



EXPERIMENTAL AND NUMERICAL INVESTIGATION
OF THE FLOW THROUGH CASCADES

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ABSTRACT

Investigation of the flow through two-dimensional cascade is considered in the present work, the relation between cascade geometric and aerodynamic parameters is handled both experimentally and theoretically. Experiments have been performed on an isolated airfoil and a cascade of airfoils. The pressure distribution along the airfoil contour was measured at different incidence angles, for the isolated airfoil, and different stagger and inlet flow angles, for the cascade; NACA 0015 blade section is employed in the measurements. The theoretical approach to calculate the frictionless flow through two-dimensional cascades was conducted using the singularity method of Schlichting, the airfoil is replaced by distributions of vortices, and sources and sinks. The direct problem is considered i.e. determination of the vortex and source distributions related to a prescribed profile geometry. A computer program, in Basic language, is developed for that purpose.

The measured pressure distributions along the blade suction and pressure sides were compared with the computed results. Moreover, the present prediction is compared with other experimental data that covers a variety of geometrical configurations and flow conditions. The comparison shows that the present computer code is stable and efficient; it is economical and yields good engineering results.

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INTRODUCTION

The quasi three-dimensional flow through the blading of a turbomachine is usually considered as a combination of two basic two-dimensional flows [1,2], the first is the mean meridional flow on stream surfaces assumed of revolution about the axis of rotation, and the second is the cascade, or the blade-to-blade, flow on these stream surfaces. The present investigation considers the flow through two-dimensional cascades. The main problem in the investigation of two-dimensional cascades is to find the relation between the geometric and aerodynamic parameters.

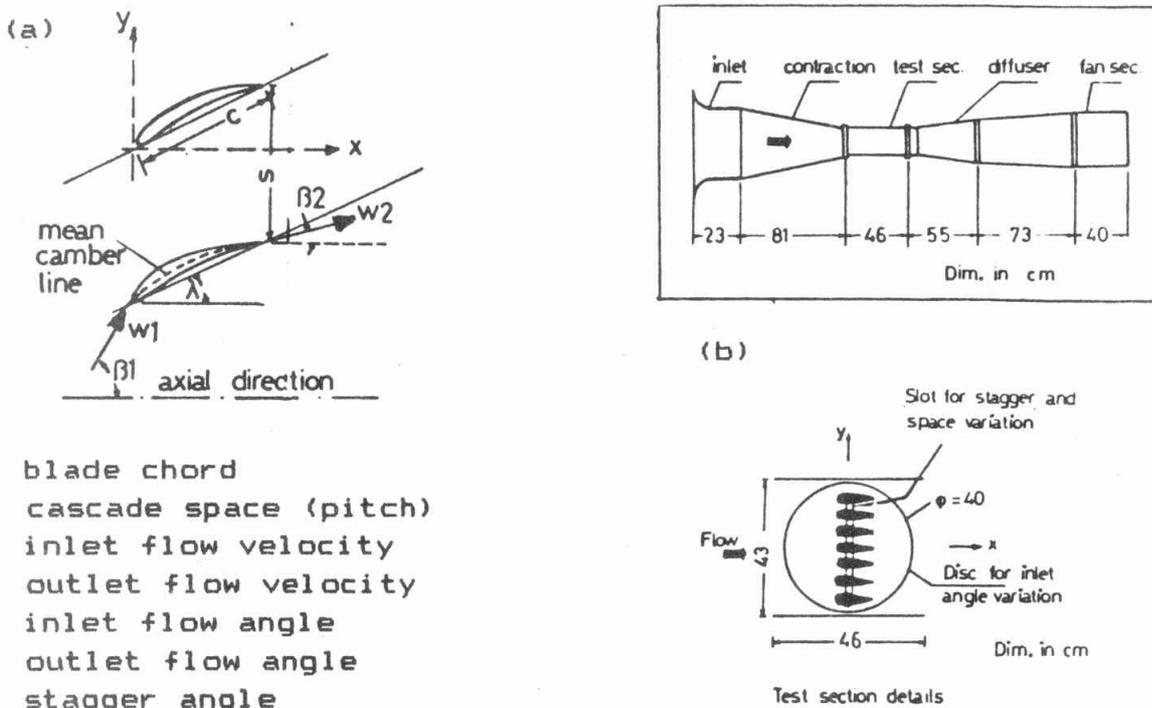


Fig. 1. (a) Cascade representation
 (b) Schematic of the wind tunnel and test section.

The geometric parameters could be obtained from airfoil shape and airfoils arrangement in the cascade as shown in Fig. 1(a). The aerodynamic parameters of a cascade are the inlet and outlet flow angles, the pressure distribution along the blade contour, and the resultant aerodynamic force. The relation between the geometric and aerodynamic parameters could be handled experimentally and/or theoretically. Experimental investigations had been carried out for a wide range of two-dimensional compressor cascade configurations at low speed, [3,4,5,6], e.g. NACA 65-series laminar airfoils. These experimental results are

used as a design basis for engineering applications in the field of axial turbomachinery. Two-dimensional cascade test data at low speed are essential for design purposes, corrections for high speed (compressibility effect) could be introduced [1,7,8]; moreover, they provide a means for testing and evaluating the existing computational methods.

The present work follows the above methodology. An experimental investigation is conducted and a computational method for the inviscid flow is adopted. Comparisons between the computed results against the present measurements and some other measurements available in the literature are also presented.

2- EXPERIMENTAL WORK

2.1- Wind Tunnel Description and Instrumentations

Experiments were conducted using a modified version of a low speed open circuit wind tunnel laboratory facility, originally dedicated to isolated airfoils. The modification was necessary to convert the circular test section into a rectangular one, thus allowing measurements to be carried out for cascade configurations. Figure 1(b) shows the actual wind tunnel facility used in the present measurements. Air is drawn into the test section by an axial flow type fan driven by a D.C. motor (1.5 kw), the air velocity can be continuously raised up by adjusting the fan speed via a control unit. The contraction is of rectangular cross-section of length 0.81 m and contraction ratio 1 to 2.6. The test section dimensions are 0.43 m width, 0.46 m length, and 0.20 m height. In order to vary the inlet flow angle, the bottom side of the test section was manufactured such that it has a circular disc on which a cascade of airfoils could be mounted and rotated as a block, i.e. at constant stagger angle. Changing the pitch between airfoils is achieved by a slot in the disc.

Regarding airfoil selection, the following points were considered. Extensive experimental data are available for laminar airfoils, e.g. the NACA 65-series [4]; this is not the situation as far as the classical 4-digits NACA series is considered. This last series is used in wind turbines and in the blades of the main rotor of helicopters. The present experimental investigation was carried out using NACA 0015 blade section. Selection of this type of symmetrical airfoil, rather than cambered one, is related to three reasons: (1) ease of manufacturing of the outer surface and of fitting the blade contour pressure holes with the necessary tubes; (2) decreasing, for small inlet and stagger

angles, the test section upper and lower walls effect (walls facing airfoil upper and lower sides i.e. walls limiting the maximum number of airfoils in the test section) that would be significant if cambered airfoils were employed; (3) use of a relatively simple theoretical flow model that could be worked out on the available Z-100 Zenith Data System microcomputer; this is achieved by employing a chord-wise singularity distribution that in the case of symmetrical airfoils is equivalent to the more accurate approach of camber-wise singularity distribution.

Six airfoils of NACA 0015 form were manufactured using wood as material, the coordinates of this airfoil are given in Ref. [9]. The chord and span lengths are taken as 0.1 m and 0.2 m, respectively. A seventh airfoil was manufactured of aluminium and was fitted with seven pressure holes on both of its sides, thus allowing static pressure determination on the blade contour. The pressure holes, drilled normal to the blade surface, are of 1 mm diameter and are located at the following positions: for the upper side $x/c = 0.05, 0.15, 0.25, 0.36, 0.5, 0.6, 0.7$, and for the lower side $x/c = 0.05, 0.15, 0.25, 0.36, 0.48, 0.56, 0.7$. A standard calibrated pitot tube of outer diameter 4 mm is employed to measure the upstream velocity and static pressure.

2.2- Test Procedure

Due to the expected test section upper and lower walls effect, the experimental work was limited to cascade configurations at low incidence angles. This limitation was imposed by the fact that for high incidence angles, the flow hits the wind tunnel wall, thus yielding a non periodic flow in the pitch direction. This limitation could be overcome by reducing the blade-chord dimension; however, this would yield a thin blade impractical for manufacturing with 14 pressure tubes connected to the holes on its contour. In addition, this would reduce the operating Reynolds number which is already limited by the maximum upstream velocity available in the wind tunnel.

The present experimental study was conducted at Reynolds number based on the chord of 160500 using NACA 0015 blade section.

For the isolated airfoil, the incidence angle was varied as 0,5,10 deg. For the cascade, the following parameter were systematically used:

- 1) Space-chord ratio: $s/c = 0.5, 0.75$
- 2) Inlet flow angle (deg.): -10, -5, 0, 5, 10, 15
- 3) Stagger angle (deg.): 0, 5, 10, 15

An additional value of unit space-chord ratio is used for the cascade at zero inlet flow and stagger angles; for the employed test section and in absence of turning, increasing cascade space had no detrimental effect as it would be the case in presence of turning. The static pressure distribution along the blade contour

was measured through the static pressure holes located along the blade suction and pressure sides. The upstream velocity and static pressure were measured using pitot tube. Reliability of the experimental data was checked by repeating the measurements under the same operating conditions. The pressure coefficient C_p is calculated as follows;

$$C_p = (P - P_1) / (0.5 \rho W_1^2)$$

where P and P_1 are the local and inlet static pressures, respectively, W_1 is the inlet velocity, and ρ is the density.

3- THEORETICAL ANALYSIS

Incompressible potential flow through two-dimensional cascades is considered as the design basis for axial turbomachines at low speeds. Compressibility effects could be, however, taken into account either directly or through corrections [1,7,8], and viscosity effects could also be handled by applying boundary layer theory, thus giving theoretical estimation for the loss coefficient. Theoretical investigations for the two-dimensional incompressible frictionless flow through cascades were reported, [10], for the case of direct, known geometric parameters, and indirect, known aerodynamic parameter, problems.

Many numerical techniques could be used to solve the direct problem; e.g. finite differences [11,12], finite elements [7,13], conformal transformation [7,14], and the singularities methods [7,15,16,17]. The field techniques, finite differences and finite elements, yield accurate results; however, their use require an experienced user to manipulate input data and control parameters, the corresponding codes may also limit the designer in the types of blade geometries, cascade configurations, and flow conditions [17]. In order to accelerate the blade design procedure and give the designer more freedom in developing blade shapes, a simple blade-to-blade flow code has to be used. The singularity method seems simple and adequate for this purpose, [7,17]. In the singularities method, the singularities may be distributed on the chord [7], on the camber [7,18], or on the blade contour [16,17]. A survey of the range of applicability of these methods showed that the chord-wise distribution approach yields results useful in engineering practice for airfoils with maximum thickness-to-chord ratio of the order 15 % [7]. The chord-wise distribution of singularities, Schlichting method [7,14], is adopted in the present analysis, a computer code in Basic is developed using Z-100 Zenith Data System microcomputer. The code was tested and proved to be simple, stable and efficient. Such a code provides a means of rapidly screening blade designs for analysis by more complex codes which have better accuracy.

3.1- Schlichting Method

According to the singularities method of Schlichting for the inviscid incompressible flow through two-dimensional cascades, each airfoil of the cascade is replaced by a continuous distribution of vortices, and sources and sinks; these singularities are situated on the chord [7,14]. The flow around an airfoil of finite thickness with camber and incidence could be found by superposition of two basic flows. The first flow, around the uncambered airfoil of finite thickness, is represented by a source and sink distributions; whereas, the second flow, around the infinitely thin camber line with incidence, is represented by a vortex distribution. The singularities could be represented by suitable series expansion with free coefficients. The basic procedure for solving the direct problem consists in the determination of the vortex and source distributions, related to a prescribed profile geometry, with the aid of the condition of tangential flow. For NACA 65-series and NACA 4-digits blades, analytical expressions are available for airfoil coordinates [7]; for other blades, curve fitting procedure is adopted to fit airfoil coordinates of both pressure and suction sides. The slopes of the camber-line and of the symmetric airfoil are estimated at different stations on the chord line. In the method of Schlichting, vortex and source distributions are expressed using the first three terms of Glauert series; this leads to six unknown coefficients. Each of these terms induces velocity contribution at pivotal points selected along the profile contour. With the help of the tangential flow conditions at the selected three pivotal points, and expressing the isolated profile and remaining cascade induced velocities as functions of the six coefficients, a system of six simultaneous linear equations is obtained. Upon solution, the unknown six coefficients, of Glauert source and vortex expressions, are known; consequently, the singularity distribution is available. The velocity distribution along the blade contour could then be determined. The angle of attack effect could be introduced in the equations to get the corresponding six coefficients.

3.2- Compressibility Effect

High speed cascade characteristics are similar to those at low speed until the critical Mach number is reached, after which the performance declines. For subsonic flows at high Mach number where compressibility effects cannot be ignored, the pressure distribution along an airfoil could be obtained from measurement, or calculation, conducted at low speed conditions, [1,7,8,]. The computer code of Schlichting method was extended to include compressibility effect. Once the incompressible velocity

distribution is calculated by the program, the equivalent compressible velocity distribution is computed according to equation (7.44) in Ref. [7].

3.3- Computer Program Description

Program input data:-

Table 1 : Cascade independent universal functions

$D(I,J)$: Values of $\cot(\theta/2)$, $\sin(\theta)$, $\sin(2\theta)$, $\sin(3\theta)$, $\cos(\theta)$,
 $\cos(2\theta)$ and $\cos(3\theta)$ for $x/c = 0.0, 0.05, 0.1, \dots, 1.0$ and
 $x/c = 0.58$ and 0.91
 where $x = c/2 (1 - \cos(\theta))$

$A(I,J)$: values of $\cot(\theta/2)$, $\sin(\theta)$, $\sin(2\theta)$ and $\sin(3\theta)$
 for $x/c = .25, .58$ and $.91$

Table 2 : Airfoil geometry

N_p : Number of points defining the profile

c : Chord length

$XE(I), YE(I)$: Coordinates of profile upper surface

$XI(I), YI(I)$: Coordinates of profile lower surface

Table 3 : Cascade specification

Inlet flow and stagger angles, and space-chord ratio

3.4- Numerical Algorithm

The numerical algorithm includes the following main steps.

- Step 1- Curve fitting of airfoil coordinates of both suction and pressure sides or use of analytical expressions [7].
- Step 2- calculation of the slopes of the camber line and the symmetric airfoil
- Step 3- calculation of universal-non-dimensional-cascade downwash functions
- Step 4- calculation of the six singularity coefficients
- Step 5- calculation of cascade outlet flow angle, contour velocity, and pressure distribution.

4- COMPARISON BETWEEN MEASUREMENTS AND COMPUTATION - DISCUSSION

The present cascade and isolated airfoil measurements, were compared with the numerical results of Schlichting method. In addition, to test the validity and accuracy of the computer program, other examples of cascade flows are considered. These examples include different airfoil geometries and different flow conditions; also examples with different Mach numbers were considered.

4.1- Comparison between the Present Measurements and Computations

The comparison between Schlichting method and the present measurements, for the employed NACA 0015 blade section, are displayed in Figs. 2 to 4. Figure 2 shows the results for four values of space-chord ratio ($s/c = 0.5, 0.75, 1$ and ∞). The static pressure coefficient distribution on both sides of the blade is shown for zero inlet flow and stagger angles. Generally, good agreement between the present measured pressure distribution and the prediction of schlichting method is observed. Also, the figure indicates that the difference between the pressure distribution of the blade in cascade and the single airfoil increases with closer spacing, vice versa, cascade calculations with increasing values of space-chord ratio tends to yield results closer to the results of a single airfoil.

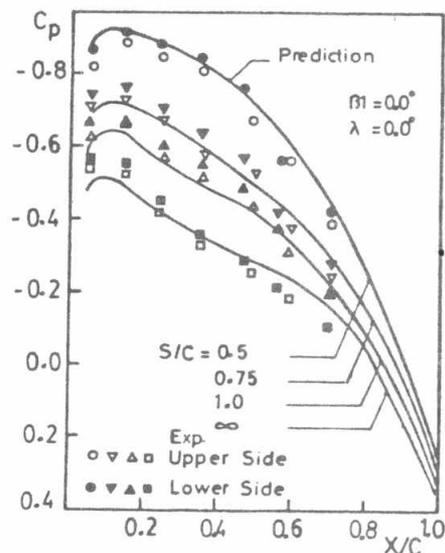
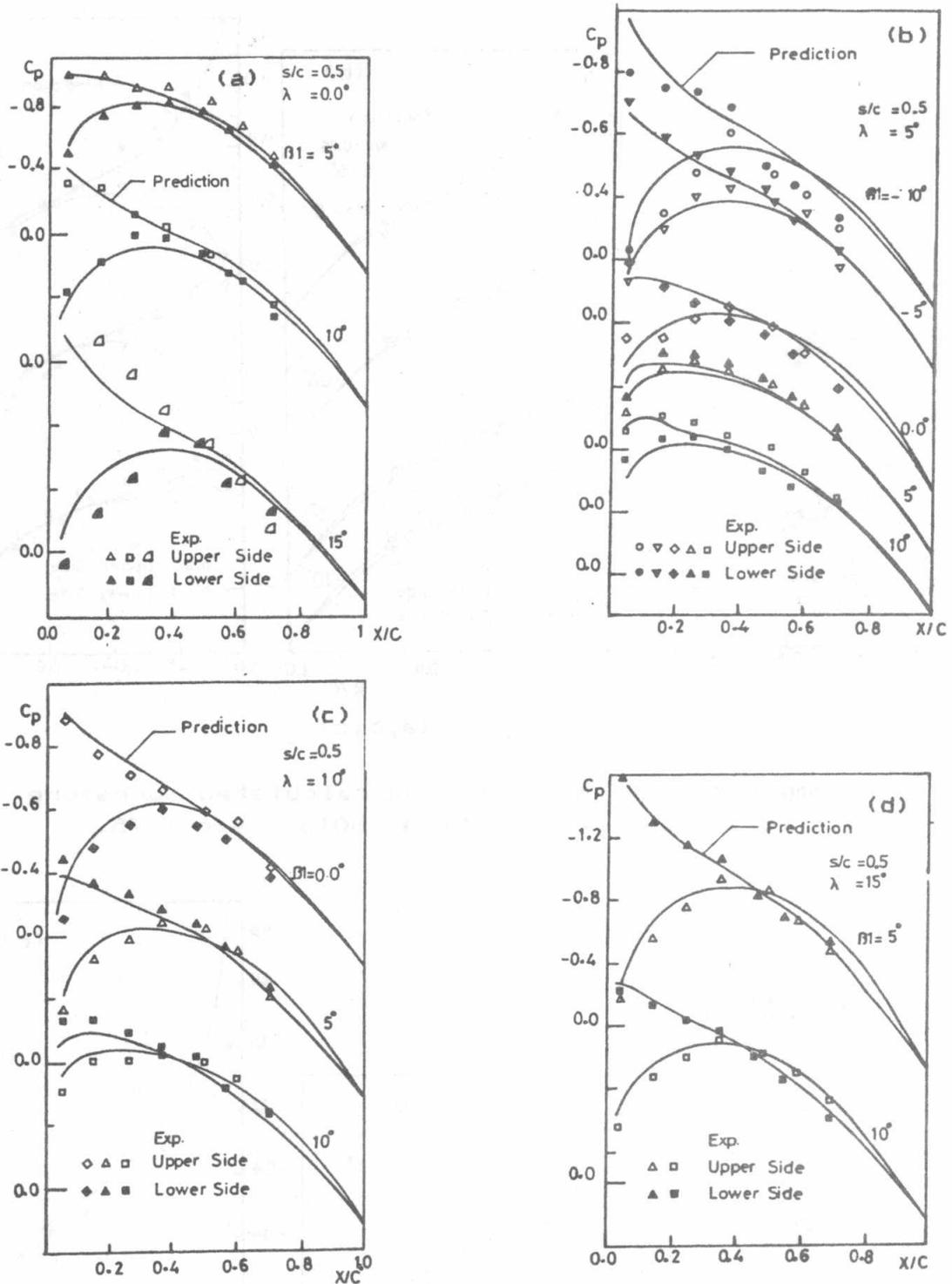


Fig. 2.

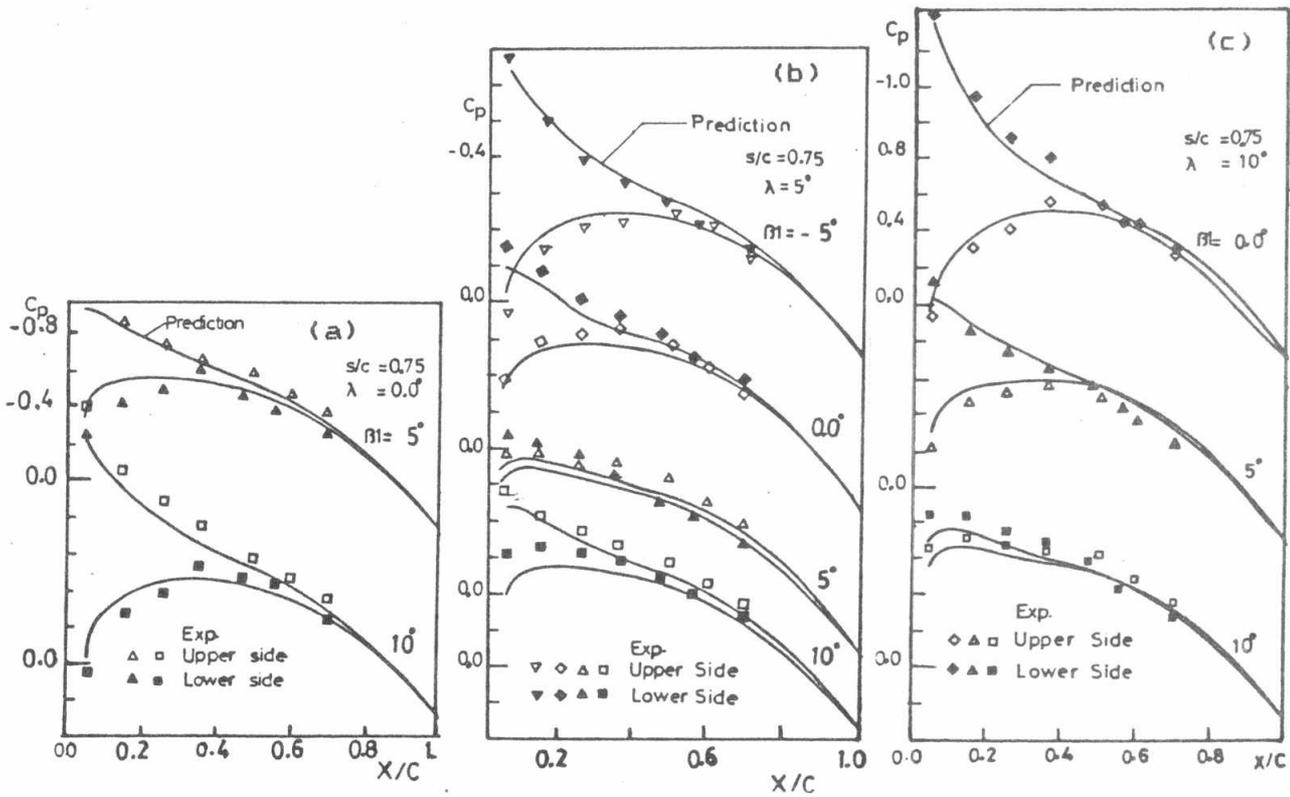
Comparison between measured and calculated pressure distributions for NACA 0015.

Figures 3(a,b,c,d) illustrate the results corresponding to stagger angles of 0, 5, 10, 15, deg. respectively, for constant space-chord ratio of 0.5 and different inlet flow angles. The inlet flow angle varies as follows (in deg.) : Fig. 3(a): (5,10,15), Fig. 3(b): (-10,-5,0,5,10), Fig. 3(c): (0,5,10), Fig. 3(d): (5,10). In general, good agreement is observed between the experimental data and the computed results; the agreement is less pronounced for high angles of incidence of the order of 15 deg. This is due to the expected effect of the test section walls facing the blade upper and lower sides. For this reason, the other experiments were limited to incidence angle of 10 deg. From Fig. 3 it could be observed that the difference between the upper and lower surface pressure coefficient increases with the increase in incidence angle. Presented in Figs. 4(a,b,c) are the results, for space-chord ratio of 0.75, corresponding to stagger angle of 0,5,10 deg., respectively, at different inlet flow angles. The inlet flow angle varies as follows (in deg.): Fig. 4(a): (5,10), Fig. 4(b): (-5,0,5,10), Fig. 4(c): (0,5,10). The discussion given for Fig. 3 applies for Fig. 4 too; moreover, the figures suggest that, at the same incidence, the area included between the upper and lower surface pressure distributions, proportional to the lift, increases as the space-chord ratio increases.



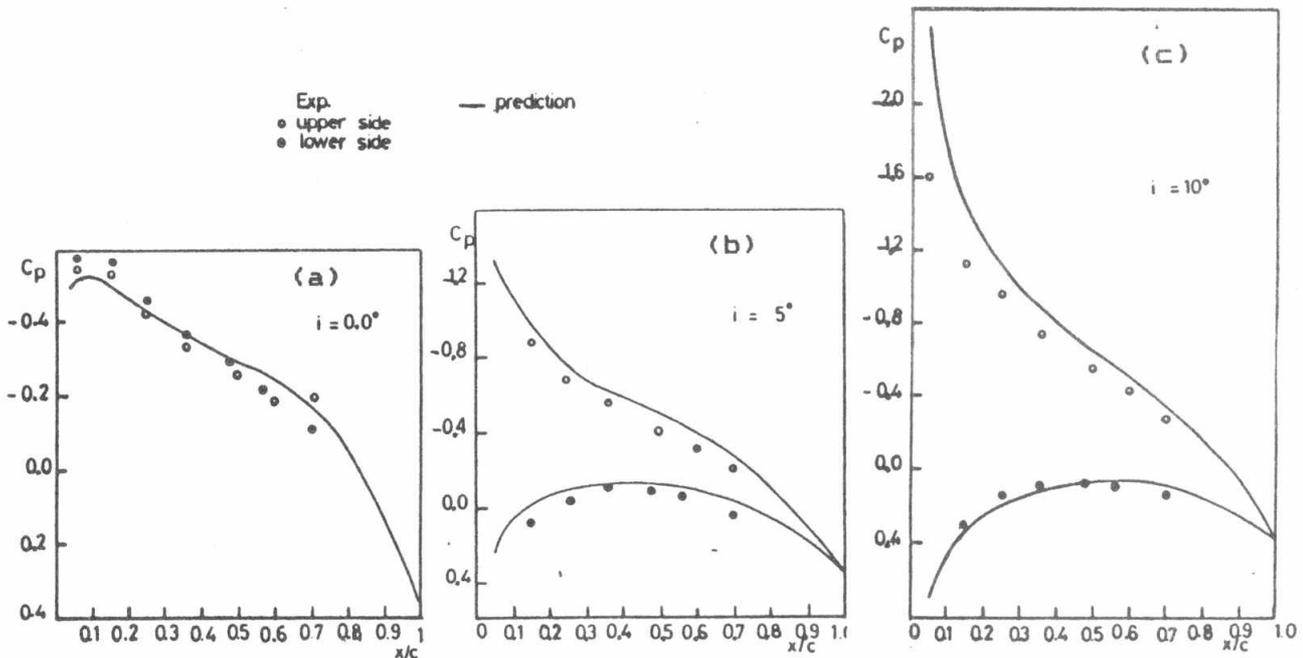
Figs. 3(a,b,c,d). Comparison between measured and calculated pressure distributions for NACA 0015, $s/c=0.5$.

In the present investigation, a series of measurements have been performed for the isolated airfoil case. Figures 5(a,b,c) illustrate the comparison between the present data and computer results at different incidence angles.



Figs. 4(a,b,c).

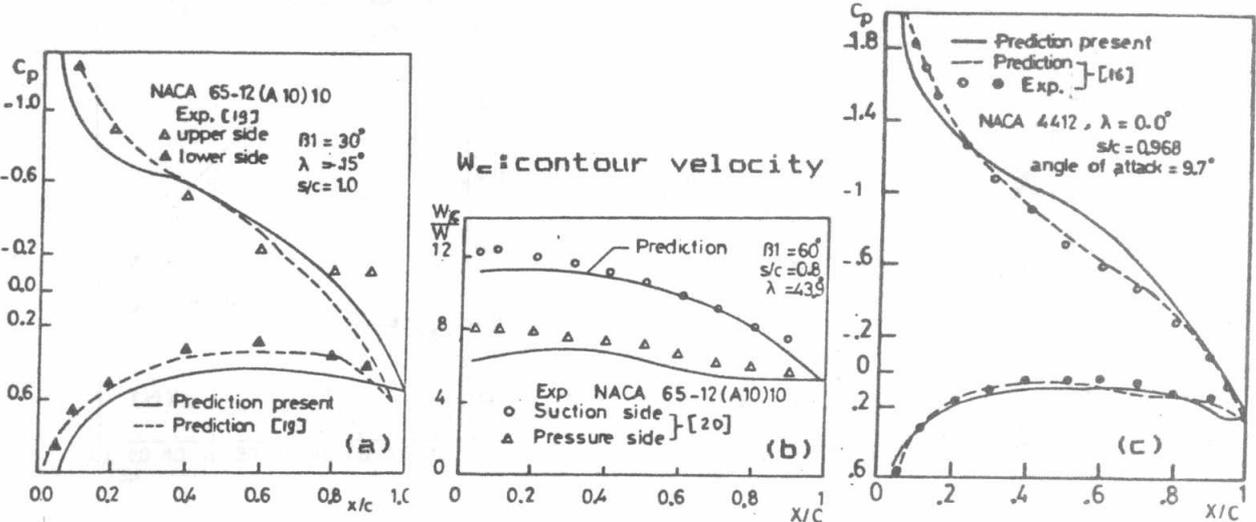
Comparison of measured and calculated pressure distributions for NACA 0015, $s/c=0.75$.



Figs. 5(a,b,c).

Comparison of measured and calculated pressure distributions for NACA 0015, isolated airfoil.

The comparison between the present measurements and the present calculation, Figs. 2,3,4,5, shows in general good agreement; however, some deviation could be observed. This difference may be attributed to the following reasons: (1) viscosity effect that is not taken into account in the computation, (2) inherent computational errors due to curve fitting procedure for the blade coordinates, (3) inherent experimental errors, (4) inherent manufacturing errors of the blade section, and (5) test section side walls boundary layer effect.



Figs. 6(a,b,c). Comparison between the present calculations against other predictions and experiments.

4.2- Comparison between the Present Computation against other Predictions and Experiments.

In order to test the present prediction, comparisons with experimental data, other than the present, and prediction methods were conducted; the comparison covers a variety of geometrical configurations and flow conditions. Figure 6(a) illustrates the comparison for NACA 65-12(A10)10 compressor blade section for the following conditions: unit space-chord ratio, -0.15 deg. stagger angle, 30 deg. inlet flow angle. The figure exhibits the pressure distribution along both sides of the blade as predicted by the present method and compares it with NACA data and with another prediction method [19]. Figure 6(b) compares the same blade section but for the following conditions: 0.8 space-chord ratio, 43.9 deg. stagger angle, 60 deg. inlet flow angle [20]. Another comparison is held for NACA 4412 blade section under the following conditions: 9.7 deg. angle of attack, zero stagger angle, 0.968 space-chord ratio. Figure 6(c) illustrates the pressure distribution along both sides of the blade as predicted by the present method and compares it with measurement and with another prediction method [16]. The comparison given in Figs.

6(a,b,c) shows that the present computer code yields good results. Furthermore, a comparison is held in Figs. 7(a,b) between the present computed results and the experimental data of [9,20], respectively, for the case of isolated airfoil.

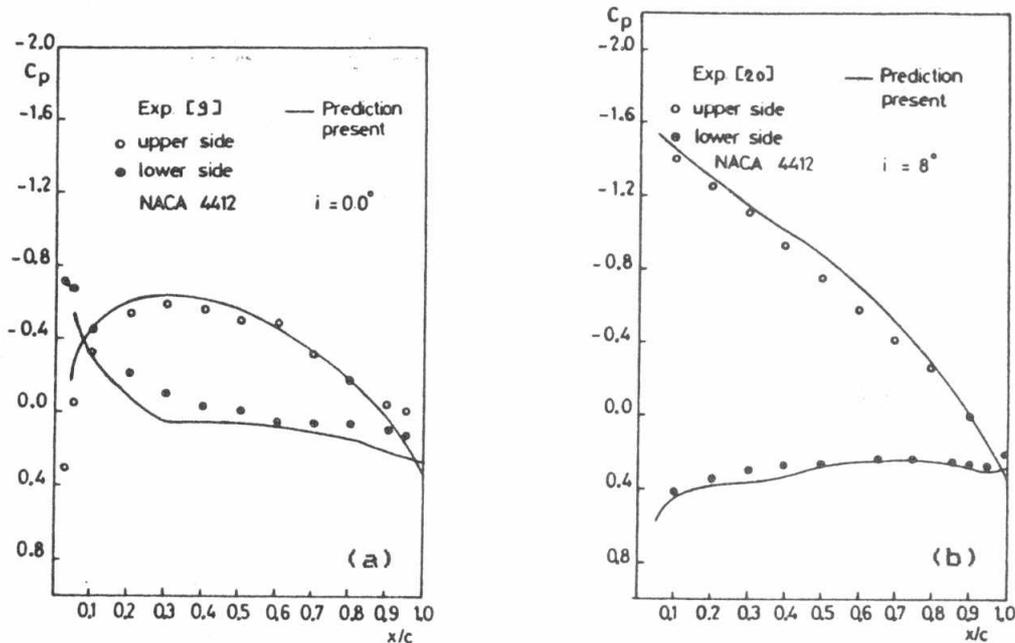
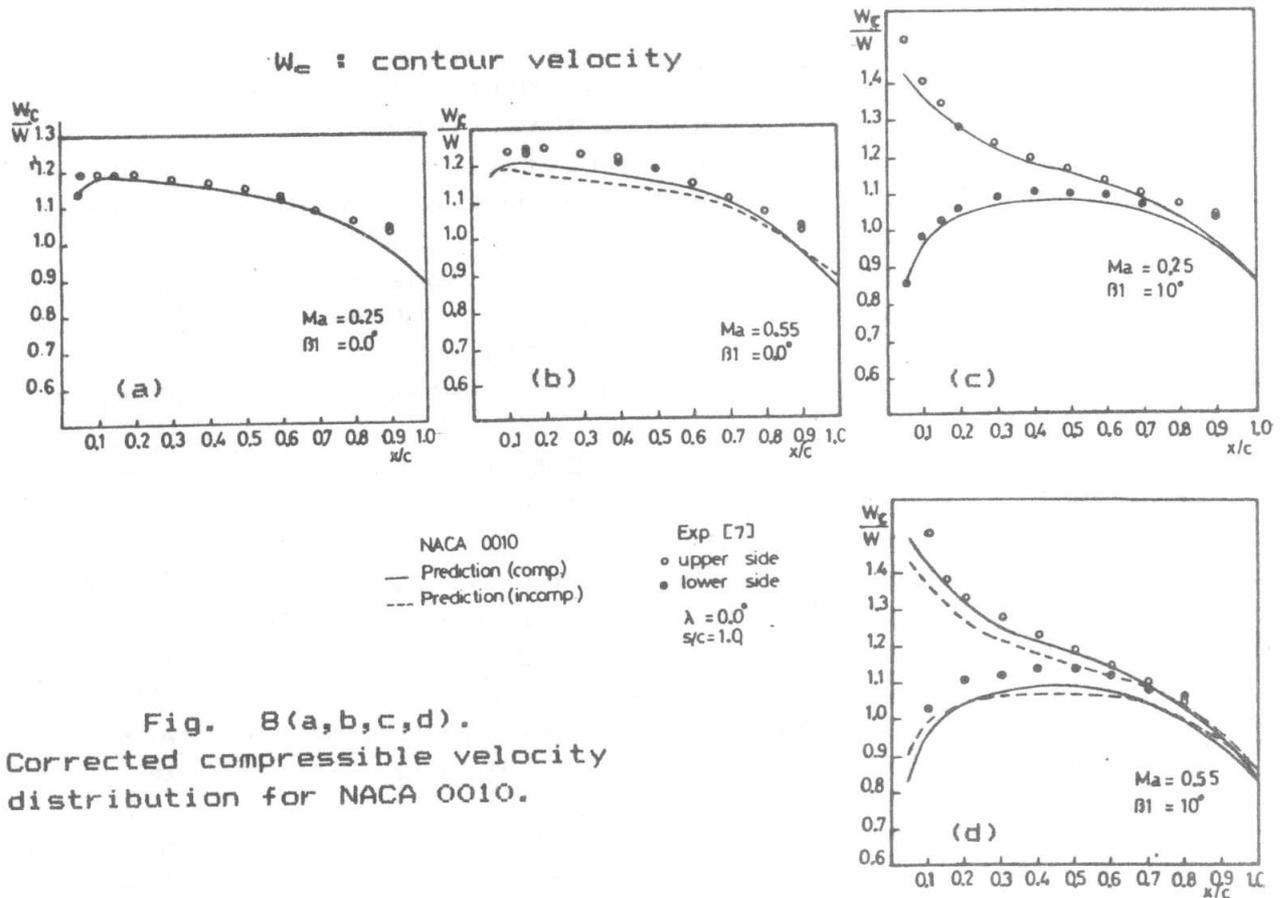


Fig. 7(a,b). Comparison of the present prediction and other experimental data for the isolated airfoil.

In general, the comparison given in Figs. 6,7 shows that the present prediction code yields good results when compared against the above mentioned experimental data. However, some discrepancy is observed between the computed and measured results. This discrepancy may be mainly attributed to viscosity effect that is not accounted for in the computer program.

The present computer code has been extended to include compressibility effects. This is achieved by first calculating the incompressible velocity distribution along the blade contour, the corrected compressible velocity distribution could then be obtained [7]. Figures 8(a,b,c,d) display the comparison between measurements [7] and prediction for a cascade based on NACA 0010 airfoil section at two inlet flow angles of 0 and 10 deg. and two different Mach numbers 0.25 and 0.55. The Figures show that better results are obtained when applying the compressibility correction. However, the figures indicate that the compressibility effect on the excess velocities is underestimated. This may be due to some extent to a greater effect of the boundary layer displacement on the local velocity at high Mach numbers, which increases it.



CONCLUSION

Experiments have been carried out to measure blade contour pressure distribution for the cases of isolated airfoil and cascade of airfoils. Different flow and cascade geometry conditions were considered (stagger and inlet flow angles, space-chord ratio). In addition, a computer program was developed based on the singularities method of Schlichting. A comparison is held between the present prediction method against : (1) the present measurements, (2) other experimental data, (3) other prediction methods. The comparison shows that the computer code is stable and efficient. The code is economical and yields good engineering results for a variety of flow conditions and cascade configurations. Better prediction could be expected if the viscosity effect (boundary layer displacement effect) has been taken into account.

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