP_1)$ of the closed-loop system is deduced from the matrix A by adding to the row ν_k the row k of the matrix P_1, where k=1,2,..,m and ν_k is defined by eqn. (16). P_1 must also satisfy the relation

$$\det\ (sI-A-BP_1) = \phi(s) \tag{23}$$

We can notice that eqn. (23) represents a set of multilinear equations function of η , from which we can get a solution P_2 of (23).

Then, the general expression of the matrix K, of output feedback, can be found from the following equation

$$K\ C = P_2 \tag{24.a}$$

This matrix equation is now consistent, thus we can solve it to get the set $\hat{K}$ of matrices K:

$$K = P_2\ C'\ (CC')^{-1} \tag{24.b}$$

where C' is the transpose of C.

Remarks

1- In some cases, we are not able to find a solution of eqn. (23), corresponding to a given set $\{\rho_i\}$of closed-loop system poles, and the specified problem has no solution. In this case we can try to overcome this special situation by slight modification of the required set of poles. However, this trial does not necessarily lead to a solution.

2- In general, eqn. (23) has infinite solutions. If it is possible to get a parametric set of solutions, this gives additional flexibility, that can be used to satisfy other design requirements.

I.3. Modal-Controller Design Algorithm

The abovementioned procedure, of designing a general set of output-feedback modal-controllers, can be summerized in the following steps

1- The system under control is assumed to be in its input-phase-canonical form. If not, it is transfered to this form via any of the known methods (e.g. Luenberger [3]).

2- Find the solution P_2 using the method mentioned in the previous section.

3. The general set $\hat{K}$ can be obtained using eqn. (24.a). There are several techniques developed to solve similar matrix equations (e.g. Porter [6] , Seraji [7]) , or we can use eqn. (24.b).

NOTE

In the case of single-input system, the required matrix P, corresponding to a given set of poles, is unique. Hence, it is impossible to find the corresponding matrix K if eqn. (6) is inconsistent.

I.4. Illustrative Example

We shall use the same example of Paraskevopulos [5] (1976 b) to explain the suggested procedure. Consider the system.

$$\dot{x} = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 4 & 2 & 3 \\ \hline 1 & 0 & 1 \end{array}\right] x + \left[\begin{array}{cc} 0 & 0 \\ 1 & 0 \\ \hline 0 & 1 \end{array}\right] u \tag{25.a}$$

$$y = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} x \tag{25.b}$$

Take

$$H = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ \alpha & \beta & \gamma \\ \hline \delta & \varepsilon & \xi \end{array}\right]$$

from eqn. (20) we get

$$P = \begin{bmatrix} \alpha-4 & \beta-2 & \gamma-3 \\ \delta-1 & \varepsilon & \xi-1 \end{bmatrix} \tag{26}$$

we can find $\bar{C}$, eqn. (21), as

$$\bar{C} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \tag{27}$$

using eqn.(21), we find

$$\left.\begin{aligned} \alpha &= 1+\gamma \\ \delta &= \xi \end{aligned}\right\} \tag{28}$$

It results from eqn.(23) that

$$\det (sI-A-BP_1) = s^3-(\beta+\xi)\ s^2+(\beta\xi-\gamma\varepsilon-\gamma-1)\ s+\xi \tag{29}$$

Let the set of closed-loop system poles be

$$\{-1;\ -2;\ -3\}$$

then,

$$\phi(s) = s^3+6s^2+11\ s+6 \tag{30}$$

From (29) and (30) we obtain the followings set of equations

$$\left.\begin{aligned} \beta+\xi &= -6 \\ \xi\beta-\gamma\varepsilon-\gamma &= 12 \\ \xi &= 6 \end{aligned}\right\} \tag{31}$$

which has the solution

$$\left.\begin{aligned} \beta &= -12 \\ \xi &= 6 \\ \varepsilon &= \frac{1}{\gamma}(-84-\gamma) \\ \gamma &= \text{arbitrary}\ ;\ \gamma \neq 0 \end{aligned}\right\} \tag{32}$$

Proceeding as explained in section (I.3) , we obtain the required set of matrices K

$$K = \begin{bmatrix} -14 & \gamma-3 \\ -\frac{1}{\gamma}(+84) & -5 \end{bmatrix}\ ;\ \gamma\neq 0 \tag{33}$$

We can notice the advantage and the simplicity of the introduced procedure in comparison with the method used by Paraskevopulos [5] (1976 b).

I.5. Concluding Remarks

We can notice that, the particular choice of the closed-loop plant matrix (A+BP) to be equal to another matrix H possessing the desired poles of the closed-loop system, simplifies the procedure and reduces the computational effort for the design of modal controllers. On the other hand, this particular choice (which can be considered as a similarity transformation with the special case of the transformation matrix to be the identity matrix) may lead to a less general solution, or in some singular cases no solution will exist.

Paraskevopulos and Tzafestas [4] developed a similar approach to that introduced in this paper, but in their algorithm the closed-loop system matrix is put into its diagonal or Jordan form. These forms have the following computational difficulties

a) calculation in the complex field when certain desired closed-loop poles are complex:

b) the non-uniqueness of the choice of Jordan form for multiple poles.

In addition, in order to search for the set $\hat{K}$, non-linear constraints are imposed on some or all the free elements of the matrix P.

Paraskevopoulos [5] (1976 c) had introduced another procedure for determining the general set $\hat{K}$ directly without passing through the determination of the general set $\hat{P}$. However, this procedure does not appear to be computationally attractive, as it requires long calculations and the solution of high number of equations. But on the other hand, it eliminates the non-linearity in determining the set $\hat{K}$ of matrices of output feedback.

It is worthy to notice that our algorithm is simpler and provides more insight of the problem. Although the approach developed here has been essentially algorithmic, it has yielded some useful theoritical results.

Part II: Dynamic Modal Controllers

II.1. Introduction

In spite of the simplicity and practical realizability of the application of static modal controllers, sometimes the requirement for dynamic modal controllers may prove to be inevitable. Great control over the closed-loop poles may be achieved using feedback through a dynamic controller as it provides more parameters than the static one. On the other hand, the introduction of a dynamic controller in the control loop increases the system order, and hence adds more poles. Many design techniques concentrate on the augmented system as a whole and overlook the stability properties of the controller as a distinct and separate dynamic system. In some cases, it would be desirable to concentrate not only on the overall closed-loop system, but also on the dynamic controller as a separate system.

In this part, we shall extend the design method of part (I) to get a dynamic modal controller -for pole assignment of the composite closed loop system-with some free parameters which can be used to assign its poles in prespecified locations in the complex plane. This approach is based on the fact that this modal control problem can be reduced to the design of an equivalent static modal controller for the augmented system.

II.2. Formulation of the problem

Having the system described by eqn. (1), the problem is to design a p-th order dynamic controller of the form

$$\dot{z}(t) = D\, z(t) + E\, y(t) \qquad (34.a)$$

$$w(t) = F\, z(t) + G\, y(t) \qquad (34.b)$$

where z is the p-dimentional controller state vector, D,E,F and G are appropriately dimentioned constant matrices. Let the system control input be such that

$$u(t) = w(t) + u_c \tag{35}$$

where u_c is a constant mxl vector of reference inputs. In view of equations (1), (34) and (35) the closed-loop system may be described by the composite state equation

$$\dot{x}(t) = A\,x(t) + B\left[F\,z(t) + G\,y(t) + u_c\right]$$

or

$$\dot{x}(t) = (A + BGC)\,x(t) + BF\,z(t) + B\,u_c \tag{36}$$

and

$$\dot{z}(t) = EC\,x(t) + D\,z(t) \tag{37}$$

Having the system (34) put in the control loop of system (1), the resultant system will have dimension (n+p) and it is defined by the following state equation

$$\begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} A+BGC & BF \\ EC & D \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_c \tag{38}$$

The closed-loop plant matrix can be written as

$$A_c = \begin{bmatrix} A+BGC & BF \\ EC & D \end{bmatrix} \tag{39}$$

or following the idea of Johnson and Athans [2]

$$A_c = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B & 0 \\ 0 & I_p \end{bmatrix} \begin{bmatrix} G & F \\ E & D \end{bmatrix} \begin{bmatrix} C & 0 \\ 0 & I_p \end{bmatrix} \tag{40}$$

Introducing the following notations

$$A^* = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad (n+p)x(n+p) \quad \text{plant-matrix} \tag{41.a}$$

$$B^* = \begin{bmatrix} B & 0 \\ 0 & I_p \end{bmatrix}, \quad (n+p)\ x(m+p) \quad \text{input-matrix} \tag{41.b}$$

$$C^* = \begin{bmatrix} C & 0 \\ 0 & I_p \end{bmatrix}, \quad (1+p)\ x(n+p) \quad \text{output-matrix} \tag{41.c}$$

$$K^* = \begin{bmatrix} G & F \\ E & D \end{bmatrix}, \quad (m+p)\ x\ (1+p) \quad \text{feedback-matrix} \tag{41.d}$$

Then, the closed-loop plant matrix is expressed as

$$A_c = A^* + B^* K^* C^* \tag{42}$$

The problem now is to find the equivalent constant output-feedback controller matrix **K*such** that the closed-loop plant matrix A_c will have (n+p) preassinged eigenvalues, and at the same time, the controller matrix D will have (p) prespecified eigenvalues . Based on the fact that the number of independent parameters in the controller must be at least equal to the number of the augmented-system poles (n+p)-necessary condition for complete pole assignment-Sirisena and Choi [8] have found an expression for the lower bound on the controller order, which is given by

$$p \geq \frac{n-m\times l}{l+m-1} \tag{43}$$

Choosing the controller order to be the smallest non-negative integer satisfying the inequality (43), then we can apply the algorithm of sec. I.3 , where the free parameters can be adjusted to satisfy the aim of control. Then, the controller set of matrices (D,E,F and G) can be obtained by appropriate partitioning of the matrix K* in accordance with equation (41.d).

II.3. Illustrative Example

Consider the following controllable and observable system

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t) \tag{44.a}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} x(t) \tag{44.b}$$

This system was studied by Ahmari [1] to design a first order compensator for assigning all four closed-loop poles at

$$\lambda_{1,2} = -1 \pm j \quad ; \quad \lambda_3 = -1 \quad ; \quad \lambda_4 = -2$$

and the compensator pole was chosen to be at $s_1 = -1$. The developed method in this paper will be applied to solve this example as follows: the set of the overall closed-loop system matrices are formed using eqn. (41) as

$$A^* = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B^* = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$C^* = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{45}$$

In order to apply the above mentioned algorithm we first put the composite system in its phase-canonical form:

$$\left.\begin{aligned} A^c &= S^{-1} A^* S \\ B^c &= S^{-1} B^* \\ C^c &= C^* S \end{aligned}\right\} \tag{46.a}$$

with the transformation matrix S found to be

$$S = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{46.b}$$

the following canonical form can be found

$$A^c = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \quad ; B^c = \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ \hline 0 & 1 \end{array}\right] \quad ; \; C^c = \begin{bmatrix} -1 & -1 & 2 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{47}$$

Following the above mentioned algorithm, the general expression for K^* can be found

$$K^* = \begin{bmatrix} -(\alpha-6) & \frac{1}{2}(\alpha-17) & \beta \\ \frac{-1}{\beta}(\alpha^2-6\alpha-4) & \frac{1}{2\beta}(\alpha^2-18\alpha-13) & \alpha \end{bmatrix} \tag{48}$$

with α and β arbitrary, with the only condition $\beta \neq 0$. From eqn. (41.d) we can get

$$D = [\alpha] \quad ; \quad E = \left[\frac{-1}{\beta}(\alpha^2-6\alpha-4) \quad \frac{1}{2\beta}(\alpha^2-18\alpha-13) \right]$$

$$F = [\beta] \quad ; \quad G = \left[-(\alpha-6) \quad \frac{1}{2}(\alpha-17) \right] \tag{49}$$

As we have recommended the compensator pole to be at $s_1=-1$, thus $\alpha=-1$. Substituting we get

$$D = [-1] \quad ; \quad E = \left[\frac{-3}{\beta} \quad \frac{3}{\beta} \right] \; ;$$

$$F = [\beta] \quad ; \quad G = [7 \quad -9] \tag{50}$$

The transfer function of the compensator is given by

$$W_c(s) = F\,(sI_p-D)^{-1}\,E + G \tag{51}$$

Substituting we get

$$W_c(s) = \frac{1}{s+1}\,[4+7s \qquad -6-9s] \tag{52}$$

This result of $W_c(s)$ is identical with that obtained by Ahmary [1] using a different method. The simplicity of the calculations used here-mainly no calculations are done in the complex field-represents a great advantage of our procedure over that one developed by Ahmary [1].

<u>Note:</u>

We can notice from the expression (52) that $W_c(s)$ is independent of (β). This is evident due to the fact that the compensator state vector (z) is arbitrary to within a non-singular linear transformation. It can be verified easily that if (D,E,F,G) constitute a solution to the controller problem, so does the 4-tuple $(TDT^{-1}, TE, FT^{-1}, G)$; where T is any non-singular pxp matrix of transformation. In the present example, where we have a controller of order p=1, the matrix T reduces to the scalar nonzero parameter β.

II.4. Concluding Remarks

The problem of pole placement in linear multivariable systems using dynamic output feedback was considered for the case where the poles of the controller are arbitrarily fixed. Providing that the order of the controller satisfies its lower bound condition, eqn. (43), complete pole placement of the composite system can be achieved. The problem is reduced to the design of an equivalent static output feedback modal controller. This fact was first introduced by Johnson and Athans [2]and have since been used by a number of other investigators (e.g. Sirisena and Choi [8]).

Our established procedure of design is computationally simple and it avoids most of the complications and disadvantages of the existing methods in the field .

OVERALL CONCLUSION

In this paper we discussed the modal-controllers design problem via the available outputs of the system, and thus directly accounts for the unattainability of all the state variables. This approach is well suited to practical control problems as it dispenses with the reconstruction and feedback of the system states. Depending upon the complexity of the required output controller, we investigated the two types of modal output controllers: static and dynamic.

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