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GENERAL MODELLING AND SIMULATION OF A DIGITAL SPEED AND CURRENT CONTROLLED D.C. MOTOR IN CONTINUOUS MODE

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ABSTRACT

In this paper a non-linear rigurous model of a d.c. motor fed from single phase thyristor bridge is presented. The digital current and speed control are realized by internal current loop with proportional integral controller and external speed loop with a non-zero proportional integral one. This model is linearized around the operating point to study the behaviour of the system for small perturbations. Analytical expressions are given for the controller parameters for any desired response. A general method of simulation to study the behaviour of the system, is also presented taking into account the possible system discontinuities.

1. INTRODUCTION.

The previous work in the field of modelling, analysis and synthesis of control sytem including static converters can be classified into three main strategies. The first one, uses linear continuous models. [1] and [2], which is correct for a limited number of applications. It depends on replacing the static converter and its firing circuit by a power amplifier. The second strategy uses a simplified non-linear discrete model [(3) and (4)]. The results obtained show that this method cannot be used for fast responses. The third strategy is the general one, uses non-linear discrete model without any assumptions. Due to the complicated analysis in this method, its applications are limited to third order systems [5] and [6]. In this paper we will present the solution of a fourth order system consisting of separately excited D.C. motor fed by single phase thyristor bridge. The thyristors are fired from a linear timing voltage firing circuit, whose input is the output of the P-I digital current controller. The digital speed of the motor is controlled using a P-I controller.

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2. DESCRIPTION OF THE SYSTEM.

The block diagram of the considered system is shown in Figure (1). The D.C. motor is fed from single phase thyristor bridge. The thyristors are fired by pulses generated from digital impulse generator whose input is the output of the digital controller. The controller have been choosen as follows:

i) Internal loop on the current with P-I controller without zero (two loops).

ii) External loop on the speed with P-I controller without zero (two loops).

3. PRELIMINARIES.

3.1 List of Symbols (Figure 2):

w : pulsation of supply.
R, L : Armature resistance and inductance.
V_m : Peak supply voltage.
F : Viscous friction coefficient.
Γ_c : Load torque.
K : E.M.F constant.
J : Moment of inertia.
ν_1, ν_2 : Integral controller.
K_1, K_2 : Propotional controller.
K_t : Constant of tachy generator.
K_i : Current transducer constant.
V_2 : Speed reference.
u_1 : Integrator output (current).
u_2 : Integrator output (speed).
Ψ_n : n^{th} firing angle.
Ψ_{n+1} : n+1 th firing angle.

3.2 Reduced Variables and Coefficients:

$\theta = wt$
$\Delta_e = wL/R$
$\Delta_m = Jw/f$
$\varepsilon_{1c} = R\,\Gamma_c/V_m K$
$C = K^2/RF$
$\varepsilon_1 = iR/V_m$
$\varepsilon_2 = \Omega K/V_m$

4. THE SYSTEM EQUATIONS.

4.1 Motor Equations:

The electrical equation of the motor can be written as follows:

$$K\,\Omega + Ri + wL\, di/d\theta = V_m \sin\theta \qquad (1)$$

The mechanical equation of the motor can be written as follows:

$$Jw\, d\Omega / d\theta + f\Omega + \Gamma_c = Ki \qquad (2)$$

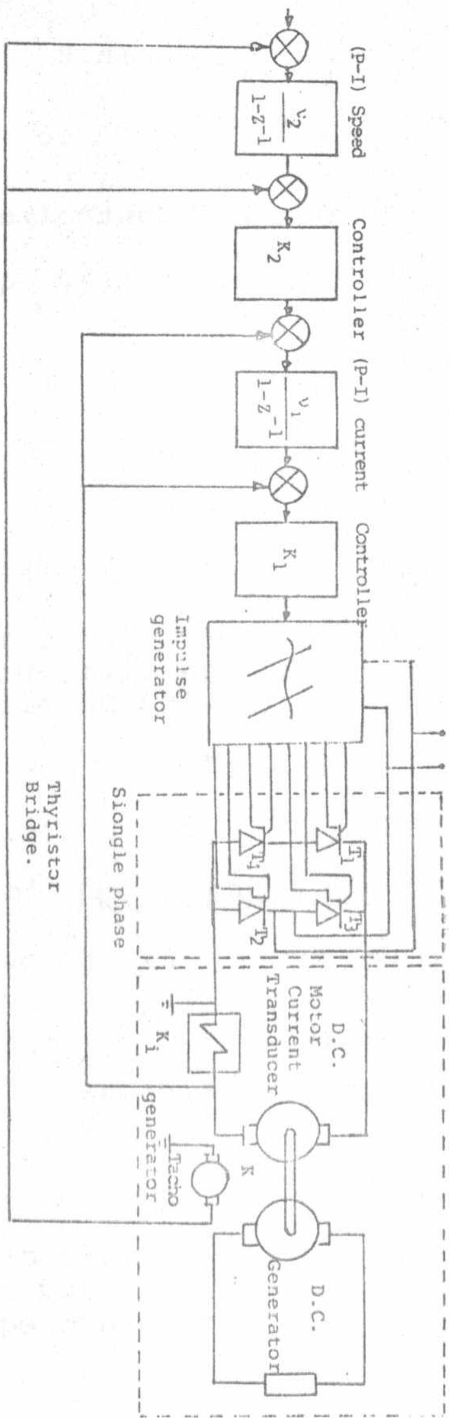

Fig. (1) Block Diagram of the System.

Using the reduced variables, the motor equations can be written in the following form:

$$\Delta_e \, d\varepsilon_1/d\theta = -\varepsilon_1 - \varepsilon_2 + \sin\theta \quad (3)$$

$$\Delta_m \, d\varepsilon_2/d\theta = c\varepsilon_1 - \varepsilon_2 - c\varepsilon_{1c} \quad (4)$$

4.2 The Digital Controller Equations:

From Figure (3) the controller equations can be easily written as follows:

$$\Psi_n = (u_1 - \varepsilon_1) K_1 \quad (6)$$

$$u_1(n) = u_1(n-1) + \nu_1 (V_B - \varepsilon_1) \quad (7)$$

$$V_B = (u_2 - \varepsilon_2) K_2 \quad (8)$$

$$u_2(n) = u_2(n-1) + \nu_2 (\varepsilon_{20} - \varepsilon_2) \quad (9)$$

where, u_1 and u_2 are the controller state variables. When using the reduced variables and coefficients, Equations (1) and (2) can be written in the following matrix form:

$$d/d\theta \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} -1/\Delta_e & -1/\Delta_e \\ c/\Delta_m & -1/\Delta_m \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} + \begin{bmatrix} (\sin\theta)/\Delta_e \\ -c\varepsilon_{1c}/\Delta_m \end{bmatrix} \quad (10)$$

5. THE NON-LINEAR MODEL OF THE SYSTEM.

For a system which has the following state variable equation:

$$d\underline{X}/d\theta = A\underline{X} + \underline{B}(\theta) \quad (11)$$

We can define a function $E(X,\theta)$ as,

$$E(X,\theta) = e^{-\theta A}\underline{X} - \int_0^\theta e^{-\lambda A} B(\lambda)\, d\lambda \quad (12)$$

For example, if we have a mode of operation starts at instant θ_1 with a state variable $\underline{X}_1$ and ends at the instant θ_2 with a state variable $\underline{X}_2$, its recurrence equation can be written as follows:

$$E(\underline{X}_1, \theta_1) = E(\underline{X}_2, \theta_2) \quad (13)$$

The considered system has one mode of operation (Figure 2), consequently its recurrence equation may be written as follows:

$$E[X(\Psi_{n+1}+\pi), \Psi_{n+1}+\pi] = E[X(\Psi_n), \Psi_n] \quad (14)$$

where,

$$\underline{X} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}, \quad A = \begin{bmatrix} -1/\Delta_e & -1/\Delta_e \\ C/\Delta_m & -1/\Delta_m \end{bmatrix} \text{ and } B = \begin{bmatrix} (\sin\theta)/\Delta_e \\ -c\varepsilon_{1c}/\Delta_m \end{bmatrix} \quad (15)$$

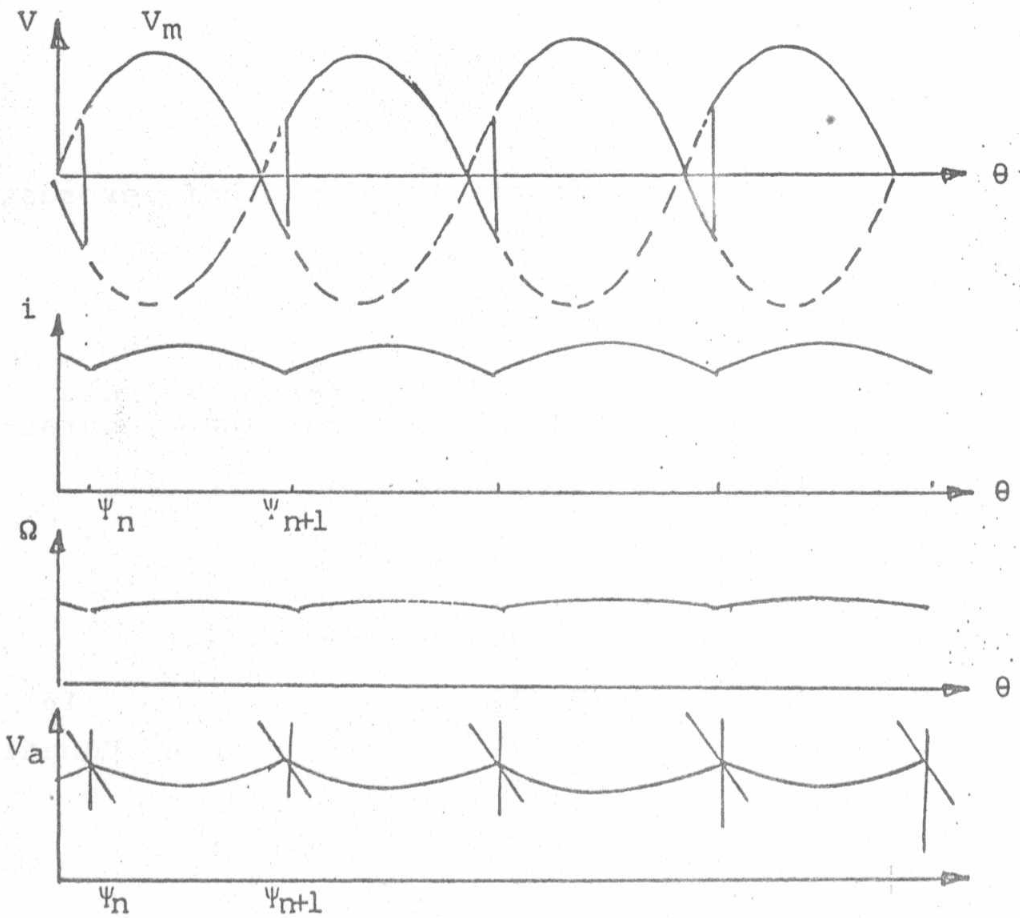

Fig. (2): Definitions of Some Variables.

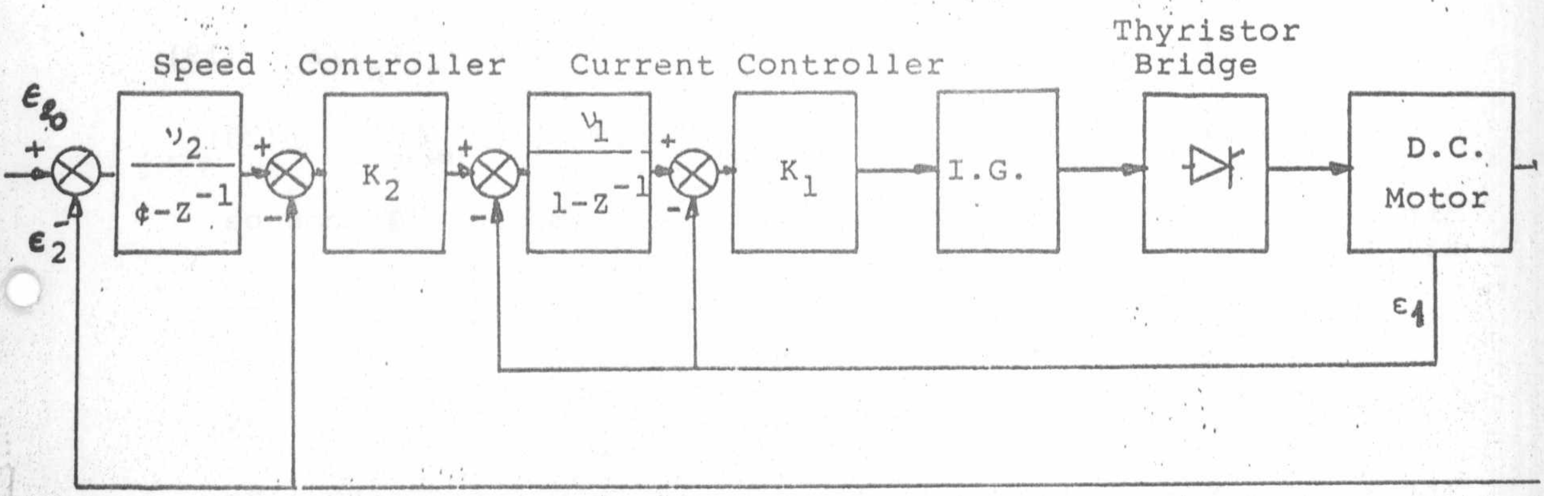

Fig. (3): Block Diagram Using Reduced VAriables.

The non-linear model of the sytem is represented by the non-linear recurrence Equation (13).

The matrix $e^{A\theta}$ can be calcualted using matrix transformation and can be written:

$$e^{A\theta} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

where, ϕ_{11}, ϕ_{12}, ϕ_{21} and ϕ_{22} are functions of θ, their expressions are given in the Appendix.

6. THE LINEARIZED MODEL.

To study the dynamical behaviour of the system, we have linearized the non-linear model around the steady state operating point which is defined by:

$$\Psi_n = \Psi_{n+1} = \Psi_\infty$$

$$X(\Psi_n) = X(\Psi_{n+1}) = X_\infty$$

The linearization of the function E(Equation 11) gives:

$$\delta E(X,\theta) = e^{-\theta A} [\delta X - (A\underline{x} + \underline{B})\ \delta\theta] \qquad (16)$$

and the recurrence Equation (14) can be linearized using Equation (16) and written in the following form:

$$e^{-(\Psi_\infty+\pi)A} \{\delta X(\Psi_{n+1}+\pi) - [A\underline{X}_\infty + \underline{B}(\Psi_\infty+\pi)]\ \delta\Psi_{n+1}\}$$

$$= e^{-\Psi_\infty A} \{\delta X(\Psi_n) - [A\underline{X}_\infty + B(\Psi_\infty)]\ \delta\Psi_n\} \qquad (17)$$

Equation (17) can be developed in the following two recurrence equations.

$$\delta\varepsilon_1(n+1) = \phi_{11}\delta\varepsilon_{1(n)} + \phi_{12}\delta\varepsilon_{2(n)} - (2\phi_{11}\sin\Psi_\infty\delta\Psi_n)/\Delta_e \qquad (19)$$

$$\delta\varepsilon_2(n+1) = \phi_{21}\delta\varepsilon_{1(n)} + \phi_{22}\delta\varepsilon_{2(n)} - (2\phi_{21}\sin\Psi_\infty\delta\Psi_n/\Delta_e \qquad (20)$$

The digital controller Equations (6), (7), (8) and (9) can be written:

$$\delta\Psi_n = (\delta u_{1n} - \delta\varepsilon_1)\ K_1 \qquad (21)$$

$$\delta u_1(n+1) = \delta u_{1n} + \nu_1(\delta u_{2n} - \delta\varepsilon_2)\ K_2 - \nu_1\ \delta\varepsilon_1 \qquad (22)$$

$$\delta u_2(n+1) = \delta u_{2n} - \nu_2\ \delta\varepsilon_2 \qquad (23)$$

The first two Equations (19) and (20) represent discrete linearized equations of the motor and they are independent of controller parameters. The other two Equations (22) and (23) depend on the motor and controller parameters.

The characteristic equation of the system can be deduced from the linearized model of the system (Equations 19 to 23) using the Z transform.

$$Z^4 - Z^3 D_1 + Z^2 D_2 + Z D_3 + D_4 = 0 \quad (24)$$

D_1 , D_2 , D_3 and D_4 are given in the Appendix.

7. CHOICE OF THE POLES OF THE SYSTEM.

The characteristic Equation (24) can be used for poles displacement. Our system consists of two loops. Current loop which is naturally fast. Its two poles can be chosen at the origin in the Z plane ($Z_1 = Z_2 = 0$). The speed loop is much slower than the current loop. Its two poles can be chosen to get the required response for motor speed [$Z_{3,4} = \exp(T_{3,4})S$. where, T is the period of converter = 10 m.s., $S_{3,4}$ are the chosen poles in the S-plane].
Then the characteristic Equation (24) may be written as:

$$Z^2 (Z - Z_3)(Z - Z_4) = 0 \quad (25)$$

8. SIMULATION PROGRAM.

A program of simulation is made taking into consideration all the discontinuties imposed by the converter, the motor and controller. The method of simulation is based on a general modelling of such a system [7]. The mathematical model is programmed.

9. RESULTS.

The considered system has the following characteristics:

The D.C. motor: 1HP, 1500 r/mine, 5 Amp, R=1.65 ohm, L=0.09 H, J=0.01985 Kgm², F=0.0055 Nm/rad.S^{-1}, Γ_c=6 N.m, K=0.518.

Speed transducer: $K_t = 0.573$ volt/rad/sec.

Current transducer: $K_i = 0.1$ volt.

The steady state operating point:

$\Psi_\infty = 58.1°$ $\qquad \Omega_\infty = 1500$ r.p.m.

Figure (4) shows the response of the system due to step change in speed reference for five values of poles, 20/135°, 60/135°, 100/135° , 200/135°. and 300/135°.

Figures (5) and (6) Experimental result for a step change in speed reference (P = 20/135°, P = 100/135°)

Figure (7) shows the current response due to step in speed reference.

Figure (8) represents the speed due to step change in motor load.

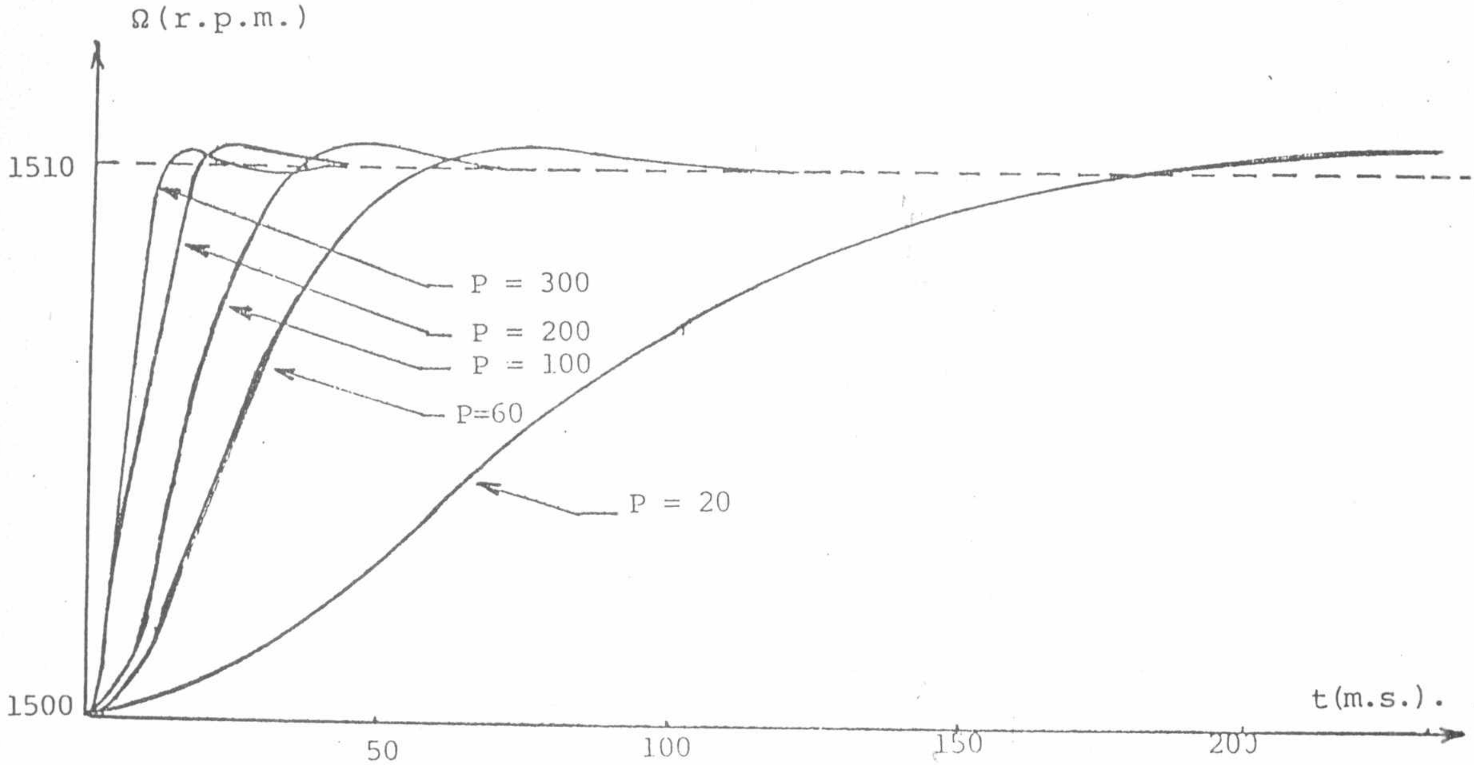

Fig. (4): The System Response Due to Step Change in Speed Reference.

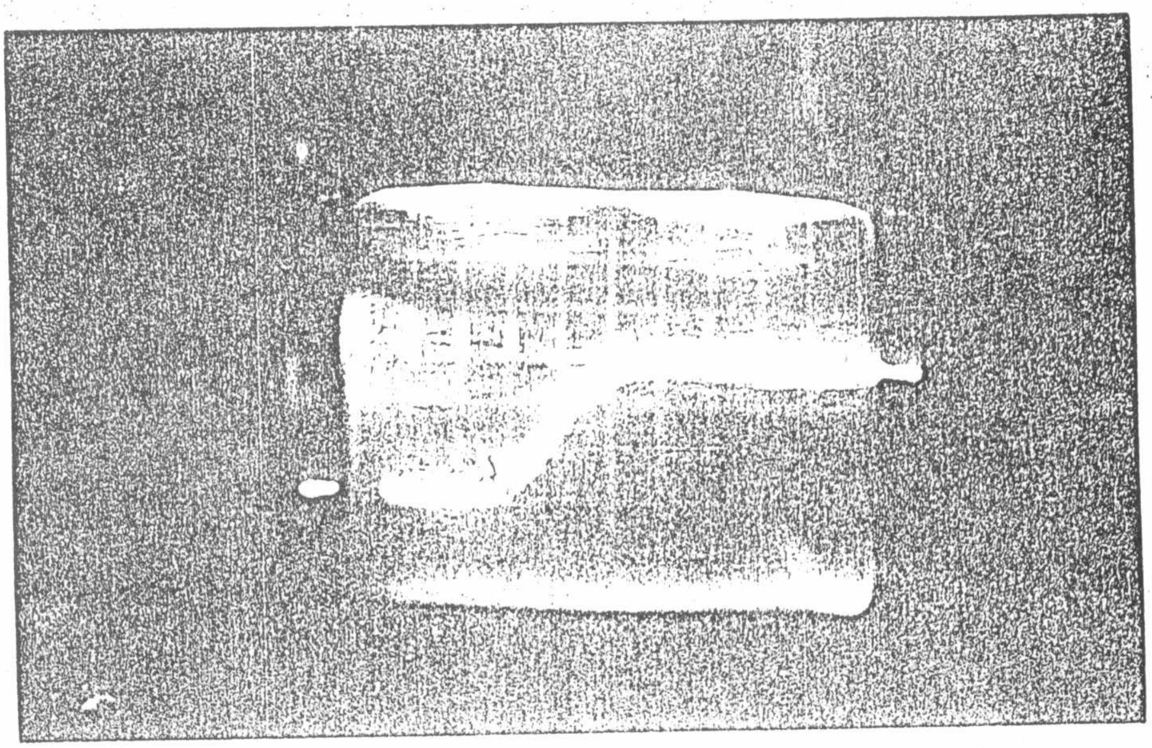

Fig. (5): Experimental Result for Astep change in Speed Reference.

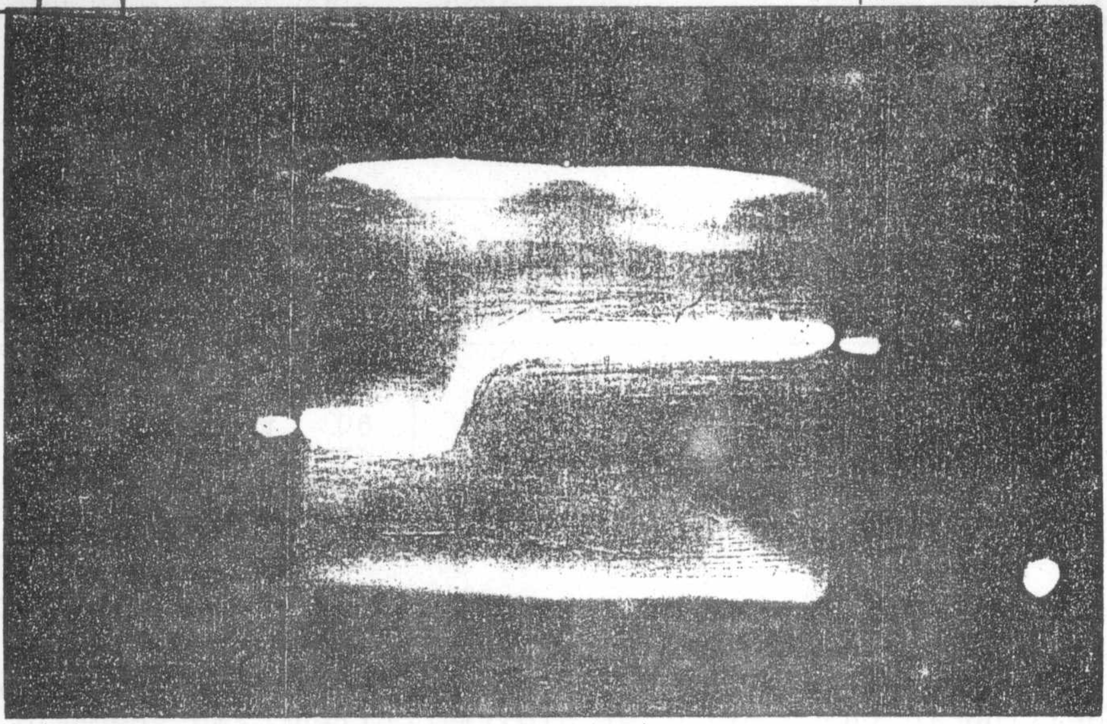

Fig.(6): Experimental result for a step change in speed referenfe (P = 100).

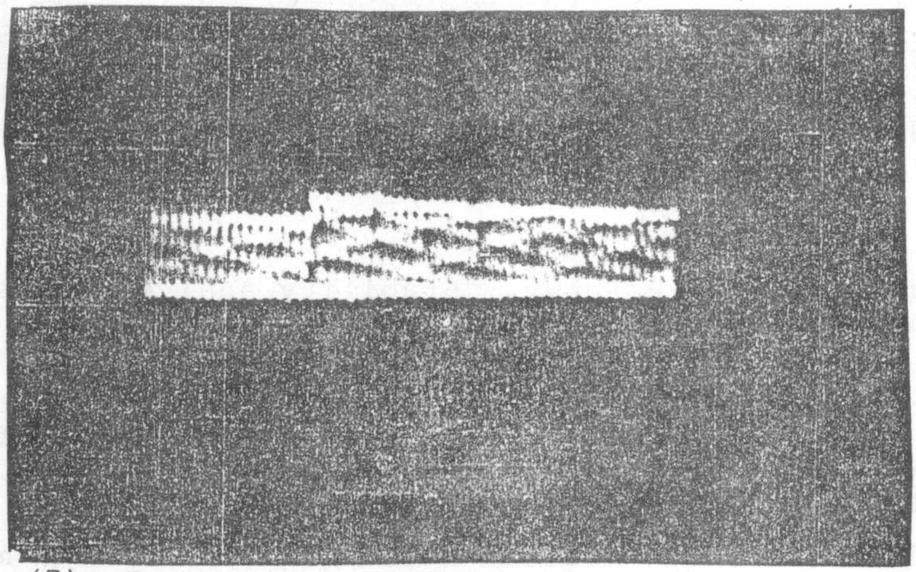

Fig. (7): Experimental current due to step change in speed reference.

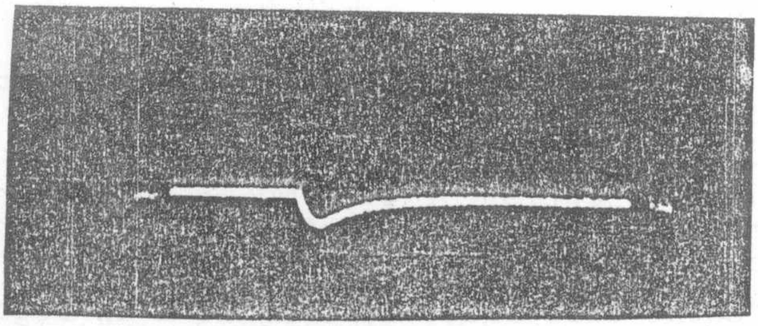

Fig.(8): Experimental speed due to step change in motor load.

The following table gives the values of the poles (P, t_r and over shoot).

Poles (P)	Mathematical		Simulation	
	$t_r=\sqrt{2}\ \pi/\ P$	overshoot	t_r	overshoot
20	222 m.s.	4.5 %	230 m.s	4 %
60	74 m.s	4.5 %	80 m.s	3.6 %
100	44 m.s	4.5 %	47 m.s	3.3 %
200	22 m.s	4.5 %	23 m.s	3.7 %
300	15 m.s	4.5 %	17 m.s	3.2 %

10. CONCLUSIONS.

In this paper we presented the application of general method of modelling in the control of digital current and speed of a D.C. motor. This method takes into consideration the non-linear discrete characteristics of the thyristor bridge. The formulae of calculating the controller parameters are given. A general programme of simulation is made to determine the speed response due to step change in speed reference. The experimental results have good agreement with those obtained by simulation.

11. REFERENCES.

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12. APPENDIX.

$$\Theta_{11} = [(1/\Delta_m+S_1)\exp(S_1\theta) - (1/\Delta_m+S_2)\exp(S_2\theta)]/2B$$

$$\phi_{12} = -[\exp(S_1\theta) - \exp(S_2\theta)]/2B\,\Delta_e$$

$$\phi_{21} = C[\exp(S_1\theta) - \exp(S_2\theta)]/2B\,\Delta_m$$

$$\phi_{22} = [(1/\Delta_e+S_1)\exp(s_1\theta) - (1/\Delta_e+S_2)\exp(S_2\theta)]/2B$$

$$B = \sqrt{[(1/\Delta_e-1/\Delta_m)^2 - 4C/\Delta_e\,\Delta_m]}/2$$

$$A = \phi_{1I}\ ,\ B = \phi_{12}\ ,\ C = 2\phi_{11}\sin\Psi_\infty/\Delta_e$$

$$A_1 = \phi_{21}\ ,\ B_1 = \phi_{22}\ ,\ C_1 = 2\phi_{21}\sin\Psi_\infty/\Delta_e$$

$$D_1 = A_3/A_2\ ,\ d_2 = A_4/A_2\ ,\ D_3 = A_5/A_2\ ,\ D_4 = a_6/A_2$$

$$\begin{aligned} A_6 = {} & A_1B-A_CK_1\nu_1K_2 - A\nu_1\nu_2K_2+C_1K_1B-C_1CK_1^2\nu_1K_2-C_1K_1\nu_1\nu_2K_2 \\ & -C_1K_1\nu_1B+CK_1^2K_2+C_1K_1\nu_1^2K_2+C_1K_1\nu_1^2\nu_2K_2+AC_1K_1\nu_1K_2+C_1K_1\nu_1K_2\nu_2 \\ & -AB_1+CC_1K_1^2\nu_1K_2+CC_1K_1^2\nu_1\nu_2K_2-CK_1B_1^2-2CC_1/C_1^2\nu_1K_2\nu_2^2-CC_1K_1^2\nu_1^2K_2 \\ & -CC_1K_1^2K_2\nu_2+B_1CK_1\nu_1. \end{aligned}$$

$$\begin{aligned} A_5 = {} & -2A_1B+ACK_1\nu_1K_2-2A_1B+2A_1K_1\nu_1K_2+2A_1\nu_1\nu_2K+2C_1K_1B+C_1CK_1^2\nu_1K_2 \\ & -2C_1K_1B+2C_1K_1^2C\nu_1K_2+2C_1K_1^2\nu_1K_2-2C_1K_1\nu_1\nu_2K_2+2BC_1K_1\nu_1 \\ & -C_1K_1^2{}_1^2K_2+BC_1K_1\nu_1-C_1CK_1^2\nu_1^2K_2-C_1K_1\nu_1^{22}\nu_2K_2-C_1K_1\nu_1K_2 \\ & -C_1K_1\nu_1K_2\nu_2+B_1-C_1K_2\nu_1K_2A+2B_1A+A-2AC_1K_1\nu_1K_2-2AC_1K_1\nu_1K_2\nu_2 \\ & +2AB_1-CC_1K_2^2\nu_1K_1+2CK_1B_1+CK_1-2CK_1^2C_1\nu_1K_2+2CK_1B_1+CC_1K_2^2\nu_1^2K_1 \\ & -2CK_1B_1\nu_1-Ck_1\nu_1+CC_1K_1^2\nu_1^2K_2+CC_1K_1^2\nu_1^2K_2\nu_2-B_1CK_1\nu_1\ . \end{aligned}$$

$$\begin{aligned} A_4 = {} & AB + 4A_1B - 2A_1CK_1\nu_1K_2 + AB - ACK_1\nu_1K_2 - A\nu_1\nu_2K_2 \\ & +C_1K_1B + 4C_1K_1B - 2C_1K_1^2C\nu_1K_2 + C_1K_1B - CK_1^2C\nu_1K_2 \\ & -C_1K_1\nu_1\nu_2K_2 - 2BC_1K_1\nu_1 + C_1CK_1^2\nu_1^2K_2 - C_1K_1\nu_1B \\ & +C_1K_2\nu_1K_2 - 2B_1-1+2C_1K_1\nu_1K_2 + 2C_1K_1\nu_1K_2\nu_2 - 2\nu B_1 \\ & -B_1A - 2A + 2AC_1K_2\nu_1K_2 - 4AB_1-2A+AC_1K_1\nu_1K_2+C_1K_1\nu_1K_2\nu_2A \\ & -AB_1 - CB_1K_1 - 2CK_1 + 2CC_1K_2^2\nu_1K_1 - 4CK_1B_1 - 2CK_1 \\ & +CC_1K_1^2\nu_1K_2+CC_1K_1^2\nu_1K_2\nu_2 - CK_1B_1 + CK_1\nu_1B_1 + 2CK_1\nu_1 \\ & - CC_1K_2^2\nu_1K_1\nu_1. \end{aligned}$$