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HYDRAULIC ACTUATORS STEP RESPONSE CONSIDERING FLUID LINES INERTIAL EFFECTS

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ABSTRACT

A convenient simple model for a high velocity fluid line, connecting a hydraulic actuator to a 4/3 zero lapped valve, is presented. The conditions under which the proposed model is valid, are deduced. The system mathematical model, in case of constant supply pressure operation and pure inertial external load, shows that the actuator response to step opening of the valve depends on two parameters; the system dimensionless inertial resistance and its dimensionless capacitance. The former equals the ratio between the fluid line inertial resistance and that of the load, while the latter accounts for the system capacitance, valve resistance, load inertial resistance, and the supply pressure. Numerical solution of the system governing equations verifies that at small values of the dimensionless capacitance, cavitation occurs at certain values of the dimensionless inertial resistance, a phenomenon that can not be predicted when the fluid line inertial resistance is ignored. At small dimensionless capacitances, the fluid line inertial resistance is shown to have considerable effect on the transient response, and can be made use of to improve the system performance. The combination of dimensionless capacitance and inertial resistance, at which minimum settling time and reasonable values of overshoot and delay and rise times occur, is determined.

INTRODUCTION

Modern developments in materials and sealing technology allowed using high pressures in hydraulic control systems. Pressures up to 40 MPa are now frequently used in order to reduce the required flow rate for a certain power transmitted. With smaller flow rates, systems are more compact, and are of less weight and initial costs.

At high pressures, flow velocities in the connecting pipes can be increased, since the resulting higher pressure losses within the pipes can be tolerated. A 12 m/s flow velocity is now allowed in many cases, a value that is about four times the maximum permissible value in the past. With small flow rates and high flow velocity, the connecting pipes cross sectional areas are considerably reduced. A small diameter fluid line has high inertial resistance and small compliance.

Most of the previous investigations of valve-controlled hydraulic actuators

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dynamics [1 - 7] dealt with systems in which the fluid lines are of large diameters. In these cases, the fluid lines inertial resistances are negligibly small and the lines compressibility effects can be accounted for by adding the volume of the fluid line to the volume of the actuator cavity connected to it. Arafa and Kassem [8] verified that the resulting model, in this case, would not yield accurate prediction of the system performance when the fluid lines inertial resistances are high. They derived the set of equations governing the system dynamics and reduced it to a linear form. Solving the obtained set of linearized equations, using Laplace transformation, they showed that the fluid lines inertial resistances might improve the system step response. They showed also that the fluid lines frictional resistances are of minor effect on the response, since their values in practice are much smaller than those of the control valve. Results obtained in [8] are qualitative since the analysis is based on a linearized model and is made for a system with certain physical parameters.

In this paper, a generalized analysis for the response of hydraulic actuators to step openings of the control valves is carried out when both of the fluid line inertial resistance and the compressibility of fluid filling the actuator are considered.

SYSTEM MODELS

The investigated system is shown in Fig. 1 . It consists of a rotary actuator, of geometric volume v_o and fluid filling volume v_a , which drives a load of moment of inertia I. The actuator is connected to a 4/3 zero-lapped control valve by means of two symmetrical fluid lines, each of length l_p and cross sectional area a_p .

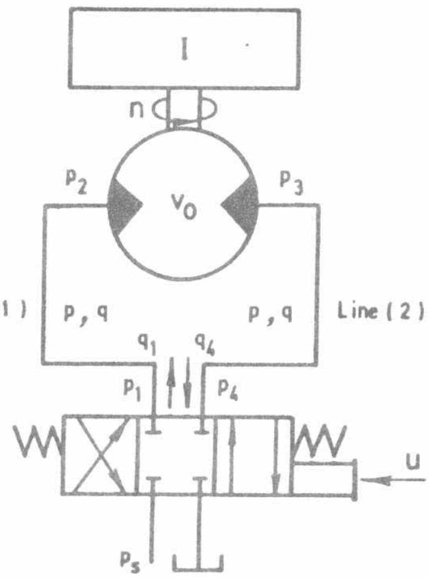

Fig. 1 Inertially Loaded Valve-controlled Actuator

The proper treatment of the fluid line is to consider it as a system with distributed parameters. In this case, the system shown above can be represented by the circuit shown in Fig. 2 , where each line is represented by an infinite number of infinitesimal capacitances and inertial resistances; dc and dh respectively.

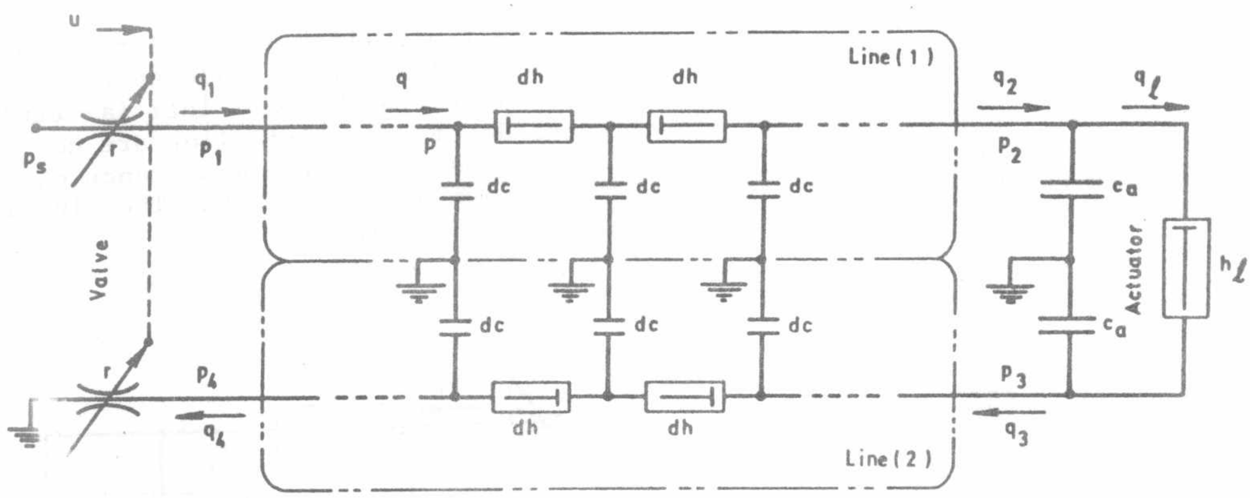

Fig. 2 System Equivalent Circuit

The equations governing this circuit are :

$$p_s - p_1 = rq_1^2 \qquad (1)$$

where r is the valve resistance which equals $0.5\,\rho/(C_d^2 b^2 u^2)$.

$$q_2 - q_\ell = c_a\, dp_2/dt \qquad (2)$$

where c_a is the capacitance of one actuator cavity, and equals $0.5v_a/\beta$.

$$p_2 - p_3 = h_\ell\, dq_\ell/dt \qquad (3)$$

where h_ℓ is the load inertial resistance which equals $4\pi^2 I/v_o^2$, and $q_\ell = v_o n$ [9] .

$$q_\ell - q_3 = c_a\, dp_3/dt \qquad (4)$$

$$p_4 = r\, q_4^2 \qquad (5)$$

$$\partial^2 p/\partial t^2 = (\beta/\rho)\, \partial^2 p/\partial x^2 \qquad (6)$$

$$\partial^2 q/\partial t^2 = (\beta/\rho)\, \partial^2 q/\partial x^2 \qquad (7)$$

The last two equations are combined with the boundary conditions :

$q(0,t) = q_1$, $p(0,t) = p_1$, $q(1_p ,t) = q_2$, and $p(1_p ,t) = p_2$

when dealing with line (1).

Equations (6) and (7) also govern the variation of pressure and flow rate along the other line; i.e. line (2). The boundary conditions in this case are $q(0,t)=q_3$, $p(0,t)=p_3$, $q(1_p,t)=q_4$, and $p(1_p,t)=p_4$.

It is worthy noting that if the rotary actuator is replaced by an equal area linear actuator; i.e. a synchronizing cylinder, the foregoing equivalent circuit and governing equations still hold. The parameters of the circuit in this case are the same, except that $h_\ell = m/a^2$ and $q_\ell = aw$, where m is the load mass, a is the cylinder effective cross sectional area , and w is the load speed.

For any input signal u(t), the system response can be obtained by solving the foregoing set of equations with the relevant initial conditions. However, the solution of this set is quite tedious, if ever possible, since it contains partial differential equations beside the nonlinear and ordinary differential equations.

The system model can be simplified when the fluid lines inertial resistances are very small. In this case, the infinitesimal inertial resistances dh are either ignored, or summed up together and added to the load inertial resistance h_ℓ , to yield h_t . The capacitances dc of each fluid line are now connected in parallel, together with the actuator cavity capacitance c_a , and their equivalent capacitance c_t is the sum of all of them. The circuit representing the system in this case is shown in Fig. 3 .

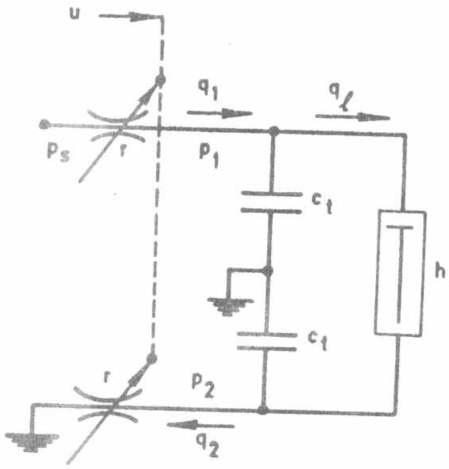

Fig. 3 Equivalent Circuit with Fluid Lines Inertial Resistances Added to the Load

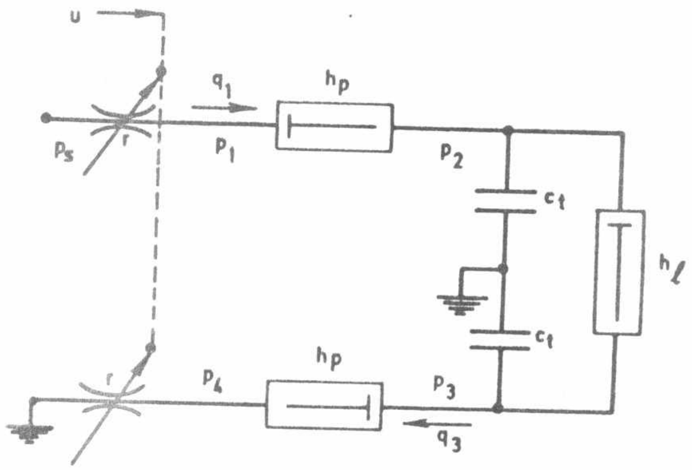

Fig. 4 Equivalent Circuit for Systems with Fluid Lines of High Inertial Resistance and Small Compressibility

The values of c_t and h_t are given by

$$c_t = c_a + c_p = c_a + l_p a_p / \beta$$

$$h_t = h_\ell + 2h_p = h_\ell + 2\rho\, l_p / a_p$$

When the fluid line inertial resistance is of considerable value while its capacitance is small, the case prevailing for a high velocity short fluid line, the line infinitesimal capacitances can be neglected, or summed up together with the actuator cavity capacitance, to yield the system equivalent circuit shown in Fig. 4 .

The equations describing the circuit shown in Fig. 4 are eqs.(1) to (5), with c_a replaced by c_t in eqs. (2) and (4), beside the following equations,

$$q_1 = q_2 \ , \ q_3 = q_4 \ , \ p_1 - p_2 = h_p \, dq_1/dt \ , \text{ and } p_3 - p_4 = h_p \, dq_4/dt \ .$$

This set of nonlinear ordinary differential equations can be solved numerically to predict the system response to the various input signals.

For the analysis of the system response to step opening of the control valve to be general, the set of the governing equations is reduced to a dimensionless form. It is to be noted that the valve resistance decreases suddenly from infinity to a constant value r_o which equals $0.5\rho/(C_d b u_o)^2$, when the valve is suddenly opened u_o . Using the reference quantities,

$$p_r = p_s \ , \ q_r = (0.5 p_s / r_o)^{0.5} \ , \ h_r = h_\ell \ , \ c_r = h_r/(2 r_o p_r) \ , \text{ and } t_r = h_r q_r / p_r \ ,$$

it can be shown that the governing equations, in a dimensionless form, are:

$Q_1^2 = 2(1 - P_1)$, $Q_1 - Q_\ell = C\ dP_2/dT$, $P_2 - P_3 = dQ_\ell/dT$, $Q_3^2 = 2P_4$,

$Q_\ell - Q_3 = C\ dP_3/dT$, $P_1 - P_2 = H\ dQ_1/dT$, and $P_3 - P_4 = H\ dQ_3/dT$

where $P_{1,2,...} = p_{1,2,...}/p_r$, $Q_{1,2,...} = q_{1,2,...}/q_r$, $T = t/t_r$,
$C = c_t/c_r$, $H = h_p/h_r$.

SYSTEM RESPONSE

Solving the foregoing set of dimensionless equations, the variation of the load flow rate with time can be obtained. These equations show that the response depends on the two dimensionless parameters C and H. The first parameter C depends on the system capacitance c_t , valve resistance r_o , load inertial resistance h_ℓ , and the supply pressure p_s . It increases linearly with the increase of either c_t , r_o , or p_s , while it is inversely proportional to h_ℓ . The second dimensionless parameter H equals the ratio between the inertial resistances of the fluid line and the load.

The effect of these two parameters on the system response is shown by solving the set of governing equations numerically, for different combinations of C and H. The initial conditions are $Q_1(0) = Q_\ell(0) = Q_3(0) = 0$, $P_1(0) = 1$, $P_2(0) = P_3(0) = 0.5$, and $P_4(0) = 0$. These are the relevant initial conditions for this case, as shown in [8] . A sample of the obtained results is shown in Fig. 5 , which shows that the load flow exhibits some oscillations during a transient period, before reaching the steady state constant value.

When H is zero, which means that the fluid line inertial resistance is either neglected or added to the load , the different pressures in the system were found not to attain zero during the transient or steady states, whatever the value of C is. At the values of H other than zero, and at certain values of C, the pressure P_2 was found to attain zero during the transient period, indicating occurrence of cavitation. The combinations of C and H at which cavitation occurs, as detected from the computer runs, are shown in Fig. 6 . Occurrence of cavitation at these combinations is explained as follows. The inertial resistance of the fluid line connecting the actuator to the supply pressure line via the control valve, is expacted to limit the flow rate that can reach the actuater, at some instances during the transient period. During these periods of limited supply flow to the actuator, the supply flow rate might be less than the load flow rate. The pressure P_2 consequently decreases due to the actuator cavity capacitance. At those combinations of C and H at which cavitation occurs, the difference between the load flow and the actuator supply flow is high, and lasts a period enough to let P_2 drop to zero or negative values. Cavitation was not detected when H is assumed zero, and this can be referred to that the drop of the pressure P_2 causes an immediate higher pressure difference across the control valve, which causes an instantaneous increase in the flow rate passing through the valve and reaching the actuator cavity. Thus the increase of the load flow than the actuator supply flow gets smaller and lasts a shorter period such that cavitation is eleminated.

The effect of C and H on the system response is shown in terms of the transient response specifications. Figure 7 depicts the variation of the maximum percentage overshoot with C and H. The lower limit of C at each value of H is determined from the cavitation limit. The maximum percentage

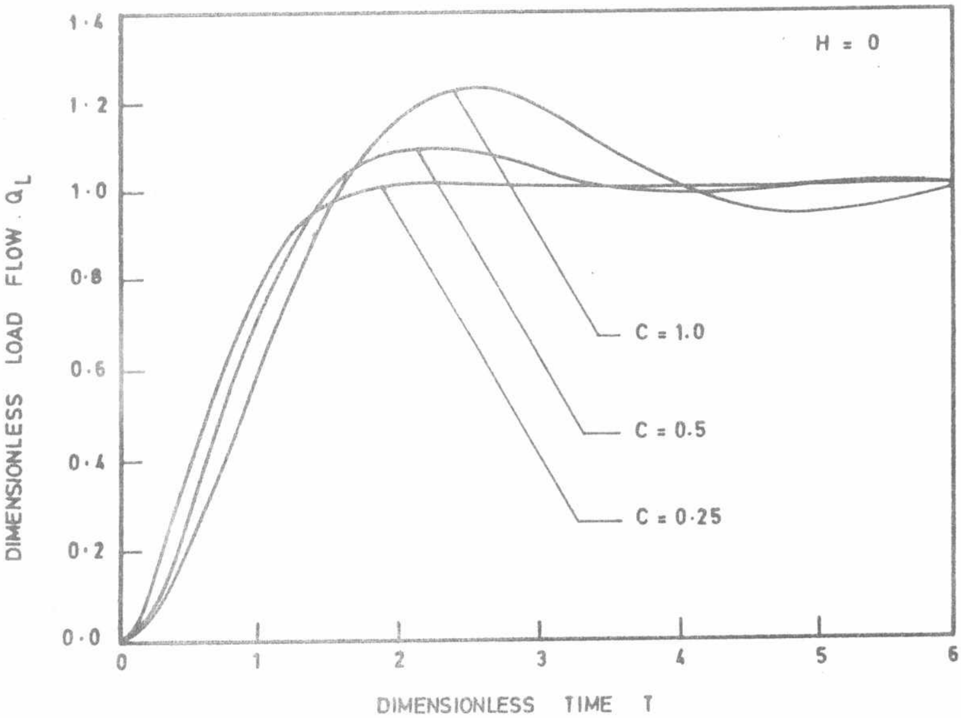

Fig. 5 Variation of Dimensionless Load Flow with Dimensionless Time at Different Values of the Dimensionless Capacitance

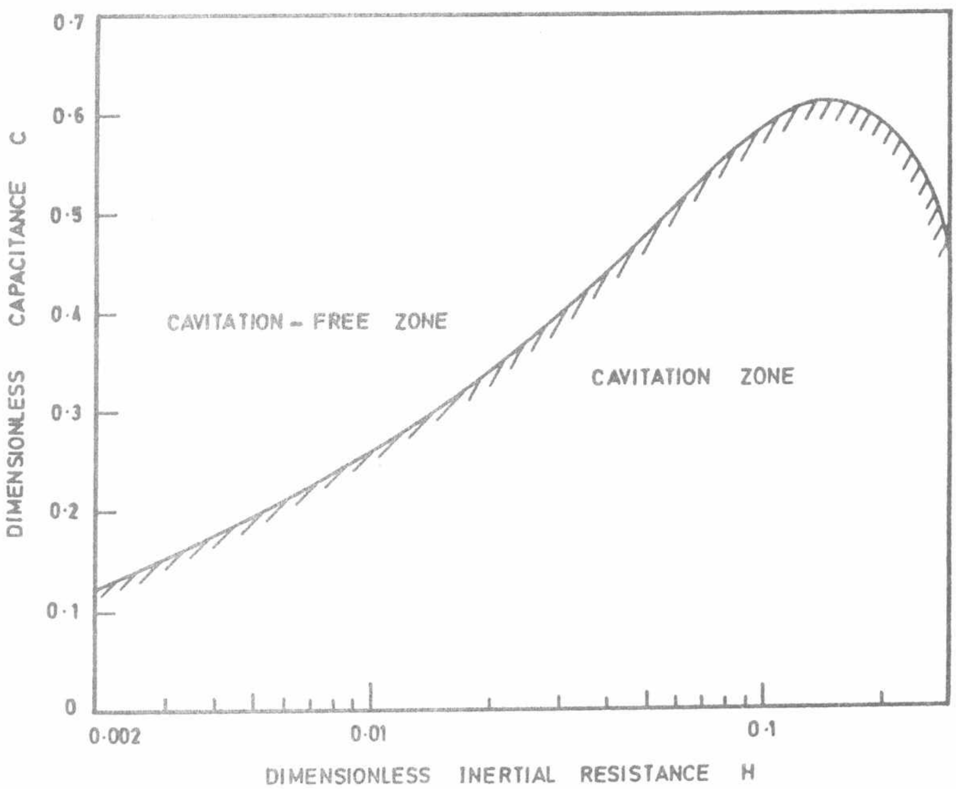

Fig. 6 Combinations of C and H at Which Cavitation Occurs

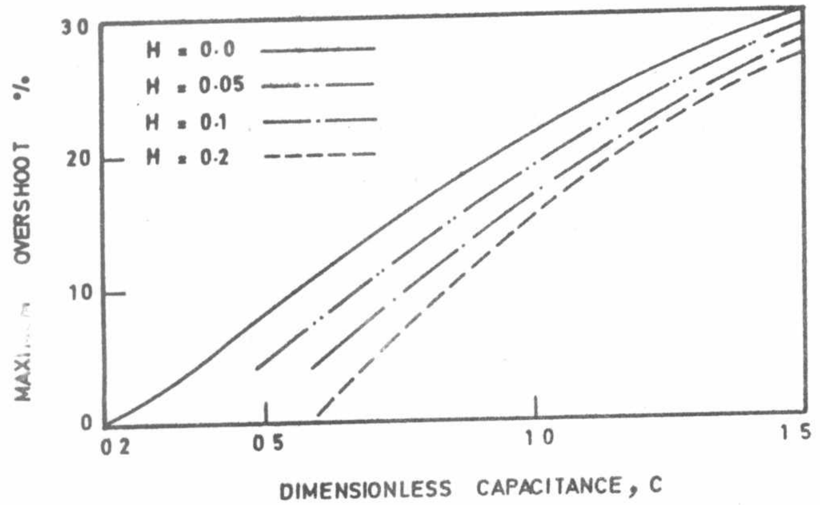

Fig. 7 Variation of Maximum Percentage Overshoot with C and H

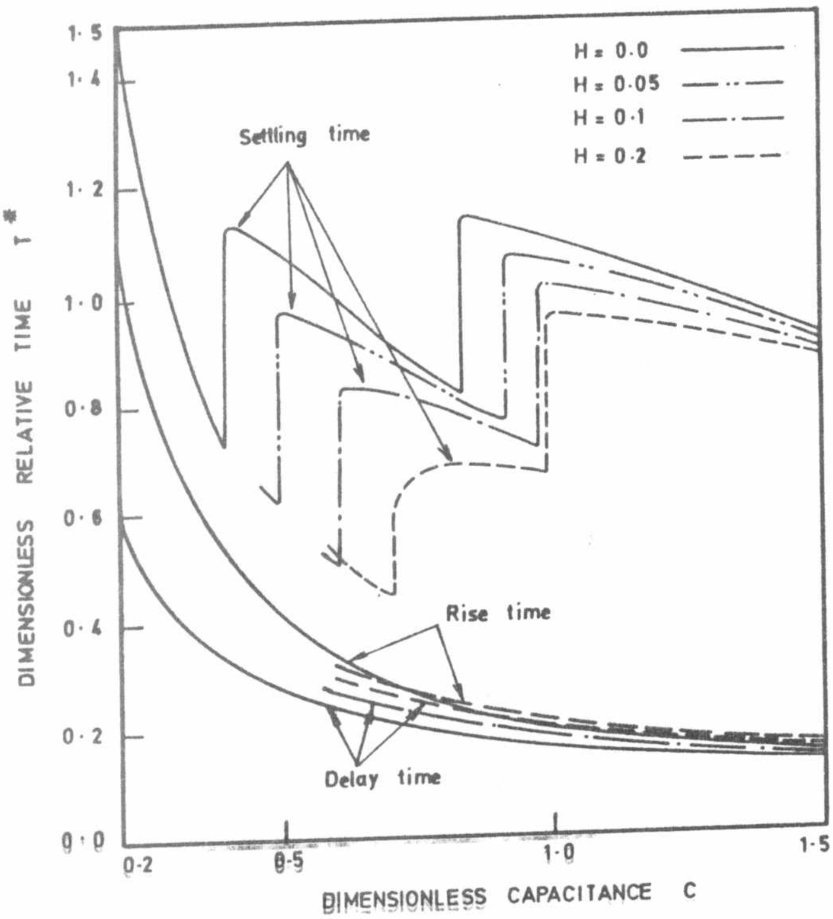

Fig. 8 Variation of Delay, Rise, and Settling Times with C and H

overshoot is seen to increase with the increase of C and decrease of H.

The variation of delay, rise, and settling times with C and H are shown in Fig. 8 . The time axis in this figure represents the relative dimensionless time; T^* , rather than the dimensionless time T. This is because the variation of C is accompanied by variations in the reference time t_r [7] , which can be explained as follows. If it is required to rotate a certain mass of moment of inertia I at a steady state speed n, when the supply pressure is p_s , then the reference time can be shown to equal $\gamma/(v_0^2\sqrt{r_0})$, and the capacitance C equals $\delta v_0^3 r_o$, where γ and δ are constants given by :

$$\gamma = 4\pi^2 I/p_s^{0.5} \quad , \text{ and } \quad \delta = \alpha\, p_s/(2\pi^2 \beta\, I) \; ,$$

α being the ratio between the actuator cavities volume v_a and its geometric volume v_o , and is assumed to be constant. The steady state rotational speed n equals $\sqrt{\{0.5p_s/(r_o v_o^2)\}}$, and in order to be constant irrespective to the variations in the system physical parameters, $r_o v_o^2$ should be kept constant. If the actuator geometric volume is changed from v_{o1} to v_{o2} , such that $v_{o1} = kv_{o2}$, the valve resistance should be changed from r_{o1} to r_{o1}/k^2 in order to keep the speed n unchanged. With these variations , the dimensionless capacitance varies from C_1 to kC_1 while the reference time varies from t_{r1} to t_{r1}/k . This means that increasing C by a certain factor means the decrease of the reference time by the same factor. Taking t_r at C = 0.2 as a reference, the scale of measuring the time at any value C_1 is changed by multiplying the dimensionless time by $0.2/C_1$. This is used for the scale of the time axis in Fig. 8 , which shows that H has a slight effect on both the delay and rise times. With the increase of C the delay and rise times decrease. The effect of C on these times is notable when $C < 1.2$. The settling time is seen to depend on both C and H. The minimum possible settling time is found to occur at the combination C = 0.7 and H = 0.18 , and equals 0.4 . The curves at this value of H are not presented since they are very close to the curves of H = 0.2. . At the mentioned combination, reasonable values of maximum overshoot and delay and rise times result. With H kept constant at 0.18 , any slight increase in the value of C above 0.7 increases the settling time considerably. At H = 0.2 , which means that the inertial resistance of the two fluid lines is 40% of the load inertial resistance, the value of C is preferably to be taken in the range 0.6 - 0.7 to obtain small values for both the settling time and maximum overshoot, beside reasonable values for the delay and rise times.

A comparison between the case of H = 0 and any other case reveals that the fluid line inertial resistance can be made use of to improve the system transient response. Figure 8 shows that the minimum settling time at H = 0 is 0.73 , and occurs at C = 0.4 , while it is 0.41 only at the combination H = 0.2 and C = 0.72 . At the former combination, the maximum percentage overshoot and the delay and rise times are 5% , 0.33 , and o.53 respectively , while they are 5.3% , 0.27 , and 0.29 for the latter combination. This shows that the fluid line inertial resistance, when properly combined with the other system parameters, improves the system transient response.

The obtained results can also be used to establish a criterion that determines the conditions under which the fluid line inertial resistance can be neglected, without introducing much error in predicting the system response. Figures 7 and 8 show that when $C > 1.4$, the variation of the transient response specifications with the variation of H is small. Thus when the system dimensionless capacitance is greater than 1.4 , the fluid line inertial resistance can be either ignored or added to the load inertial resistance, which renders the system equivalent circuit as shown in Fig.3.

For the values of C less than 1.4 , the system transient response is seen to depend strongly on the value of H , and the fluid line inertial resistance should be taken into account as shown in Fig. 4 , in order to predict with reasonable accuracy the system response.

On the other hand, it is more convenient to account for both the compressibility and inertial resistance of a relatively long, small diameter fluid line, by representing the line by two inertial rasistances center-tapped by a capacitance. Each inertial resistance equals half that of the fluid line, and the capacitance equals the fluid line capacitance. The mathematical model obtained for this case showed that the response depends on the ratio between the volume of the two fluid lines and the actuator volume; $V = 2a_p l_p / v_a$; beside the dimensionless capacitance C and the dimensionless inertial resistance H . When the system, in this case, was simulated on a digital computer, the results proved that the foregoing model,shown in Fig. 4,as well as the conclusions,are valid so long as $V < 0.3$.

Consequently, if the ratio between the volumes of the two fluid lines and the actuator; V , is less than 0.3 , and the system dimensionless capacitance C is greater than 1.4 , the fluid line inertial resistance can be ignored or accounted for by adding it to the load. In this case the system mathematical model is to be based on the equivalent circuit shown in Fig. 3. When V is less than 0.3 , and C is less than 1.4 , the fluid line should be represented by an inertial resistance and a capacitance which is to be added to the capacitance of the actuator cavity connected to the line, to get the system equivalent circuit as shown in Fig. 4 .

CONCLUSION

Analysis of the step response of an inertially loaded hydraulic actuator to step opening of the control valve, when the volume of the fluid lines is less than 30% of the actuator volume, shows that the response depends on the ratio between the inertial resistances of the fluid line and the load; H, and on the system dimensionless capacitance C which is directly proportional to the system capacitance, valve resistance, and supply pressure, and is inversely proportional to the load inertial resistance.

It is verified that when C is greater than 1.4 , the fluid line inertial resistance can be neglected or added to the external load, without introducing much error in predicting the system response. The system model in this case is the conventional simple model, at which the fluid line compressibility effects only are considered by adding the volume of each fluid line to the volume of the actuator cavity connected to it.

When C is less than 1.4 it is verified that the conventional simple model is inaccurate, since the fluid line inertial resistance, whatever small is its value , affects strongly the system performance. The fluid line in this case should be represented by an inertial resistance, while its capacitance is to be added to the capacitance of the actuator cavity connected to it. In this case, and at certain combinations of C and H , cavitation occurs, a phenomenon that can not be predicted when H is assumed zero. Further, the transient response specifications show to depend on the value of H. By the proper choice of the values of C and H, the system transient response can be improved , and the minimum possible settling time can be obtained.

Charts are presented, that can be used to choose the values of C and H that yield the required transient response.

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NOMENCLATURE

a_p	cross sectional area of fluid line
b	valve port width
c_a	capacitance of one actuator cavity
c_t	total capacitance of a fluid line and actuator cavity connected to it
c_p	fluid line capacitance
C_d	Coefficient of discharge
dc	capacitance of an infinitesimal elemant of length dx of the fluid line, $dc = a_p\, dx/\beta$
h_ℓ	load inertial resistance
h_p	fluid line inertial resistance
h_t	total inertial resistance of load and two fluid lines
dh	inertial resistance of an infinitesimal element of length dx of the fluid line, $dh = \rho\, dx/a_p$
I	mass moment of inertia of load
l_p	length of fluid line
n	rotational speed
p, q	pressure, and flow rate at any section of the fluid line
$p_{1,..,4}$, $q_{1,..,4}$	pressure and flow rate at locations 1,.., 4
p_s	supply pressure
q_ℓ	load flow
r , r_o	valve resistance, valve resistance at a constant opening u_o
t	time
u , u_o	valve opening , valve step opening
v_a, v_o	volume of fluid filling actuator , actuator geometric volume
x	coordinate along fluid line axis
c_r , h_r , p_r , ... , etc	reference quantities
C , H , P , ... , etc	dimensionless quantities
β , ρ	fluid bulk modulus , and density , respectively
α, γ, δ	constants