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A SIMPLIFIED APPROACH FOR STEADY-STATE ANALYSIS OF THE ISOLATED SELF-EXCITED INDUCTION GENERATOR

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ABSTRACT :

This paper presents a new approach for analysing the steady-state performance of the isolated self-excited induction generator feeding R-L load using the conventional equivalent circuit of the machine. The approach is shown to be very efficient in analysing such systems under steady-state operation. Effects of various system parameters on the steady-state performance have been studied. It is shown that the analysis can be used to predict the terminal capacitance required to maintain a constant terminal voltage. The presented simulation results provide guidelines for optimum design of such systems.

1. Introduction :

The capacitor self-excited induction generators are becoming attractive for generating electric power from unconventional energy sources such as wind, biogas, small hydroheads...etc., specially in isolated and remote areas. These isolated power sources may meet the needs of the local people in the absence of the utility grid. Since induction generators can convert mechanical power to electrical power over a wide range of rotor speeds, they have been considered as high reliable generators for critical locations like aircrafts and fire-fighting equipments [1].

Such generators offer numerous advantages over synchronous generators like brushless construction (cage rotor), reduced size, no separate d.c. source for excitation, least maintenance and above all low cost. The flux build-up (self-excitation) in the air-gap depends essentially on the stator current. The capacitors help the stator voltage to build-up, hence allowing more and more stator current to flow. Some initial energy has to be present in the system. This can take the form of residual magnetizm in the rotor or in the form of a pulse of current through the stator while the machine is running [3].

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Owing to the increasing emphasis on the application of such generators, their steady-state analysis are gaining greater importance. If the terminal voltage and frequency are known, as in the case of a machine connected to an utility grid the prediction of performance is straightforward [7,8]. In a capacitor self-excited induction generator used as an isolated power source, both the terminal voltage and frequency are unknown and have to be computed for a given speed, capacitance, and load impedance. The analysis is complicated due to the magnetic saturation in the machine and the need to choose suitable parameters corresponding to the saturated condition. Steady-state analysis of such generators has been recently reported [2-6]. References [3,6] use the principle of conservation of real power and VARS. Newton-Raphson itteration technique has been adopted in references [2,5]. In reference [4] the analysis depends on the concept of operational equivalent circuit (complex frequency approach).

This paper proposes a simplified approach for analysing the steady-state performance of isolated self-excited induction generator feeding R-L load. The resultant equation is rewritten as a 7^{th}-degree polynomial in p.u. frequency and is solved by a numerical method. The approach proved to be simple and elegant if compared with the earlier approaches. The developed computer algorithm is used in studying the effect of the variation of generator and load parameters and excitation capacitor on the generated voltage and frequency.

2. Steady-State Analysis :

The analysis of the capacitor self-excited induction generator under steady-state operation can be made using the conventional equivalent circuit of the induction machine. However, as the operating frequency of the generator varies with the driving speed and load, the equivalent circuit has to be modified as shown in Fig.1, where all the parameters are referred to the rated frequency [6,8]. The following assumptions are made in this analysis :

i. Only the magnetizing reactance X_m is considered to vary with the level of saturation, all other parameters being constant.
ii. The magnetomotive force (MMF) space harmonics and time harmonics in the induced voltage and current waveforms are ignored.
iii. The core loss in the machine is neglected.

The steady-state equivalent circuit shown in Fig.1, can be simplified as follows :
Let:

Z_1 = stator Impedance per phase

$$= ((R_1/F) + R_L) + j(X_1 - X_L) \quad (1)$$

where :

$$R_L = R\,X_C^2/(F(RF)^2 + (XF^2 - X_C)^2) \quad (2)$$

$$X_L = X_C(R^2 + X(XF^2 - X_C))/((RF)^2 + (XF^2 - X_C)^2) \quad (3)$$

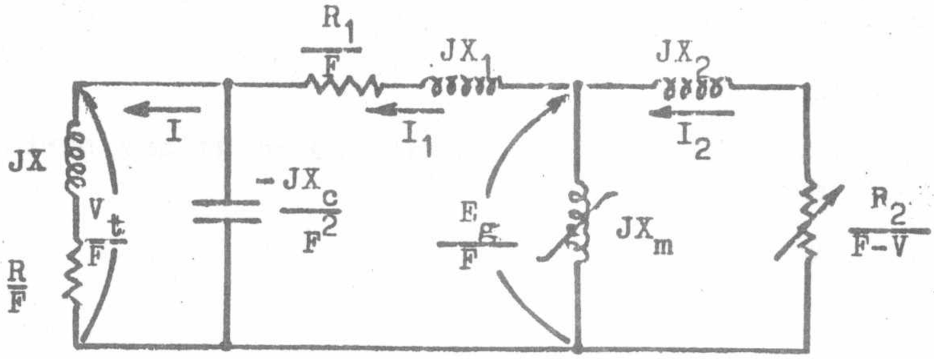

Fig. 1. Per-Phase Equivalent Circuit of Induction Generator at Excited Frequency.

and

Z_2 = rotor impedance per phase referred to stator.

$= (R_2/(F - V) + jX_2)$ (4)

Thus, the previous equivalent circuit can be reduced to :

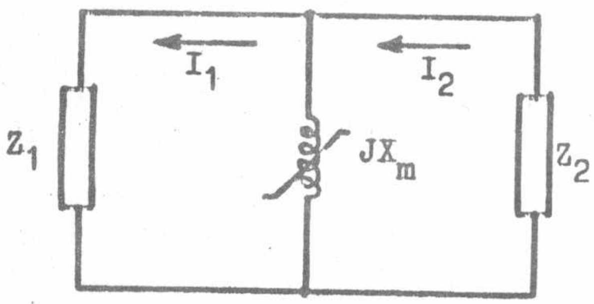

Fig. 2. Reduced Equivalent Circuit.

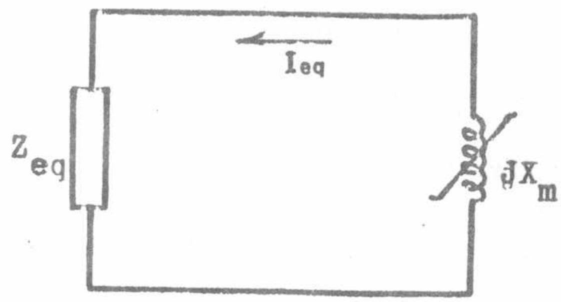

Fig. 3. Simplified Equivalent Circuit.

Since the three branches in Fig. 2, are in parallel, another reduction can be made by combining Z_1 and Z_2 into one impedance Z_{eq}, where

$$Z_{eq} = Z_1 Z_2/(Z_1 + Z_2) \qquad (5)$$

and the equivalent circuit of Fig. 2, reduces to that shown in Fig. 3.

This equivalent circuit does not contain any emf source or current source. Therefore, the loop equation of the current I_{eq} can be written as :

$$(Z_{eq} + jX_m) I_{eq} = 0 \qquad (6)$$

But, under steady-state self-excitation, I_{eq} does not equal to zero. Therefore, from equation (6),

$$Z_{eq} + jX_m = 0 \tag{7}$$

which implies that both the real and imaginary parts would be separately equal zero.

$$R_{eq} + jX_{eq} = - jX_m \tag{8}$$

From which :

$$R_{eq} = 0 \tag{9}$$

$$X_m = - X_{eq} \tag{10}$$

Since R_{eq} and X_{eq} are the real and imaginary parts of Z_{eq} respectively, Z_{eq} can be rewritten in the form :

$$Z_{eq} = R_{eq} + jX_{eq} = \frac{B + jG}{P + jQ} = \frac{(BP + GQ) + j(PG - BQ)}{P^2 + Q^2}$$

From which and referring to equations (9) & (10)

$$R_{eq} = \frac{BP + GQ}{P^2 + Q^2} = 0$$

$$\text{or} \quad BP + GQ = 0 \tag{11}$$

Also

$$X_{eq} = \frac{GP + BQ}{P^2 + Q^2} = - X_m$$

$$\therefore \quad X_m = (BQ - GP)/(P^2 + Q^2) \tag{12}$$

where :

$$B = C_{11}\ F^3 + C_{12}\ F^2 + C_{13}\ F + C_{14} \tag{13}$$

$$G = C_{21}\ F^4 + C_{22}\ F^3 + C_{23}\ F^2 + C_{24}\ F + C_{25} \tag{14}$$

$$P = C_{31}\ F^4 + C_{32}\ F^3 + C_{33}\ F^2 + C_{34}\ F \tag{15}$$

$$Q = C_{41}\ F^3 + C_{42}\ F^2 + C_{43}\ F + C_{44} \tag{16}$$

where the values of the coefficients C_{11} to C_{44} are given in Appendix (2).

Substituting the values of B, G, P & Q as given in equation (13) to (16) into equation (11), rearranging terms and simplifying yields :

$$C_1F^7 + C_2F^6 + C_3F^5 + C_4F^4 + C_5F^3 + C_6F^2 + C_7F + C_8 = 0 \tag{17}$$

where the values of the coefficients C_1 to C_8 are given in Appendix (3).

Equation (17) is the 7th-degree polynomial in one variable F which can be solved using a numerical technique. Values of the parameters R, X and C, and wide speed range showed that equation (17) has only one positive real root and three pairs of complex conjugate roots. Only the positive real root of (17) is physically acceptable. Substitution by the value of F in equation (12), yields the value of saturated magnetizing reactance X_m.

Having determined X_m and F, the next step is to calculate the air-gap voltage E_g and the terminal voltage V_t. For this purpose, one can use information regarding the variation of X_m with the quantity E_g/F as mentioned in reference [2]. This information can be obtained experimentally by driving the induction machine at synchronous speed (V = 1.0) corresponding to the supply frequency, i.e. F = 1.0, and measuring the magnetizing reactance, for different input voltages at supply frequency. A curve of E_g/F against X_m can be plotted using the experimental results. From this curve, E_g/F can be obtained for the steady-state saturated values of X_m. Knowing F, the air-gap voltage E_g can be computed.

With E_g, X_m, F, X_c, V, R, X and machine parameters known, the terminal voltage V_t, load current I, and power output P_{out} can be obtained from the equivalent circuit shown in Fig. 1. A summary of the corresponding expressions is given as follows :

$$I_1 = E_g/(F(R_1 + R_L) + j(X_1 + X_L))) \qquad (18)$$

$$I_2 = -E_g/(F(R_2/(F - V) + jX_2)) \qquad (19)$$

$$I = - jX_c\, I_1/(RF + j(XF^2 - X_c)) \qquad (20)$$

$$V_t = I(R + jXF) \qquad (21)$$

$$P_{in} = 3|I_2|^2 R_2/(F - V) \qquad (22)$$

$$T_{in} = P_{in}/w_s \qquad (23)$$

$$P_{out} = 3|I|^2 R \qquad (24)$$

$$\text{Total losses} = 3(|I_1|^2 R_1 + |I_2|^2 R_2) \qquad (25)$$

$$\text{Efficiency} = P_{out}/P_{in} = (1 - \text{losses}/P_{in}) \qquad (26)$$

3. No-LOad Condition :

In the equivalent circuit shown in Fig. 1, if R = ∞ and X = 0, the equivalent circuit reduces to :

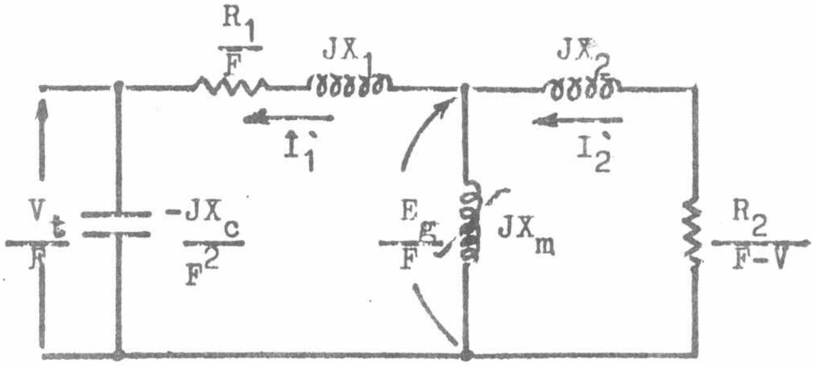

Fig.4. No-Load Equivalent Circuit.

A similar treatment for no-load condition of operation results in the following equations :

$$Z'_{eq} = (B' + JG')/(P' + JQ') \tag{27}$$

$$X'_m = (B'Q' - G'P')/(P'^2 + G'^2) \tag{28}$$

$$R'_{eq} = (B'P' + G'Q')/(P'^2 + Q'^2) = 0 \tag{29}$$

from which

$$B'P' + G'Q' = 0 \tag{30}$$

where :

$$B' = C'_{11} F^3 + C'_{12} F^2 + C'_{13} F + C'_{14} \tag{31}$$

$$G' = C'_{21} F^2 + C'_{22} F + C'_{23} \tag{32}$$

$$P' = C'_{31} F^2 + C'_{32} F \tag{33}$$

$$Q' = C'_{41} F^3 + C'_{42} F^2 + C'_{43} F + C'_{44} \tag{34}$$

The values of the coefficients C'_{11} to C'_{44} are given in Appendix (4).

Substitution by the values of B' , G' , P' and Q' as given in equations (31) to (34) yields the 5^{th}-degree polynomial in F.

$$C'_1F^5 + C'_2F^4 + C'_3F^3 + C'_4F^2 + C'_5F + C'_6 = 0 \tag{35}$$

where the values of the coefficients C'_1 to C'_6 are given in Appendix (5).

To determine the value of F at no-load, equation (35) is solved using the same numerical technique. Having determined the value of F, substitution in the equation (28) yields the value of X_m at no-load. Since X_m and F are determined, the calculation of the air-gap voltage E_g, terminal voltage V_t, currents in rotor and stator and output power are striaghtforward, and they are summarized below :

$$I_1 = (E_g . F)/(R_1 F + j(X_1 F^2 - X_c)) \tag{36}$$

$$I_2 = -(E_g/F)/(R_2/(F - V) + jX_2) \tag{37}$$

$$V_t = - jX_c \; I_1/F \tag{38}$$

$$P_{in} = - 3|I_2|^2 \; R_2/(F - V) \tag{39}$$

4. Machine Parameters, Results and Discussion :

To verify the proposed analysis given in this paper, the Mawdsley's generalized machine data given in ref. (2) is used for computation. The machine was connected as an induction machine with a 4-pole, 50 Hz, delta-connected stator winding rated 230 V, 8.2 A and 2.9 hp. Theoretical computations are carried out in per unit, using the following particulars of the machine :

V_{base} = rated phase voltage = 230 V
I_{base} = rated phase current = 4.73 A
Z_{base} = V_{base}/I_{base} = 48.582 ohm
P_{base} = Base power = V_{base} I_{base} = 1088.883 watt
N_{base} = base speed = 1500 rpm.

Induction machine parameters required in the analysis were determined experimentally using standard techniques. the parameter values thus obtained were :

R_1 = 3.0 ohm (0.062 p.u.)
R_2 = 3.4 ohm (0.07 p.u.)
X_1 = X_2 = 4.5 ohm (0.093 p.u.)

Since we need to know the value of E_g/F for a particular value of X_m, that value has been taken as the independent variable. The variation of E_g/F with X_m will be nonlinear due to magnetic saturation. To simplify the analysis the variation under the saturated region was linearized as follows :

$$\begin{aligned} E_g/F &= 1.64673 - 0.3246\, X_m \qquad 0 \le X_m \le 1.3894 \\ &= 1.76562 - 0.4102\, X_m \qquad X_m \ge 1.3894 \end{aligned} \tag{40}$$

These expressions are incorporated into the developed computer program for the analysis of isolated self-excited induction generators.

Fig. 5, shows calculated values of the generated voltage and frequency at no-load versus speed for C = 15, 20 and 30 μf. It is interesting to note that at fixed value of terminal capacitance, the terminal voltage increases with speed almost linearly. Also the difference in the value of terminal capacitance does not affect the frequency.

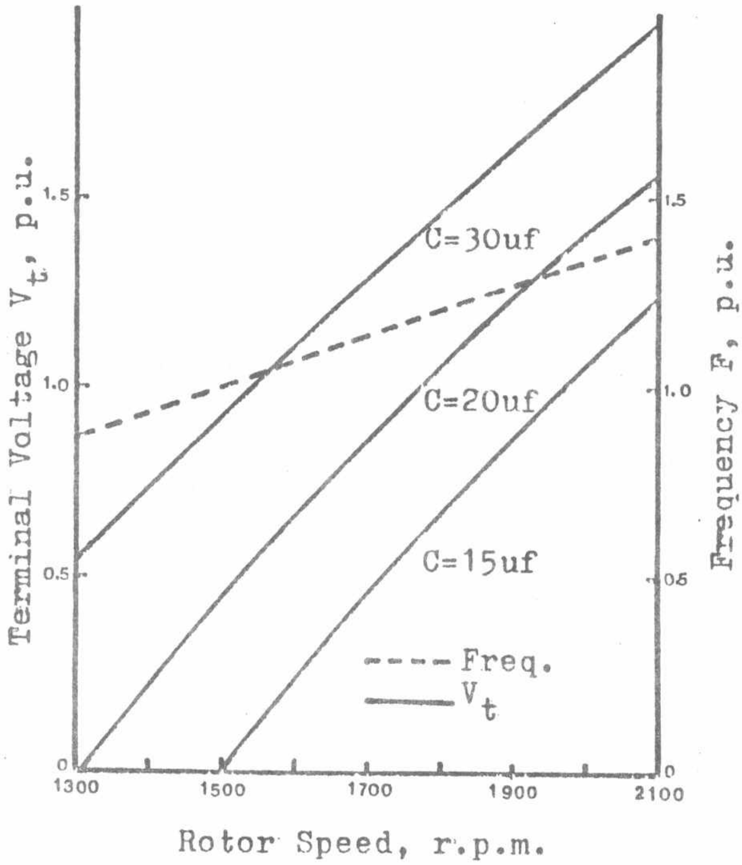

Fig.5. Generated Voltage and Frequency at No-Load.

Fig. 6, indicates the calculated no-load terminal voltage and frequency as a function of terminal capacitance at constant speed. Capacitance values were chosen above the minimum value required (which was 15 μf.) to cause self-excitation at V = 1.0 p.u. It will be noted that at a fixed speed the terminal voltage increases with capacitance. Several runs of the computer programs showed that the frequency is speed dependent and capacitance independent.

Fig. 7, presents a family of load characteristics computed for various values of C atV = 1.0 p.u.. It can be seen that the characteristics are almost parallel indicating the proportional increase of V_t with C. The frequency drop with load was not very much affected by the capacitance. At 1.0 p.u. speed, C has to be varied from 40 μf to 50 μf. to provide constant terminal voltage of 1.0 p.u. for output power varying from 1.0 to 1.94 p.u..

Fig. 8, gives the computed load characteristics of an induction generator that indicates the variation of terminal voltage with output power for a fixed C at constant speed and varying load power factor. Better voltage regulation is observed with leading p.f. loads. With lagging p.f. loads, the regulation becomes poorer, as the exciting capacitors must, in this case, supply the reactive load requirements, in addition to the magnetizing vars of the generator. For improved voltage regulation in this case, additional shunt capacitance would be necessary.

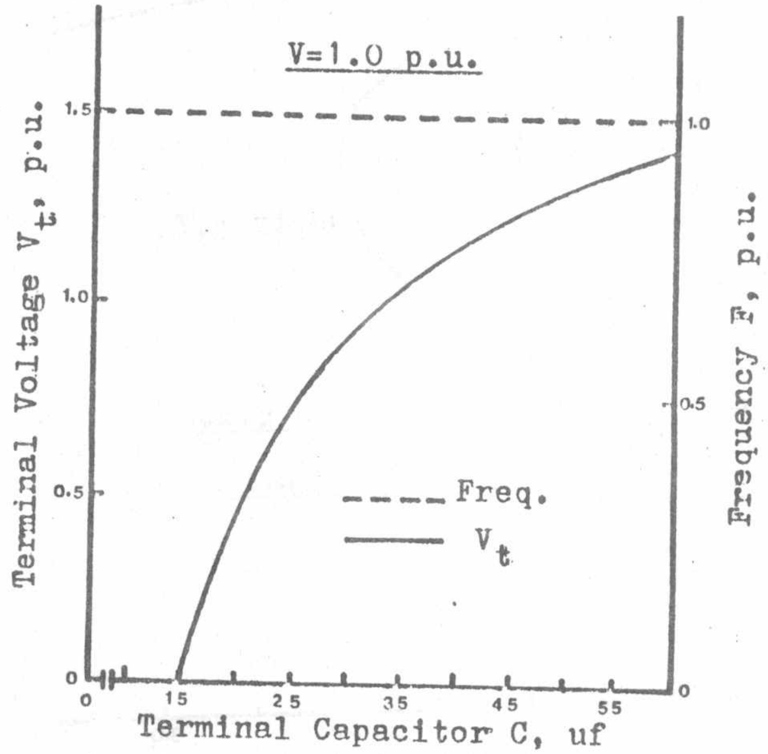

Fig.6. Generated Voltage and Frequency at No-Load.

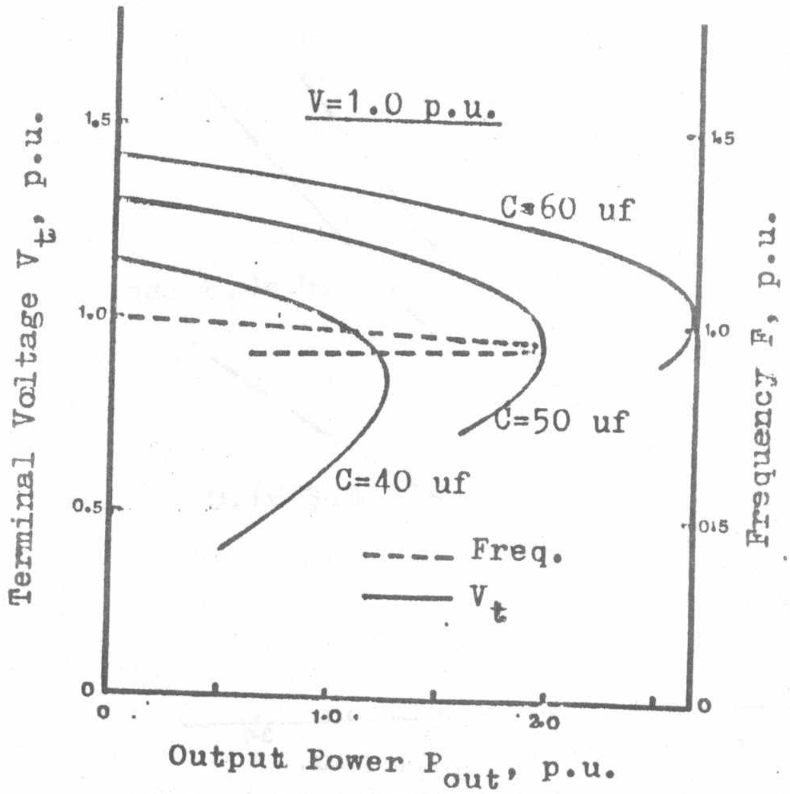

Fig.7. Load Characteristics.

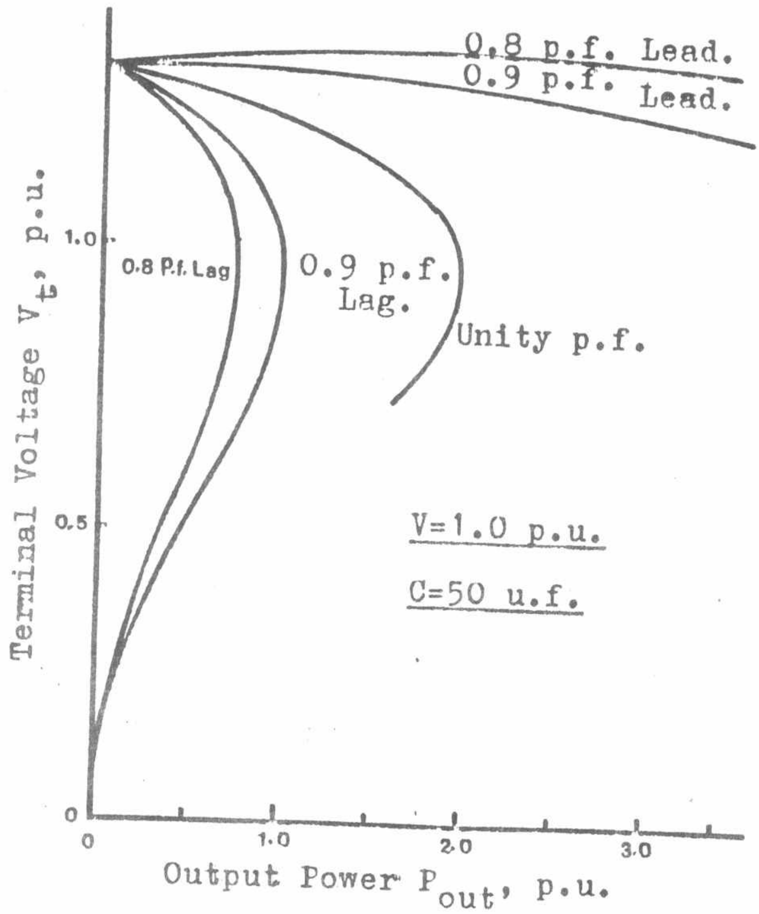

Fig.8. Effect of Load Power Factor.

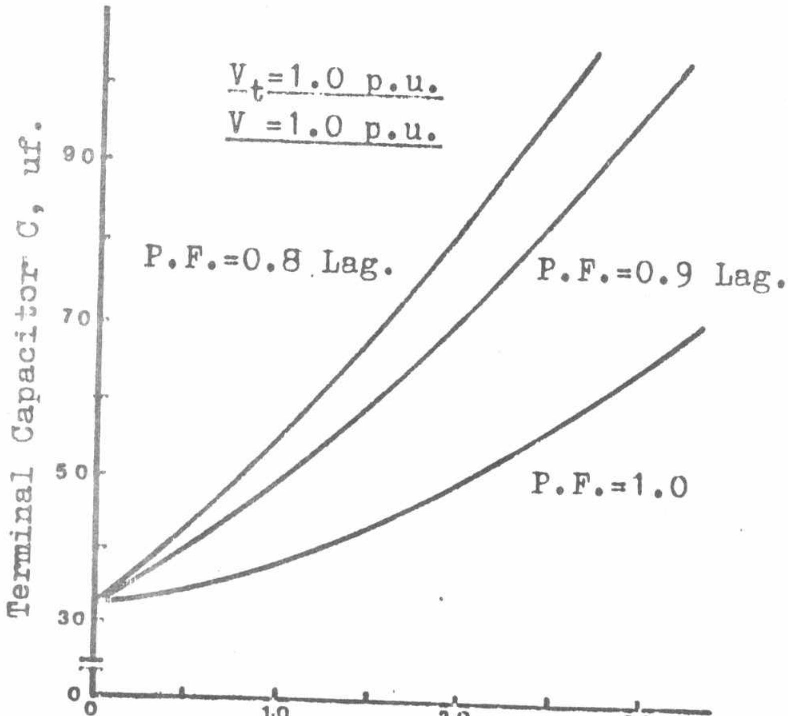

Fig.9. Capacitance Requirements to Maintain a Const. V_t Under Var. Induct. Load.

Fig. 9, shows the effect of load power factor on the value of capacitance C in order to maintain a constant terminal voltage of 1.0 p.u. at a speed of 1.0 p.u.. From this figure it is clear that the requirement of terminal capacitance from no load to output power of 2.0 p.u. at 0.8 power factor lagging lies in the range of 32.7 to 82.5 μf corresponding to a speed of one per unit.

5. Conclusion :

A new approach for the steady-state analysis of a capacitor self-excited induction generators feeding a general impedance load has been developed. The developed computer algorithm for computing the roots of n^{th}-degree polynomial and prediction of performance brings out the elegance of the proposed approach. The algorithm was used in studying the effect of the variation of generator and load parameters and excitation capacitor on the generated voltage and frequency at no-load and full-load.

Families of characteristics, showing the effect of various system parameters, are presented. As these curves are presented in per unit values, they may provide guidelines for appropriate design of the system. Variation of capacitance with load to maintain constant terminal voltage has been also determined. This will help in developing suitable voltage regulator. The results obtained indicate that capacitance requirement increases with decreasing speed, load impedance and p.f. (lagging loads).

Such steady-state analysis of induction generators, would not only facilitate the performance computation under different load situations, but also would be useful from both the design and operational point of view.

6. References :

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7. Appendix :

7.1. List of principal symboles :

R_1 , X_1 Stator resistance, leakage reactance per phase.
R_2 , X_2 Rotor resistance, leakage reactance per phase(referred to stator).
X_m Magetizing reactance per phase.
X_c Reactance of terminal capacitor C per phase.
R , X Load resistance, reactance per phase.
F , V Per-unit frequency and speed respectively.
E_g Air-gap voltage per phase.
V_t Terminal voltage per phase.
I_1 Stator current per phase.
I_2 Rotor current per phase.
I Load current per phase.
P_{in} Total input power.
P_{out} Total output power.
w_s Synch. speed in mechanical rad./sec.
T_{in} Total input Torque.

7.2. Coefficients of Equations (13) to (16) :

$$C_{11} = -X(X_1R_2 + X_2R_1) - RX_1X_2 \tag{41}$$
$$C_{12} = VX_2(XR_1 + RX_1) \tag{42}$$
$$C_{13} = RR_1R_2 + X_c(R_2(X + X_1) + X_2(R + R_1)) \tag{43}$$
$$C_{14} = -VX_2X_c(R + R_1) \tag{44}$$
$$C_{21} = -XX_1X_2 \tag{45}$$
$$C_{22} = XX_1X_2V \tag{46}$$
$$C_{23} = R(R_1X_2 + R_2X_1) + X(X_cX_2 + R_1R_2) + X_cX_1X_2 \tag{47}$$
$$C_{24} = -VX_2(RR_1 + X_c(X + X_1)) \tag{48}$$
$$C_{25} = -X_cR_2(R + R_1) \tag{49}$$
$$C_{31} = -X(X_1 + X_2) \tag{50}$$
$$C_{32} = X(X_1 + X_2)V \tag{51}$$
$$C_{33} = R(R_1 + R_2) + X_c(X + X_1 + X_2) \tag{52}$$
$$C_{34} = -V(RR_1 + X_c(X + X_1 + X_2) \tag{53}$$
$$C_{41} = X(R_1 + R_2) + R(X_1 + X_2) \tag{54}$$
$$C_{42} = -V(XR_1 + R(X_1 + X_2)) \tag{55}$$
$$C_{43} = -X_c(R + R_1 + R_2) \tag{56}$$
$$C_{44} = VX_c(R + R_1) \tag{57}$$

7.3. Coefficients of Equation (17) :

$$C_1 = C_{11}\,C_{31} + C_{21}\,C_{41} \tag{58}$$
$$C_2 = C_{11}\,C_{32} + C_{12}\,C_{31} + C_{22}\,C_{41} + C_{21}\,C_{42} \tag{59}$$
$$C_3 = C_{11}\,C_{33} + C_{12}\,C_{32} + C_{13}\,C_{31} + C_{23}\,C_{41} + C_{22}\,C_{42} + C_{21}\,C_{43} \tag{60}$$
$$C_4 = C_{11}\,C_{34} + C_{12}\,C_{33} + C_{13}\,C_{32} + C_{14}\,C_{31} + C_{24}\,C_{41} + C_{23}\,C_{42} + C_{22}\,C_{43} + C_{21}\,C_{44} \tag{61}$$
$$C_5 = C_{12}\,C_{34} + C_{13}\,C_{33} + C_{14}\,C_{32} + C_{25}\,C_{41} + C_{24}\,C_{42} + C_{23}\,C_{43} + C_{22}\,C_{44} \tag{62}$$
$$C_6 = C_{13}\,C_{34} + C_{14}\,C_{33} + C_{25}\,C_{42} + C_{24}\,C_{43} + C_{23}\,C_{44} \tag{63}$$

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$$C_7 = C_{14}\, C_{34} + C_{25}\, C_{43} + C_{24}\, C_{44} \tag{64}$$
$$C_8 = C_{25}\, C_{44} \tag{65}$$

7.4. Coefficients of Equations (31) to (34) :

$$C'_{11} = -\, X_1\, X_2 \tag{66}$$
$$C'_{12} = V X_1\, X_2 \tag{67}$$
$$C'_{13} = R_1\, R_2 + X_2\, X_c \tag{68}$$
$$C'_{14} = -\, V\, X_2\, X_c \tag{69}$$
$$C'_{21} = R_1\, X_2 + R_2\, X_1 \tag{70}$$
$$C'_{22} = -\, V\, R_1\, X_2 \tag{71}$$
$$C'_{23} = -\, R_2\, X_c \tag{72}$$
$$C'_{31} = R_1 + R_2 \tag{73}$$
$$C'_{32} = -\, V\, R_1 \tag{74}$$
$$C'_{41} = X_1 + X_2 \tag{75}$$
$$C'_{42} = -\, V(X_1 + X_2) \tag{76}$$
$$C'_{43} = -\, X_c \tag{77}$$
$$C'_{44} = V\, X_c \tag{78}$$

7.5. Coefficients of Equation (35) :

$$C'_1 = C'_{31}\, C'_{11} + C'_{21}\, C'_{41} \tag{79}$$
$$C'_2 = C'_{31}\, C'_{12} + C'_{32}\, C'_{11} + C'_{21}\, C'_{42} + C'_{22}\, C'_{41} \tag{80}$$
$$C'_3 = C'_{31}\, C'_{13} + C'_{32}\, C'_{12} + C'_{21}\, C'_{43} + C'_{22}\, C'_{42} + C'_{23}\, C'_{41} \tag{81}$$
$$C'_4 = C'_{31}\, C'_{14} + C'_{32}\, C'_{13} + C'_{21}\, C'_{44} + C'_{22}\, C'_{43} + C'_{23}\, C'_{42} \tag{82}$$
$$C'_5 = C'_{32}\, C'_{14} + C'_{22}\, C'_{44} + C'_{23}\, C'_{43} \tag{83}$$
$$C'_6 = C'_{23}\, C'_{44} \tag{84}$$