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A NEW APPROACH FOR THE DESIGN OF AN AEROSPACE SYSTEM

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ABSTRACT

A new approach is given for the design of an aerospace knowledge based expert system.

We deal with the critical problems in connection with optimum methods of utilizing the firing means against located targets. With the assumption that the probability of the target's hitting is a function of its stay in fire system, it was possible to convert the solution of optimal control into generalized assignment problem for aerospace system and obtain the optimum distribution of targets.

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I- INTRODUCTION

The process of fire control of ground air defence means can be divided, in rough, into the following stages :

1-Detection of targets.
2- Acquisition of information on one's own firing means.
3- Data communications of targets and own firing means to control center.
4- Processing of these information.
5- Elaboration of decision concerning the optimum method of utilization of firing means against located targets
6- Decision on opening the combat activity.
7- Data communication (in the form of an order) of the assignment of targets to concrete firing means.
8- Opening of fire against the assigned targets.

All the stages of processes of the command and control(except the stage 6) must be carried out automatically. It is possible only in this way to achieve the necessary continuity, speed and reliability of operation of the command system. Man can enter the control process at one point only: on basis of automatically obtained information on situations and on the basis of machine designed optimum decision he may answer "Yes" or "No" to the question whether the combat activity is to be stared (i.e. to satrt the stage 7).

The design of automatic system of command control requires, on the one hand, to solve a number of the critical problems connected with the control process and,on the other hand, to provide the necessary technical equipments(first of all suitable computers).

II- EFFICIENCY OF A CENTRALIZED AND DECENTRALIZED FIRE CONTROL

The higher effeciency of a centralized fire control system will be seen when comparing the expected results(mathematical expectations) of both types of fire control, for equal effeciency of fire subsystems.

THE CENTRALIZED SYSTEM

Will distribute targets uniformly for subsystems(assuming that targets are of equal importance). Against each airtarget the centralized system will use fire of (K) subsystems. For simplicity let us assume that there are two targets (N=2), and constant effeciency of each subsystemis (p).

Mathematical expectation(M) will be given as following :

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Probability of destroying the 1st target:

$$W_1 = 1-(1-p)^k$$

Probability of destroying the 2nd target:

$$W_2 = 1-(1-p)^k$$

The value of the mathematical expectation (M) is :

$$M = W_1 + W_2 = 2-2(1-p)^k,$$

$$M = 2-(1-p)^{k-1}.2(1-p) \qquad (2.1)$$

DECENTRALIZED SYSTEM

Deducing now the mathematical expectation for this case(M') we shall assume that 1st target will be engaged by (k+1) subsystems and 2nd target by (k-1) subsystems, The effeciency of fire against the 1st target is :

$$W_1 = 1-(1-p)^{k+1}$$

and for the 2nd target is:

$$W_2 = 1-(1-p)^{k-1},$$

then (M')is :

$$M' = 2-(1-p)^{k-1}.(2-2p+p^2) \qquad (2.2)$$

From (2.1) and (2.2) we find :

$$\Delta M = M-M' = p^2.(1-p)^{k-1} \qquad (2.3)$$

The analysis of this difference for centralized fire system and decentralized fire system control can give the following :

- The increase of (M) for centralized control is exponential and the maximum difference is reached when $p \rightarrow 1$ and is equal to $\Delta M = 37\%$.

- The reason of centralized fire control system is that the effeciency of each subsystem is considerably high, for subsystems being of lower effeciency we have to organize a centralized command, warning system, protection and supply; but decentralized fire control

- The centralized fire control will give the above mentioned results under conditions of its reliabe function.

III- TOTAL PROBABILITY OF DESTROYING

The total probability of destroying the target by one rocket(R_1)presents the main basic criterion of the fire effeciency. Having $\phi(r)$-probability density of radial error, G(r)-conditional probability of destroying, then $\phi(r)dr$-is elementary probability, and $G(r)\phi(r)$-is the probability of destroying by one rocket(for the case r,r+dr).

The total probability of destroying the target by one rocket is :

$$R_1 = \int_0^\infty \phi(r)G(r)dr \qquad (3.1)$$

The accuracy of guidance described by Gaussian distribution law is used as circular law of errors and is given by :

$$\phi(r)=(r/\sigma^2)e^{(-(r_o^2+r^2)/2\sigma^2)}.I_o(r.r_o/\sigma^2) \qquad (3.2)$$

Where : r- is the radial error at target,

r_o- is the final dynamic error,

σ -is the standered deviation,and

$I_o(x)$- is Bessel's function.

The parameters (σ)and (r_o)are dependent on conditions of fire : (manoeuvre,altitude,speed,jamming, etc.)

The function G(r) is given by :

$$G(r)=1-e^{(-\alpha_o^2/r^2)} \qquad (3.3)$$

Where : α_o- is a parameter depending on the altitude and target's type

Expression (3.3)can also be written as :

$$G(r) =e^{(-r^2/2B^2)} \qquad (3.4)$$

where : $B= 1.06\alpha_o$

using(3.2) and (3.4) in (3.1) we get :

(1). for zero fixed error (r_o=0.)

$$R_1=\int_0^\infty (r/\sigma^2).e^{(-r^2/2\sigma^2)}.e^{(-r^2/2B^2)}\,dr,$$

Given the parameter: $t^2=r^2(B^2+\sigma^2)2B^2\sigma^2$

we shall find :

$$R_1(0)=(B^2/(B^2+\sigma^2)).\int_0^\infty 2t.e^{-t^2}\,dt \qquad (3.5)$$

As the integral value 1., we can obtain:

$$R_1(0)=(B^2/B^2+\sigma^2) \qquad (3.6)$$

(2). For general case ($r_o \neq 0$)

When the value of fixed error of guidance is to be taken in consideration, then we can have the following final expression :

$$R_1=R_1(0).e^{-x} \qquad (3.7)$$

Where: e^{-x} always shows the influence of fixed error (r_o) on (R_1), and

$$x= ((r_o^2 / 2(B^2+\sigma^2)) \qquad (3.8)$$

IV- MONTE-CARLO METHOD

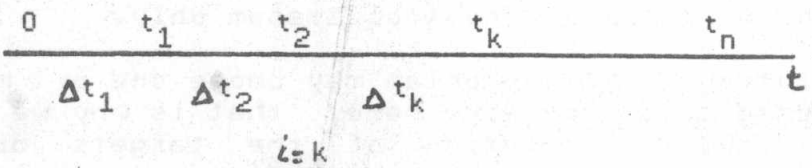
Analytical methodes of analysis are acceptable for a 'simple model' of operation ,however under more complicated conditions the analytical wayes are less reliable or practically impossible (many factors,random events) due to the fact,analytical expressions are not yet drived or they are more complicated for analysis. The method of the statistical experiment which can be used on computer for the statistical modeling is the Mont-Carlo Method. This method is based on ability of modeling the random events, variables or random functions.

EXAMPLE: MODELING OF TARGETS FLOW

Assume simple flow of air targets with density (λ) target / min. (Δt_i) is random interval between targets ;then :
Probability density for Δt is :

$$f(\Delta t)=\begin{cases} \lambda e^{-\lambda \Delta t} & \text{for } \Delta t > 0 \\ 0 & \text{for } \Delta t < 0 \end{cases}$$

Time of arrival of individual air targets ($t_1, t_2 \ldots\ldots t_k \ldots\ldots t_n$) is according to the following scheme :



$$t_k= \sum_{i=1}^{i=k} \Delta t_i$$

As $f(\Delta t)$ is exponential distribution, we shall generate values of (Δt_i) using (ν_i) tables having uniform distribution [0,1], and :

$$F(\Delta t)= \int_0^{t_i} f(\Delta t)dt =-\left| e^{-\lambda \Delta t} \right|_0^{\Delta t_i}$$

$$F(\Delta t)=1-e^{-\lambda \Delta t_i}$$

Let $\nu_i=1-e^{-\lambda \Delta t_i}$

$$-\lambda \Delta t_i=\ln(1-\nu_i)$$

$$\Delta t_i=(-1/\lambda)\ln(1-\nu_i)$$

If $(1-\nu_i)$ is random in the interval [0,1], then (ν_i) is also random and we use :

$$\Delta t_i=-(1/\lambda).\ln \nu_i$$

V. PROBLEM OF DISTRIBUTION AIR TARGETS AMONG FIRING MEANS

-The choice of targets to be destroyed by individual firing means is difficult due to complicated and variable air situations . The decision should be taken at the moment, preventing the 2nd side to change the reliability and correctness of the taken decision. In such away , the decision for targets distribution are to be realized automatically , having high accuracy and the shortest time standerds..

-There are several possible criteria to be used ,for different conditions, demands ,battle missions etc. Beside that, the criteria can oppose each other i.e. the demand to minimize the rocket consumption is in counter version with the demand to maximize the effeciency of the system. The demand of maximum number of engaged targets will be in counter version with the demanded high effeciency of each individual subsystem.

-There are difficulties to allocate the various importance of targets. The solution of distribution problem may be simple enough for targets of equal importance only,but for determination of target's importance this can be solved with limited reliability and by automized control system only.

-The final solution of this problem may chose one or more of the criteria ,assumed to be decisive here, that is why we shall show the possible ways of solution of the targets distribution problem.

(1)Distribution of targets according to mathematical expectation of the number of destroyed air targets.

Having(n) complexes and(N) targets to be destroyed, we call this " task nxN".

The ith complex will engage the jth target reaching the efficiency described by the probablity of destroying(P_{ij}),table (1).

The task is to find the optimum distribution of targets among fire subsystems . The criteria of efficiency will be presented by the mathematical expectation of the number of destroyed air targets M(x).

i \ j	1	2	3	.....N
1	P_{11}	P_{12}	P_{13}	P_{1N}
2	P_{21}	P_{22}	P_{23}	P_{2N}
n	P_{n1}	P_{n2}	P_{n3}	P_{nN}

Table (1)

During each fire against one(jth)air target we can achieve the results:

$$x_j = \begin{cases} 0 \text{.....} \text{ with probability } (1-w_j) \\ 1 \text{.....} \text{with probability } w_j \end{cases}$$

where : w_j.....is the final efficiency of fire against jth target. If one comlpex (ith) will be only used then: $w_j = P_{ij}$
If there are (k) units used against(jth) target, then :

$$w_j = 1 - \sum_{i=1}^{i=k} (1-P_{ij})$$

(2). Distribution of targets accoprding to Mathematical Expectation taking into consideration the various importance.

We know that some air targets will be of higher importance and we have to destroy them first of all. The possible way is to

take into consideration their various importance (carrier of radar, heavy bomber, source of interference, target aiming towards important object etc.) , consisting in the use of coefficients (weight) of importance : $D_1, D_2, \ldots\ldots\ldots, D_j, \ldots\ldots, D_n$.

The mathematical expectation of number of destroyed air targets when the efficiency against jth target is(W_j) may be expressed as :

$$M(x) = D_1W_1 + D_2W_2 + \ldots\ldots + D_NW_N$$

$$= \sum_{j=1}^{j=N} D_jW_j$$

the solution of the matrix of efficiency is similar to the pervious distribution but the columns are to be multiplied by the coefficient of target's imporatance.

(3). Distribution of targets according to M(x) under condition that the number of engaged targets are maximum.

The case when n=N is simple here, each one will engage one target. The total number of variants (for nxN matrix) is : N(n-1)(N-2).....(N-n+1). The optimum variant is corresponding to the maximum M(x) as the previous distribution. For greater number of variants it is necessary to use the mtehods of linear programming. Denoting the complex by(i) the target by (j), and (P_{ij}) is the probability of destroying the (jth) target by the ith complex then the mathematical expectation of the number of destroyed air targets is :

$$M(x) = \sum_{i=1}^{i=n} \sum_{J=1}^{j=N} b_{ij} \cdot p_{ij}.$$

Where :

$$b_{ij} = \begin{cases} 1 \ldots \text{if } i\underline{th} \text{ complex is against } j\underline{th} \text{ target} \\ 0 \ldots \text{if } i\underline{th} \text{ complex is not against the } j\underline{th} \text{ target.} \end{cases}$$

As each complex is engaging one target only , we have :

$$\sum_{j=1}^{j=N} b_{ij} = 1 \qquad , i=1,2,\ldots\ldots,n$$

$$\sum_{i=1}^{i=n} b_{ij} = 1 \qquad , j=1,2,\ldots\ldots,n$$

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For the case when $n < N$, we may concentrate fire by several complexes against each target and the problem can not be solved by linear programming. The solution can be carried out by gradual choice (step-by-step) or twice centered matrix.

VI. GENERALIZED ASSIGNMENT PROBLEM(GAP)

PROBLEM FORMULATION

We have at our disposal (m) complexes to be used against (n) air targets. Therefore we are looking for the optimum combinations of assignement of individual firing elements, i.e to create optimum arranged pairs (i,j) , when ith firing complex (i=1,2.....,n) we assume :

(a).We know the parameter of target path $d_{ij} \geqslant 0$, i.e. the distance of the center of firing position of the ith firing element (i=1,2.....,m) from extrapolated projection of path of the jth target (j=1,2.....,n) to horizontal plane .

(b).It is possible to assign just one target to one firing element.

(c).Probability of hitting jth target by the ith firing element is inversely proportional to parameter (d_{ij}) i.e. with the increasing of d_{ij} decreasing the probability of hit.

Noting that the probability of hit is therefore, assumed to be a function of time for which the target remains in the area of effective operation of the complex (we shall not consider the target parameters i.e. speed,or the flight heigh etc.)

By fulfilling the mentioned assumption it is possible to convert the solution of the problem of optimum fire control to the solution of generalized assignment problem.

Let to each arranged couple(i,j),wher : i=1,2.....,m and j=1,2,....,n. The corresponding value (C_{ij})is determined as following :

(a). $C_{ij}=d_{ij}$ if $d_{ij} \leqslant R$
coupled(i,j) is feasible, i.e. it may be a part of the solution of the problem. The corresponding field in table (2) is called 'nucleus'.

(b). $C_{ij}=M$ if $d_{ij} > R$
couple (i,j) is unfeasible, and(R) is the effective operation of the complex, and (M) is an arbitrary negative number it indicates that the ith complex cannot engage the jth target.
We arrange the jth values(C_{ij}) into the following table :

i \ j	1	2	.....n
1	c_{11}	c_{12}	c_{1n}
2	c_{21}	c_{22}	c_{2n}
m	c_{m1}	c_{m2}	c_{mn}

Table (2)

Where : i=1,2,.....m. order of complex, and
j=1,2,.....n. order number of air targets,
c_{ij}.....value.

From the character of the problem it followes that the number of the complex (m) need not to be equal to the number of targets(n) (i.e. $m \gtrless n$).

The maximum possible number of created couples, therefore, will equal to number (a), where :

$$a=\min (m.n)$$

The quality of the solution is evaluated according to the following criteria :

Criterion(1) :

Maximization of number of created couples(i.e. maximization of number of engaged targets):

$$F_1 = \max_A b$$

Criterion(2) :

Minimization of total sum of distance. With regared to assumption F_1 is thus maximization the total probability of success :

$$F_2 = \min_A c_{ij}$$

Where : F_1, F_2.... are functions

A.....feasible solution set(i.e. solution respecting to one complex is assigned one target only

b.... number of formed couples

Generally : $b \gtrless a$.

Let us define the quantity as the ratio of the number of unfeasible couples to the total number of couples :

$$Q = p/m.n$$

Where : p...number of unfeasible couples.
If Q= 0,i.e. all couples are feasible, and the requirement of maximization of formed couples is automatically fulfilled (b=a) thus criterion(2) become the only criterion for the solution. The classical assignment problem, therefore, is a special case of generalized assignment problem(for m=a, Q=0.).

VII-RESULTS

A sample of computer results are shown in tables 3 and 4

CONCLUSION

The results of the previous part can give the information where and by what means it is possible to hit the target using the following procedure :

The target parameters of flight,necessary to apply the methodical approach we have developed. , should be recalculated for any charactirstic means to its position in the field. Having characterised lunching and destruction zones we can estimate according to (GAP)'s criterion solution the most effective means to be used.

This method can be applied to the automatic command system and can give significant results especially if the following recommandations are fulfilled :

(1). The effeciency of the whole system is conditioned on the elasticity and accomodation of close cooperation among all used subsystem.

(2). The time standards as well as parameters of accuracy and reliability of the whole system are to be analysed and calculated using the statistical and analytical methods.

(3). High accuracy and information capacity of the used computers will ensure the high combat activity of the whole system.

(4). Effective action by commanding system will be reached under the conditions that all elements of the system are secured and protected against active and passive counter measures.

REFERENCES

1) Dantzig, G.,"Linear Programming and Extensions ",Princeton Universty Press ,Princeton , N.J.,1963.

2) Manne , A., "A Target-Assignment Problem ", Opns. Res. 6,pp 346-351, 1958.

3) Gass, S., " Linear Programming Methods and Applications ", McGRAW HILL , 1969.

4) El-Arabaty ,M.,"Mathematical Modeling for Assignment Problem" ,3rd Int. Conference On Math. Modeling University of Southern California , 1981.

5) El-Arabaty ,M.,"Differential Games Solution with Numerical Methods ",21st Confrence on Decision and Control,IEEE,1982,pp 191-194.

6) El-Arabaty,M.,"Recent Trends In diffrential Games and extended Applications",The Inaugural Lecture of the AMSE International Conference ,Pomona ,California, U.S.A., Dec.1987,pp.47-60 vol. I.

7) El-Arabaty, M.,and M.Ratib," The optimal Rendez-Vous of Space Crafts, An Application for Telecommunications Systems",Int.Conf. of AMSE, Pomona ,California, U.S.A., DEC.1987,PP-127-139 Vol.IV.

8) El-Arabaty ,M.," New Investigations for Modern Pursuit Evasion Games and Extended Applications ",SIAM Annual Meeting, Minneapolis,Minnesotea , July ,pp 11-15,1988, CP3.

AT TIME 3

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CHECK TARGET NO. 1 WITH BATTALIONS

X	Y	Z	R	H	TYPE	BN	
.111E+02	.139E+02	.488E+00	.1620E+02	.4880E+00	DETECT	1	
.157E+02	.148E+02	.638E+00	.1860E+02	.6382E+00	DETECT	2	
.204E+02	.158E+02	.788E+00	.2119E+02	.7883E+00	DETECT	3	
.251E+02	.168E+02	.938E+00	.2391E+02	.9385E+00	DETECT	4	
.297E+02	.177E+02	.109E+01	.2055E-02	.1089E+01	N.DETE	5	**

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THE SYSTEM EFFICIENCY EFF1 = 60.00000000 %

Table 3.- A sample of computer results

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CHECK TARGET NO. 4 WITH BATTALIONS

X	Y	Z	R	H	TYPE	BN	
.204E-02	.244E+02	.863E+00	.3017E+02	.8634E+00	DETECT	1	
.251E+02	.254E+02	.101E-01	.3046E+02	.1014E+01	N.DETE	2	**
.287E+02	.263E+02	.116E-01	.2817E+02	.1164E+01	N.DETE	3	**
.344E+02	.273E+02	.131E+01	.2831E+02	.1314E+01	N.DETE	4	**
.391E+02	.283E+02	.146E+01	.2055E+02	.1464E+01	N.DETE	5	**

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THE SYSTEM EFFICIENCY EFFI = 80.00000000 %

THE SYSTEM TOTAL EFFICIENCY EFF = 50.00000 %

THE EFFICIENCY IS STILL SMALL

THE PROGRAM WILL CONTINUE THE OPTIMIZATION PROCESS

Table 4.- A sample of computer results