A STABLE ADAPTIVE ALGORITHM FOR LINEAR SYSTEMS PARAMETER IDENTIFICATION

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ABSTRACT

This paper develops an updating algorithm for identifying the parameters of a known structure linear system. As it is known that recursive least squares algorithms are the best for identifying the parameters of time invariant systems. However, in case of having time varying system, the above algorithm can not track the variation in parameters in a good shape. Therefore, two approaches have been developed in the literature to resolve this problem i.e., the forgetting factor approach and the finite window one. This paper unifies the above two approaches in a stable fashion by having an algorithm that contains a variable forgetting factor and a constant window. The effectiveness of the proposed algorithm in tracking the system variable parameters is demonstrated by an example.

INTRODUCTION:

The effectiveness of adaptive controllers in realizing the objective they have already designed for depends to a large extent on the way considered for identifying the plant parameters. When these controllers are applied to time-varying systems old information used in the identifier should be discarded in favor of the recent ones in order that the resulting estimates are close to the actual ones and to guarantee high performance for the installed controller. Therefore, the recursive least squares method is no longer appropriate for such systems since it employs the whole old and new data in giving an estimate even if some of these are irrelevant to the actual values of the system parameters. It is incapable of discounting or discarding these old irrelevant data.

Several methods to discard these old data have been suggested in the literature. The most common proposed two methods are the *Forgetting factor or the moving window* approaches [5]. In the forgetting factor one, an exponential weighting is given to the old data while the recent one is given a weight of one. However, for a moving window one old data are completely removed.

In case of having a constant forgetting factor less than unity, the gain matrix approaches the unity one resulting in an algorithm similar to the normal gradient one i.e. slow -

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rate of convergence. However to have a variable one means that the gain is rich enough to guarantee a quadratic rate of convergence.
When more information about the variation of the plant parameters are available, more sofisticated adaptive algorithms can be considered such as variable forgetting factor or swinging window [5]. In case of having variable forgetting factor, a constant trace for the information matrix P^{-1} is assumed and upon which the value of the new forgetting factor can be obtained [4].

PROBLEM FORMULATION:

The system to be identified is assumed to be a single-input/single-output, deterministic, discrete-time, with time-varying parameters described by;

$$A(q^{-1})\ y(t) = q^{-d}\ B(q^{-1})\ u(t) \tag{1}$$

where q^{-1} is a unit delay operator, $\{u(t)\}$, and $\{y(t)\}$ are the input and output sequences, respectively, and d is the system delay. The polynomials A and B are defined by;

$$A(q^{-1}) = 1 + a_1\ q^{-1} + \ldots + a_n\ q^{-n} \tag{2}$$

$$q^{-d}B(q^{-1}) = \quad b_d\ q^{-d} + \ldots + b_m\ q^{-m} \tag{3}$$

where $b_d \neq 0$. The following assumptions are made;

A1: The delay d is known (consider d=1)
A2: The orders n, m are known
A3: Both zeros of A and B are time-varying but *ALL* the time inside the unit circle.

Having equation (1), the plant output at instant t can be expressed as;

$$\begin{aligned} y(t) &= -a_1(t)y(t-1) - a_2(t)y(t-2) - \ldots - a_n(t)y(t-n) + \\ &\quad + b_1(t)u(t-1) + b_2(t)u(t-2) + \ldots + b_m(t)u(t-m) \\ &= [-a_1(t), -a_2(t), \ldots, -a_n(t), b_1(t), b_2(t), \\ &\qquad \ldots, b_m]. \ w(t) \\ &= \theta'(t).\ w(t) \end{aligned} \tag{4}$$

where ' denotes the transposition of the vector and w(t) is the transpose of [y(t-1), y(t-2), ..., y(t-n), u(t-1), u(t-2), ..., u(t-m)].

A following model having a structure similar to that of the plant under consideration, but with an adjustable set of parameters is given by;

$$\hat{y}(t) = \hat{\theta}'(t)\, w(t) \tag{5}$$

Using equations (4) and (5), an error equation is generated as;

$$\begin{aligned} e(t) &= y(t) - \hat{y}(t) \\ &= (\theta(t)-\hat{\theta}(t))' w(t) \\ &= \emptyset'(t).w(t) \end{aligned} \tag{6}$$

where $\emptyset(t)$ is the parameter error vector and $e(t)$ is the measurable output error.

The problem is formulated as follows; what is the proper adaptive law for updating $\emptyset(t)$ such that $e(t)$ be close as much as possible to zero in case of any type of variations in $\emptyset(t)$?.

THE ADAPTIVE LAW:

Consider the following adaptive law for updating the model following parameters;

$$\Delta \emptyset(t) = -P(t+1).e(t).w(t) \tag{7}$$

$$P^{-1}(t+1) = \lambda_t P^{-1}(t) + s(t)s'(t) \tag{8}$$

where;

$$s(t)s'(t) = w(t)w'(t)/v1 - w(t-N)w'(t-N)/v2 + + (1-\lambda_t)\, aI \tag{9}$$

$$\begin{aligned} a &= 1/(w'(t)w(t)) \\ v1 &= 1 + 1/a^2 \\ v2 &= (w'(t).w(t-N))^2 \end{aligned} \tag{10}$$

$$\lambda_t = 1/(w'(t)P^{-1}(t)w(t) - 1)$$

Adopting such an algorithm, the tracking error would tend to zero in case of a sudden change or to a minimum in case of slowly varying parameters. The variation of parameter vector can be expressed in the form;

$$\theta(t+1) = \theta(t) + f(t+1) \tag{11}$$

where $f(t+1)$ is a forcing function vector representing the rate at which $\theta(t)$ could vary. For a sudden change in the parameters, $f(t+1)$ is chosen as a step function vector acting at an instant $t=t_j$. For a smooth varying one, the vector $f(t+1)$ could be a sinusoidal set with different frequencies bounded from above by a bound far from that corresponding to the system time constant. For a random variation in the system parameters, $f(t+1)$ is chosen as normally distributed gaussian with zero mean and a variance σ^2 to give a random walk for the parameter variations [3].

ADAPTATION OF P(T):

In recursive least squares algorithms, the adaptive gain is updated according to;

$$P^{-1}(t+1) = P^{-1}(t) + w(t)w'(t) \qquad (12)$$

which can be represented by the integrator shown in Fig. 1, and this in turn produces a growing information matrix [3] $P^{-1}(t)$ and a corresponding decaying gain matrix $P(t)$. In order to prevent this gain matrix from going to zero, a forgetting factor less than one is employed as depicted in Fig. 2.

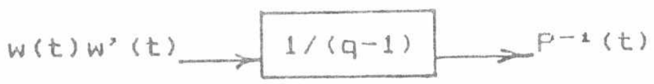

Fig. 1

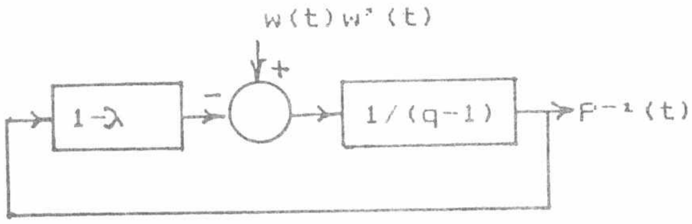

Fig.2

Using Fig. 2, the matrix $P^{-1}(t+1)$ is given by;

$$P^{-1}(t+1) = \lambda P^{-1}(t) + w(t)w'(t) \qquad (13)$$

where $\lambda \in (0,1)$. In [1], a variable forgetting factor is proposed to get better tracking for the varying parameters. On the other hand, a finite window or a swinging one is proposed in [5] to guarantee the boundedness of the information matrix $P^{-1}(t)$ using;

$$P^{-1}(t+1) = P^{-1}(t) + w(t)w'(t) - w(t-N_i)w'(t-N_i) \qquad (14)$$

where N_i is a swinging window i.e. $N_i = N$ or $N_i = 2N$ and having both back and forth, such that the proper amount of information is retained.

The approach suggested in this paper, considers the adaptation of the matrix $P^{-1}(t)$ according to Fig. 3 with the exception of a time varying λ_t instead of the constant one as shown in figure.

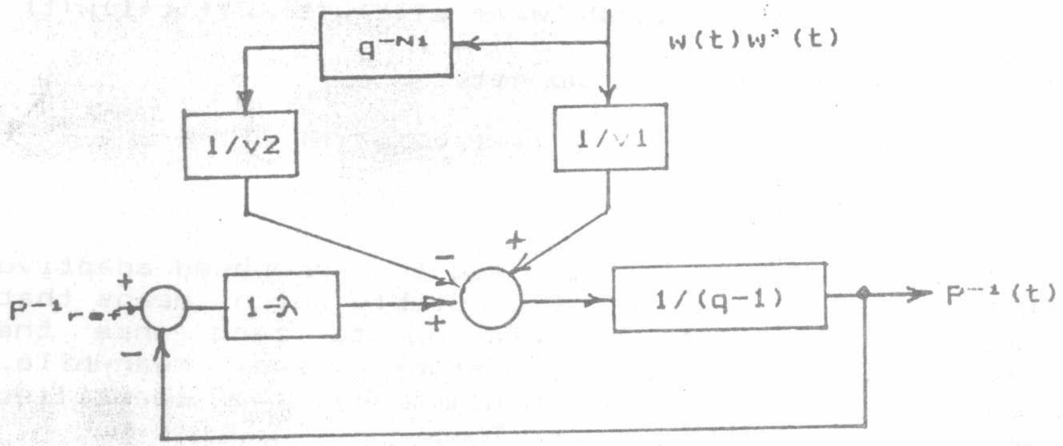

Fig. 3

i.e.

$$P^{-1}(t+1)=\lambda_t P^{-1}(t)+[w(t),\ w(t-N)]\begin{bmatrix}1/v1 & 0\\ 0 & 1/v2\end{bmatrix}\begin{bmatrix}w'(t)\\ w'(t-N)\end{bmatrix}+$$

$$+\ (1-\lambda_t)\ P^{-1}_{ref} \quad (15)$$

Assuming that $P^{-1}_{ref} = \ \in I$ with $\in \ll$, then using the matrix inversion lemma [7], we get;

$$P(t+1)= P(t)/\lambda_t - P(t)/\lambda_t.(w*(t)[w*'(t)P(t)w*(t)+diag(v1,$$

$$-v2)]^{-1}\ w*'(t)P(t) \quad (16)$$

where $w^*(t) = [w(t),\ w(t-N)]$

STABILITY OF THE PROPOSED APPROACH:

Consider the following *Lyapunov* candidate for the adaptation algorithm given in (9) and (10);

$$V_t = \emptyset_t' P^{-1}(t)\emptyset_t \quad (17)$$

$$\Delta V_t =: V_{t+1} - V_t$$

$$= [\ \emptyset_t + \Delta\emptyset_t\ ]'\ P^{-1}(t+1)\ [\ \emptyset_t + \Delta\emptyset_t\ ] -$$

$$- \emptyset_t' P^{-1}(t)\emptyset_t$$

$$= \emptyset_t' P^{-1}(t+1)\emptyset_t - \emptyset_t' P^{-1}(t)\emptyset_t +$$

$$+\ 2\ \emptyset_t' P^{-1}(t+1)\Delta\emptyset_t + \Delta\emptyset_t' P^{-1}(t+1)\Delta\emptyset_t$$

Using the adaptation law for the gain P_{t+1}^{-1} (8) and the error equation (6), we get;

$$\Delta V_t = -(1-\lambda_t)V_t - (2-1/v1)e^2(t) - e^2(t,N)/v2 +$$

$$+ (1-\lambda_t)a\phi_t'\phi_t + e^2(t)w(t)'P^{-1}(t+1)w(t)$$

Applying the results in (10), we get;

$$\Delta V_t < -(1-\lambda_t)V_t - (1+1/v2)e^2(t)$$

$$< 0$$

i.e. V_t is a Lyapunov function and the proposed adaptive algorithm is uniformly asymptotic stable which means that the tracking error e(t) should go to zero once the identified parameters reach a constant value. Meanwhile, this tracking error would be minimum if the identified parameters are still varying.

NUMERICAL EXAMPLE:

A second order discrete-time linear system is considered and one of its two unknown parameters is given a step change of 50% decrease is assumed to happen after 15 samples from the initiation of the identifier. The system dynamic equation is;

$$y(t) = -zy(t-1) - .35y(t-2) + u(t-1)$$

where z = 0.8 and 0.4

The identifier used is having a series-parallel model type and fed by the system input and output, it is represented as;

$$ym(t) = -b_1y(t-1) - b_2y(t-2) + u(t-1)$$

with zero initial values for b_1 and b_2. Assuming an input of the form;

$$u(t) = \sin(4.21t) + \sin(.223t)$$

and initial value for the gain matrix P(0)=1000I. The time history of parameter identifications using (a) Recursive Least Squares method is given in Fig. 4. (b) Forgetting Factor only is given in Fig. 5. (c) Forgetting Factor with A window of 35 samples is shown in Fig. 6

From the study of the time histories of the different methods, it can be concluded the superiority of the proposed algorithm given that the whole degrees of freedom are availble.

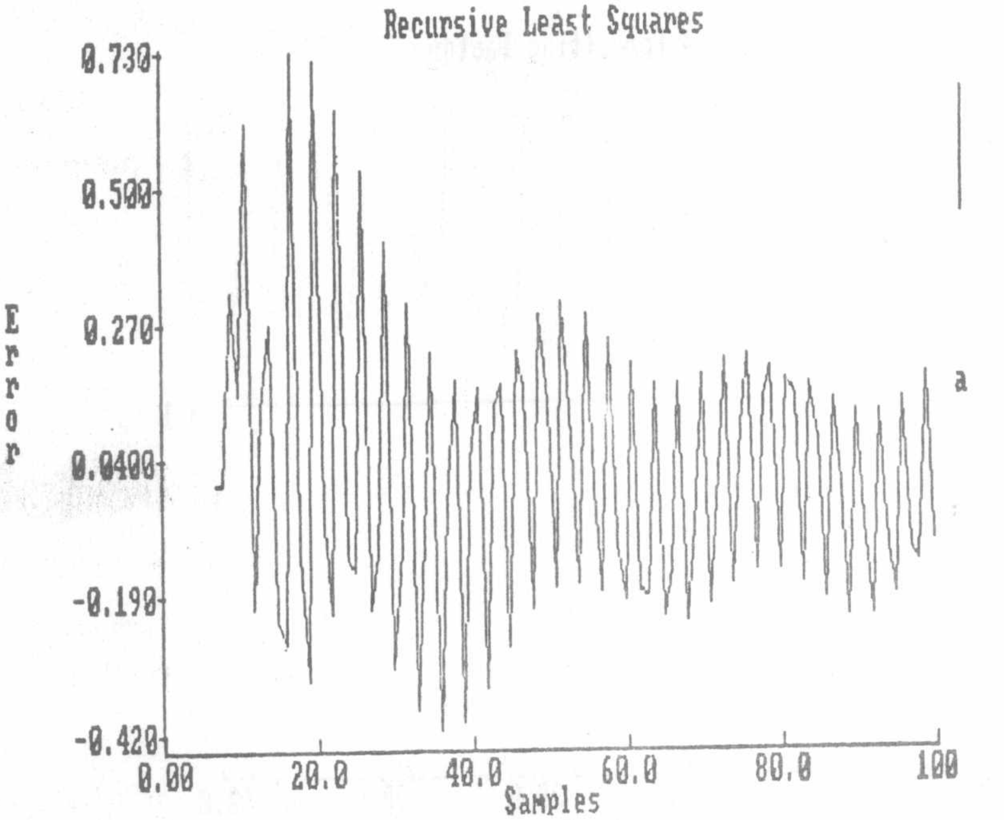
Recursive Least Squares
0.730
0.500
0.270
0.0400
-0.190
-0.420
Error
0.00
20.0
40.0
60.0
80.0
100
Samples
a

Fig. 4-a

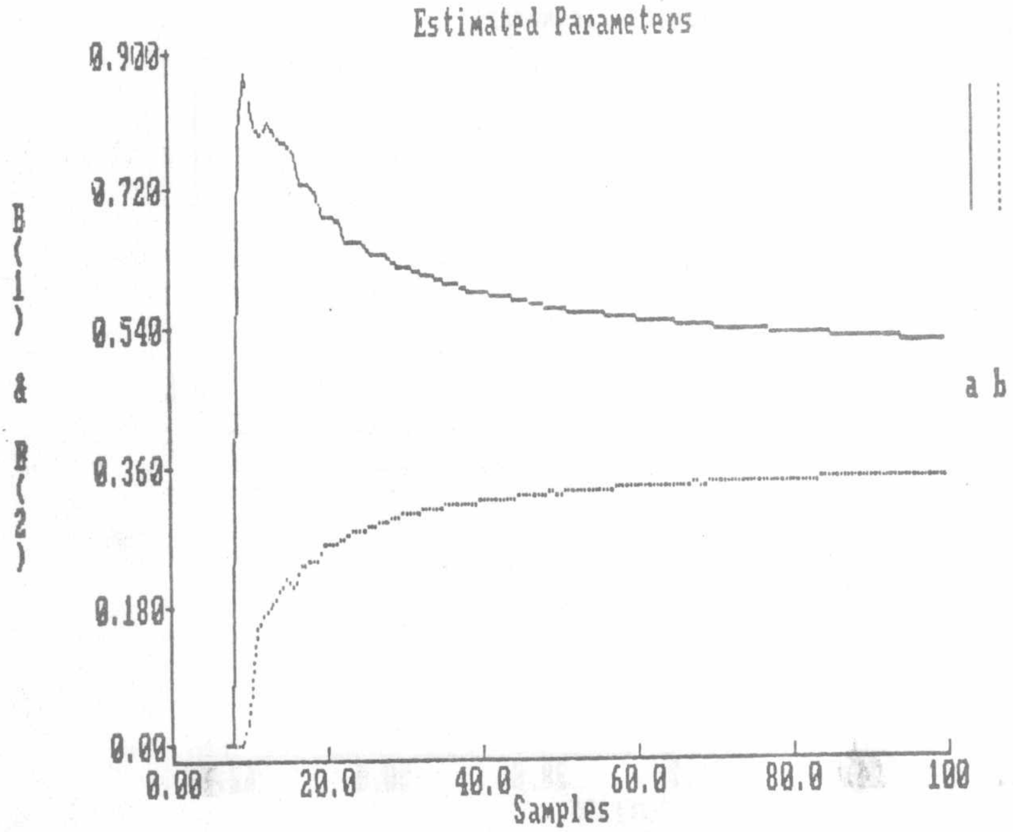
Estimated Parameters
0.900
0.720
0.540
0.360
0.180
0.00
B(1) & B(2)
0.00
20.0
40.0
60.0
80.0
100
Samples
a b

Fig. 4-b

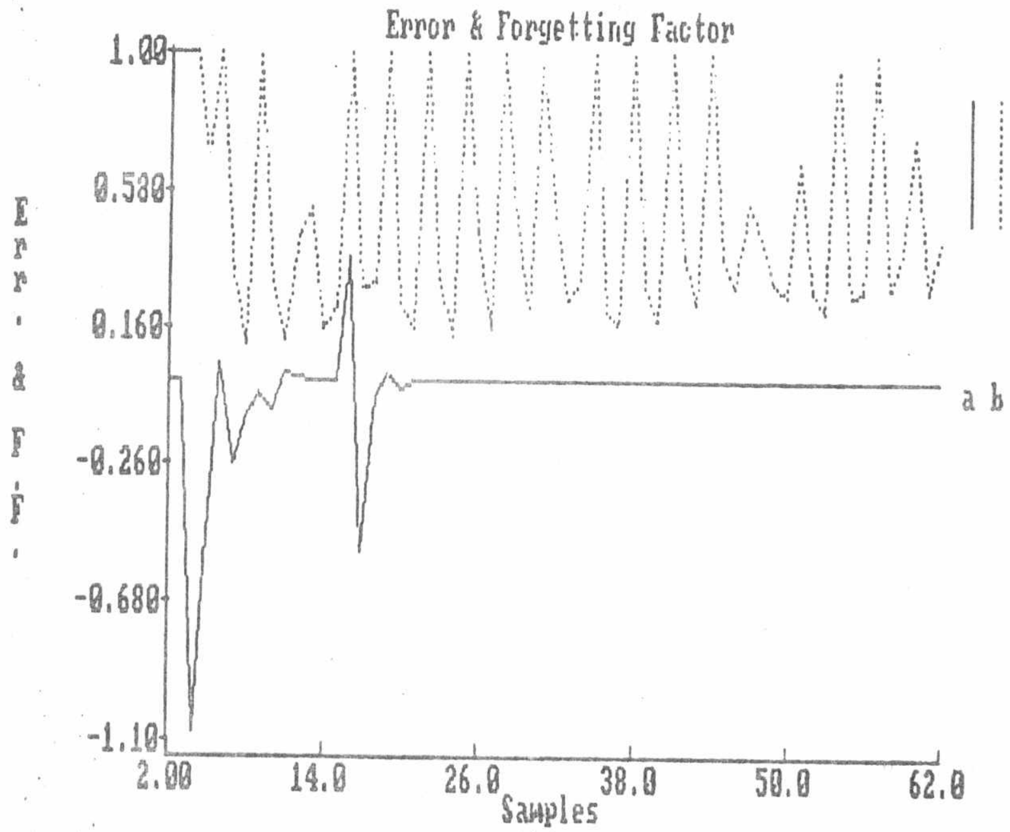
Error & Forgetting Factor
1.00
0.580
0.160
-0.260
-0.680
-1.10
Err. & F.F.
2.00
14.0
26.0
38.0
50.0
62.0
Samples
a b

Fig. 5-a

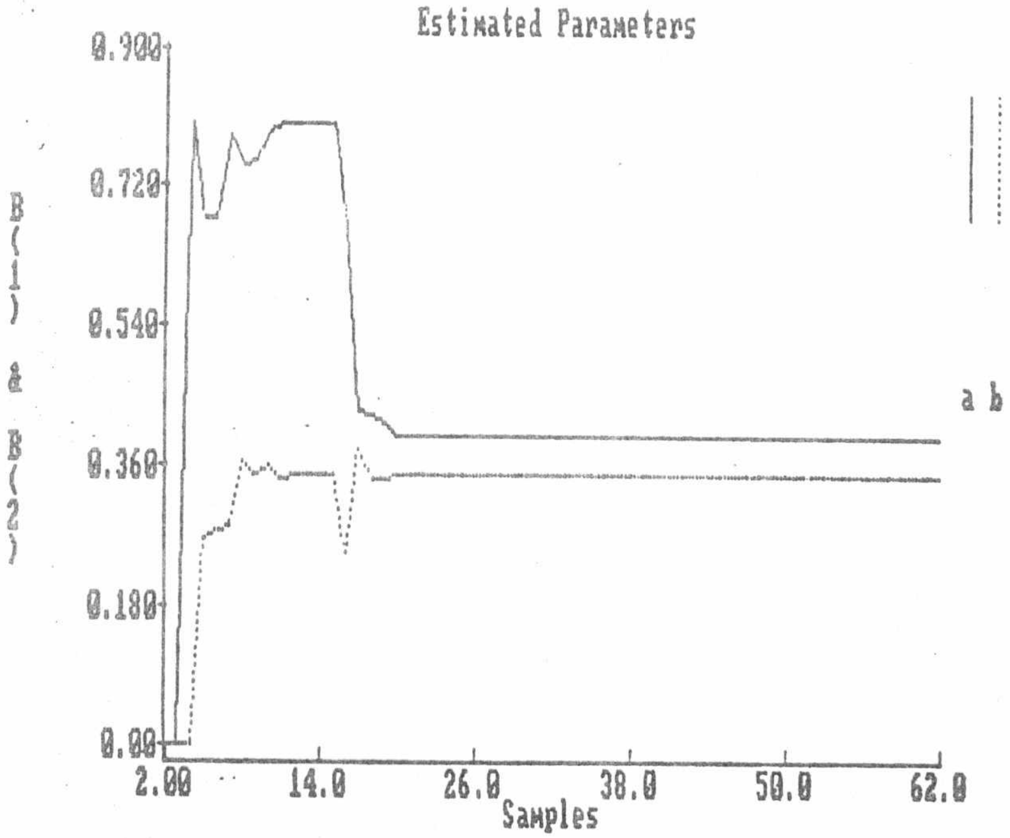
Estimated Parameters
0.900
0.720
0.540
0.360
0.180
0.00
B(1) & B(2)
2.00
14.0
26.0
38.0
50.0
62.0
Samples
a b

Fig. 5-b

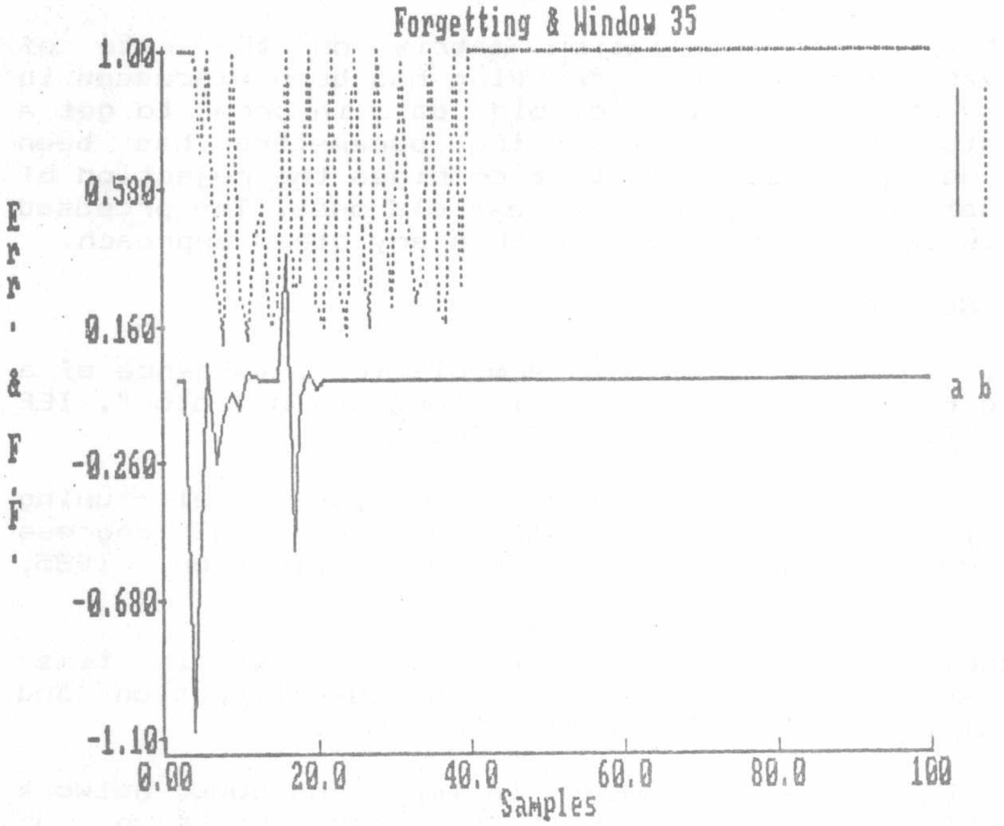
Forgetting & Window 35
1.00
0.580
0.160
-0.260
-0.680
-1.10
Err. & F.F.
a b
0.00
20.0
40.0
60.0
80.0
100
Samples

Fig. 6-a

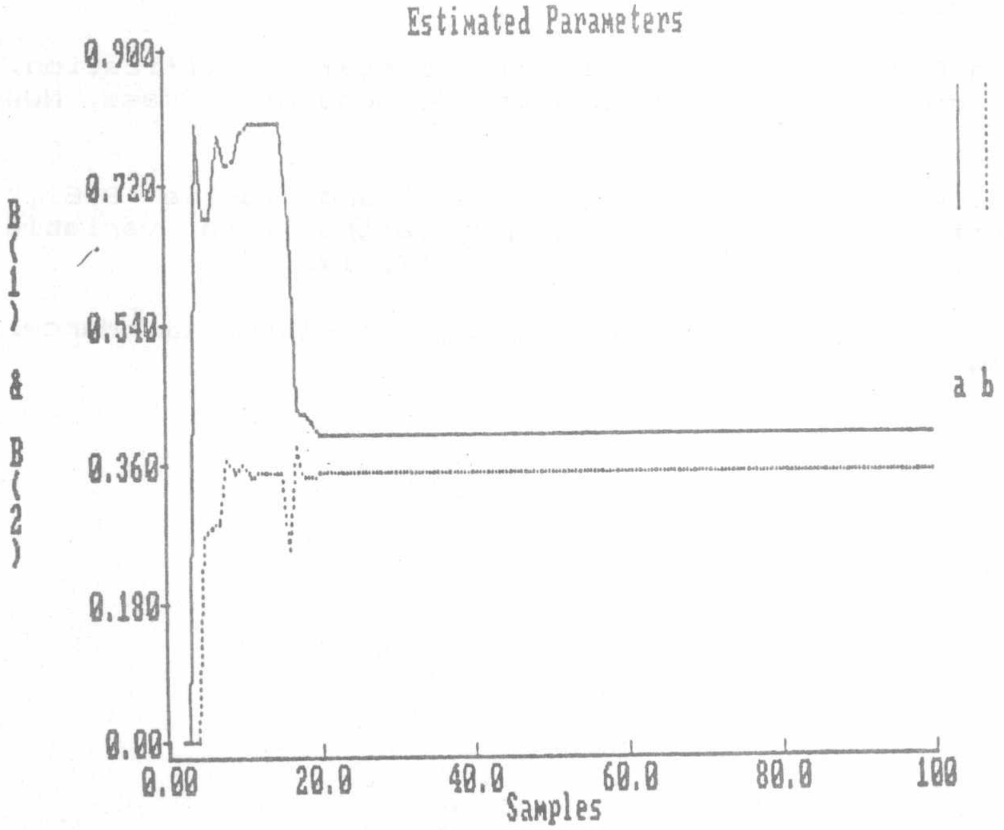
Estimated Parameters
0.900
0.720
0.540
0.360
0.180
0.00
B(1) & B(2)
a b
0.00
20.0
40.0
60.0
80.0
100
Samples

Fig. 6-b

CONCLUSIONS:

The effect of the information matrix on the rate of parameter variation and their tracking has been discussed in this paper. The discounting of old data in order to get a true estimate for the plant varying parameters has been considered using an approach that combines the rejection of very old data and weigting the near old ones. The proposed algorithm is shown to be superior than any other approach.

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