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1 # LAMINAR BOUNDARY LAYER CONTROL

2 # ABOUT A SPHERE IN A STREAM

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4 ## ABSTRACT

This paper investigates the effect of uniform suction on the characteristics of the developing laminar boundary layer about a sphere which is subjected to a steady stream of fluid. The study includes the developing meridional velocity profiles, the separation angle, the wall shear stress and the frictional drag coefficient (C_D).

Making limitation of suction region on sphere surface leads to reduction in frictional drag. However, it reduces the separation angle (θ_s) with slight effect. The present regime succeeded in obtaining numerical results up to $\theta = 180°$.

INTRODUCTION

There are in existence several methods which have been developed for the purpose of artifitially controlling the behaviour of boundary layers. The purpose of these mehtods is to affect the whole flow in a desired direction by influencing the structure of the boundary layer. One of these controlling methods is suction. The effect of suction consists in removal of decelerated fluid particles from the boundary layer before they are given a chance to cause separation. Controlling the value of suction velocity and under favourable conditions, separation may be prevented completely. Simultaneously, the amount of pressure drag is greatly reduced owing to the absence of separation; the reduction in form drag is achieved by delaying the separation of fluid. A more detailed analysis reveals that the influence of suction on transition from laminar to turbulent flow is due to two effects. First, suction reduces the boundary layer thickness and thin boundary layer is less prone to become turbulent. Secondly, suction creats a laminar velocity profile which possesses a higher limit of stability than a velocity profile with no suction.

Early Prandtl [1] described several experiments in which the boundary layer was controlled. One of his experiments was devoted to the flow past a circular cylinder with suction applied on one side of it through a small slit. The application was later widely used in the design of air craft-wings. By

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applying suction, lower absolute pressure on the upper side of aerofoil are obtained at large angels of incidence and consequently, much larger maximum lift values. Schrenk [2] investigated a large number of different arrangements of suction slits and their effect on maximum lift. An experimental proof of the fact that it is possible to maintain laminar conditions in the boundary layer with the aid of suction was first given by Holstein [3]. Ringleb [4] computed the incompressible boundary layer on a flat plate with the approximate profile for the velocity tangent to the boundary layer and this was applied by him with and without suction. Wilson [5] studied theoretically the flow between a stationary and a rotating disk with suction and studied the effect of suction parameter on the rotation direction of the core between the two disks. Hyun [6] presented numerical solutions for the flow driven by a spinning disk which forms an end wall of a finite closed cylinder. He also investigated the effect of imposing a uniform suction (or blowing) through the spinning disk in finite configurations. Libby [7] introudced an experimental investigation on the isothermal laminar boundary layer on a porous flat plate. He measured the transition Reynolds number and velocity profiles and compared them with the results of laminar boundary layer analysis that have been carried out in the past. Ulrich [8] has shown that suction greatly increases and injection (blowing) decreases the stability of incompressible laminar boundary layers. The amplification of the unstable disturbances for the asymptotic profile has been calculated by Pritch [9]. The highest degree of amplification obtained in his calculation (taking suction into account) was about 10 times smaller than that for the falt plate (Blasius flow).

The present work is concerned with the study of the flow about a sphere in a stream with uniform suction or blowing from its surface. The limiting case of stream flow on a stationary sphere without suction was analysed by many investigators. The boundary layer equations over such a body were first derived by Boltze [10]. He solved the equations by a method similar to that used by Blasius for the two dimensional case. The same problem was solved by employing the series solution techniques by Hoskin [11] and Moore [12]. Improvement of the solution were carried out by Smith and Clutter [13] and Sheridan [14] who employed finite difference appraoches. El-Shaarawi et al [15] and [16] solved the boundary layer equations for this case using an implicit finite difference method. The resulting algebraic equations were linearized and the system of linear algebraic equations were solved without any iterations. The present study includes the velocity development with suction or blowing and the effect of suction or blowing velocity (WS) on separation, wall shear stress, and frictional drag coefficient.

GOVERNING EQUATIONS

The steady laminar flow of an incompressible fluid with constant physical properties in the region outside a sphere is considered. Let x, y and z be a rectangluar curvlinear fixed coordinate system. The x-axis is measured from the stagnation point along a meridional section, the y-axis along circular corss section of the sphere by a plane perpendicular to the direction of the stream of flow, and z-axis is at right angles to the tangential x-y plane. Under the above mentioned assumptions and in the absence of body forces, the boundary layer equations for the problem at hand are:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{u}{r}\frac{dr}{dx} + \frac{\partial w}{\partial z} + \frac{2w}{a+z} = 0 \qquad (1)$$

x-momentum equation:

$$u\frac{\partial u}{\partial x} - \frac{v^2}{r}\frac{dr}{dx} + w\frac{\partial u}{\partial z} = u^*_o\frac{\partial u^*_o}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} \qquad (2)$$

y-momentum equation:

$$u\frac{\partial v}{\partial x} + \frac{uv}{r}\frac{dr}{dx} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} \qquad (3)$$

In the above equation u, v and w are meridional, azimuthal and radial velocity components, respectively, u^*_o is the potenital meridional velocity component calculated on the surface of the shpere, a is the sphere radius, r is the radial distance from the sphere surface and ν is the kinematic viscosity.

BOUNDARY CONDITIONS AND METHOD OF SOLUTION

Equations (1) through (3) are subjected to the following boundary conditions:
at $z = 0$ (sphere surface) and $x > 0$, $u = 0$, $v = 0$ and $w = ws$ (velocity of uniform suction or blowing). (4-a)

at $z \gg \delta$ and $x>0$: $u = u^*$, $v = 0$ and $w = w^*$. (4-b)

at $x = 0$ and $z > 0$: $w = w^*$, and $u = v = 0$. (4-c)

where δ is the boundary layer thickness, u^* is the potential velocity component in the x-direction, and w^* is the radial velocity component for potential flow.

To get expression for u^* and w^*, the potential flow created by the superposition of a uniform parallel flow, a space doublet and sink located at the center of the doublet is used. The stream function ψ of such potential flow which represents the flow around a sphere with uniform suction on its surface is:

$$\psi = \tfrac{1}{2}\, U_\infty r^2 \sin^2\theta \left(1 - \frac{a^3}{r^3}\right) - m\cos\theta$$

where m is the strength of the sink, θ is the center angle measured from the axis of the sphere in the flow direction and U_∞ is the free stream velocity. w^* is expressed as follow:

$$w^* = -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} = -U_\infty\cos\theta\left(1 - \frac{a^3}{r^3}\right) - (m/r^2)$$

Therefore, $m = -a^2\, ws$, and then the dimensionless radial potential velocity component taking into consideration the suction effect is:

$$W^* = \left(\frac{a^3}{r^3} - 1\right)\cos\theta + \frac{a^2}{r^2}\, WS, \qquad (4\text{-d})$$

where $WS = ws/U_\infty$ and $W^* = w^*/U_\infty$

Thus at $r = \infty$, $W^* = -\cos\theta$ (i.e. the suction effect due to ws disappears). The meridional potential velocity is expressed from the following equation:

$$u^* = \frac{1}{r \sin\theta} \frac{\partial\psi}{\partial r} = U_\infty[1 + \frac{a^3}{2r^3}] \sin\theta \qquad (4\text{-e})$$

As seen from the above equation, u* is unaffected by the existance of suction. The governing equations (1) to (3) subjected to the boundary conditions (4) were numerically solved using the implicit non-iterative finite difference scheme of Ref [15] and [17] after putting these equations into non-dimensional forms.

The matching between the meridional velocity profile obtained by solving the boundary layer equations and the corresponding potential profile, was based on the solution criterion:

$\frac{U^* - U}{U^*} < 1\%$, where $U^* = u^*/U_\infty$ and $U = u/U_\infty$

This criterion gave better results than that based on the slope of the two profiles as employed by [17], it gives matching at short radial distances from the surface. On the other hand, using the latter criterion (based on the slope of the two profiles) gives matching at larger radial distances with more difference between U and U* than that based on the first criterion ($\frac{U^*-U}{U^*}$). Also making the matching based on ($\frac{W^*-W}{W^*}$) < a reasonable value, needs more radial distances and thus consuming more computer time. However, such difficulties may be reduced as suction velocity increases since it was found that matching based on (W*-W)/W* occurs at lower radial distances due to the decrease in the boundary layer thickness (due to suction).

RESULTS AND DISCUSSION

The present results are obtained for value of Reynolds number $Re = (2\, aU_\infty/\nu) = 10000$. These results intend to investigate the effect of the parameter ws on the flow development. The profiles of the velocity component are drawn versus the radial dimensionless parameter Z, $Z = (z/a)$ at various selected values of θ. WS was choosen in the range of 0.01 to -0.1; a constant positive value of WS means uniform blowing while a negative value means uniform suction, which are the two cases considered.

Figure (1) illustrates the effect of WS on the development of meridional component U with the dimensionless parameter Z. As can be seen from the figure the value of U is low near the stagnation point (for all values of WS) and graws downward in X (i.e. θ) direction. By observing all profiles shown in figure (1), it may be concluded that, in general the rate of boundary layer growth decreases with suction. By the application of severe suction as for example at WS = - 0.1 (i.e. uniform suction velocity ws is 10% of U_∞) it can be seen that the rate of boundary layer growth is so slow that the variation of the boundary layer thickness "δ" with θ becomes unremarkable over a wide range of θ. Also, in the same case the boundary layer separation is completely avoided. At large values of θ (θ > 90°) the meridional velocity component U starts to decay to approach zero value every where at θ = 180°. It is worth mentioning that numerical results could be obtained in this case up to θ = 180°. This previously mentioned phenomenon is completely reversed at WS = 0.01 (blowing), where "δ" increases in the θ-direction at

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high rat until it reaches its maximum value at the separation point (which occurs in this case near $\theta = 90°$) i.e. the blowing effect is similar to that of an adverse pressure gradient in θ-direction beyond the equatorial plane (i.e. $\theta > 90°$).

Figure (2) shows the development of the dimensionless meridional shear stress, T_x

$$\left(T_x = \frac{t_x \sqrt{\frac{1}{2} Re}}{\rho U_\infty^2}, \text{ where } t_x = \frac{\partial u}{\partial z}\Big|_{z=0} \right)$$

at different values of WS. For all values of WS, T_x is zero at the stagnation point and returns to its original value at the separation angle, if any or at $\theta = 180°$. The effect of WS on T_x can obviously be deduced by observing the slope of the different meridional velocity profiles at $Z = 0$ in Figure (1). As can be seen, the value of dimensionless shear stress (T_x) increases as the suction velocity increases. However, it should be emphasized that at large suction velocities such an increase in the frictional drag would be compensated by the prevention of separation. This means that although there is an increase in the frictional drag due to suction, separation is prevented and it is anticipated that the total drag without separation would be smaller (due to the decrease of the form drag). The limiting value of WS which prevents boundary layer separation was found to be -0.06.

Figure (3) shows the effect of WS on both the separation angle θ_s and frictional drag coefficient C_D,

$$C_D = 4 \int_0^{\theta_s} \frac{\partial u}{\partial z} \quad R \sin\theta \, d\theta$$

where Θ_s is the angle corresponding to the meridional distance measured from the stagnation point to the point of separation and $R = (2r/a \cdot Re)$ is the dimensionless radius). As shown for values of WS $\geq$ 0.03 the separation angle is unaffected and remains equal 90°. For values of WS < -0.06, separation does not exist. This may be attributed to the fact that the outward momentum is insufficient to overcome the inward suction momentum and this fluid particles remain adherent to the sphere surface and separation could not occur. With negative values of WS separation occures at more downward angles than with positive values of WS as suction causes the removal of the decelerated fluid particles from the boundary layer before they are given a chance to cause separation. The variation of laminar frictional drag with WS is to some extent similar to that of the angle of separation with WS but it (C_D) asymptotically vanishes as WS increases (increase of blowing).

An important method for reducing the frictional drag in the presence of suction is to control the suction region. In the present case, adjusting the suction to start at $\theta > 90°$ and continue up to $\theta = 180°$, the frictional drag could considerably decreased with remarkable low effect on the separation angle. This is decleared in figures (4) and (5). The figures present the effect of limiting the suction region on the development of meridional velocity component for two values of WS = -0.1 and -0.01, respectively. For the purpose of comparison the corresponding profiles for the case with suction through the whole surface are presented. It is worth noting that the profiles for the case of no suction (WS = 0) coincide with those for the case with limited suction (WS = -0.1 or -0.01) in the range of $\theta = 0$ to 90°. Obviously it is clear that suction which starts at $\theta = 90°$ has no effect on the flow for $\theta < 90°$. Also from figures (4) and (5) it is clear that the suction velocity increases (through the presepecified region). It makes the flow more stabel and adhering to the surface, Separation is completely avoided at WS = -0.1 while it occurs at $\theta = 115.5°$ for WS = -0.01.

Figure (6) shows the development of the meridional shear stress, where in each case with a limited suction region, a sudden increase occurs in the curve of WS = 0 at $\theta = 91°$ to approach that curve representing the corresponding case with suction all over the whole surface.

Figure (7) gives the coefficient of drag and the separation angle against the suction velocity WS for the two cases under consideration (suction all over the sphere surface and limited suction). As can be seen, both the values of drag coefficient and the separation angle are decreased by limiting the suction region. It is of interest to note that to avoid separation completely, it is required to apply more suction in the latter case than it is required in the original case. This may be attributed to the decrease in the radial momentum of the flow towards the sphere surface and hence there is an accumulation of decelerated particles more than that exists in the case of suction all over the sphere surface.

CONCLUSION

From the results and the discussions presented in this paper, the following important conclusions are drawn:

1) The effect of suction is to delay the separation point, and there is a limiting value of the suction velocity (WS) above which separation may be avoided. It is expected that this limiting value of WS depends on the value of Reynolds number.

2) Imposing suction, leads to an increase of the frictional drag. However it is anticipated that the total drag (form + friction) could be reduced due to the delay of the separation point.

3) Starting suction at the begining of the adverse pressure region up to the separation point (limiting suction) leads to considerable reduction in frictional drag. This leads to the conclusion that limiting suction is recommended in practice.

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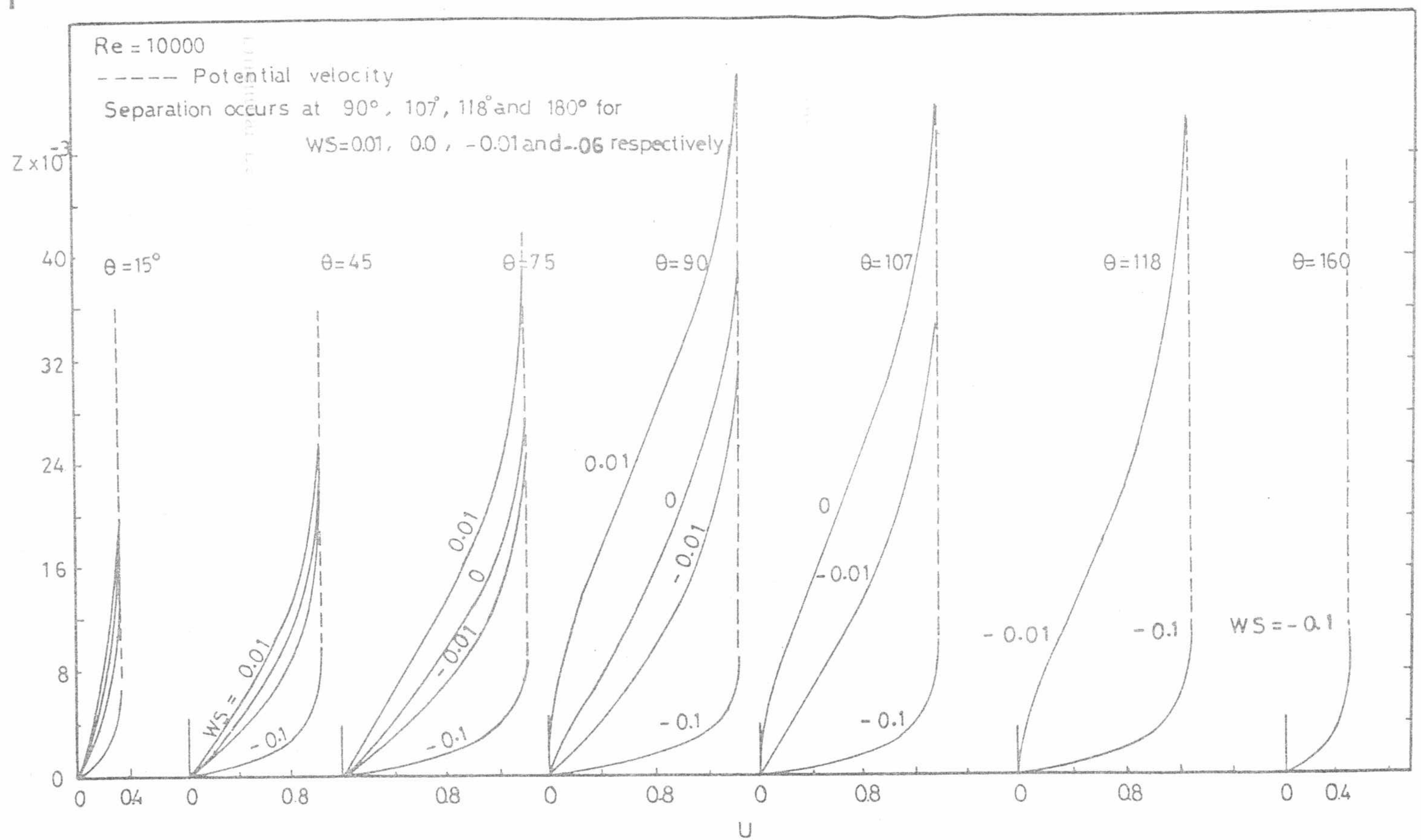

Fig.1 : Suction and Blowing Effect on the Development of Meridional velocity component.

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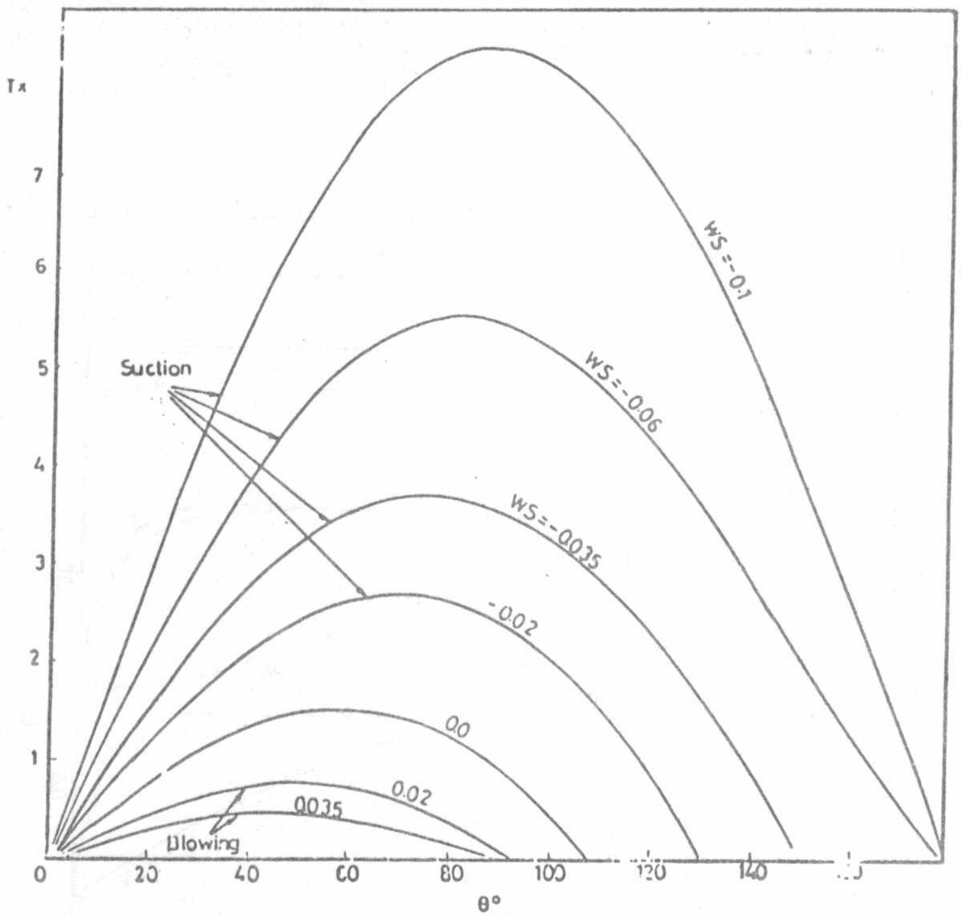

Fig. 2: Effect of Suction or Blowing on the Meridional shear stress.

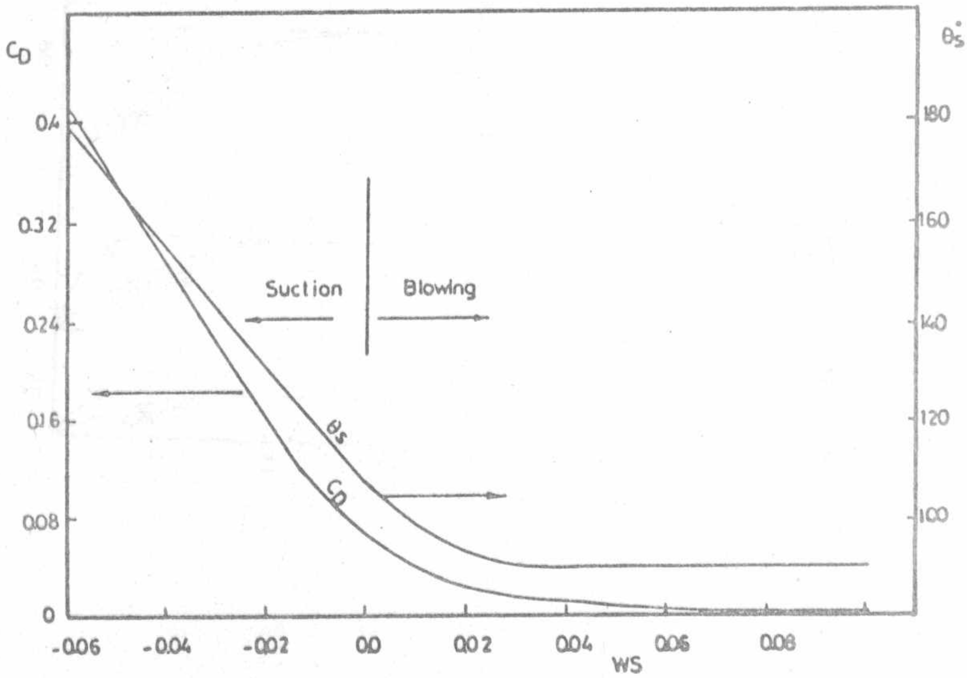

Fig. 3: Variation of Angle of Separation and Drag Coefficient with WS.

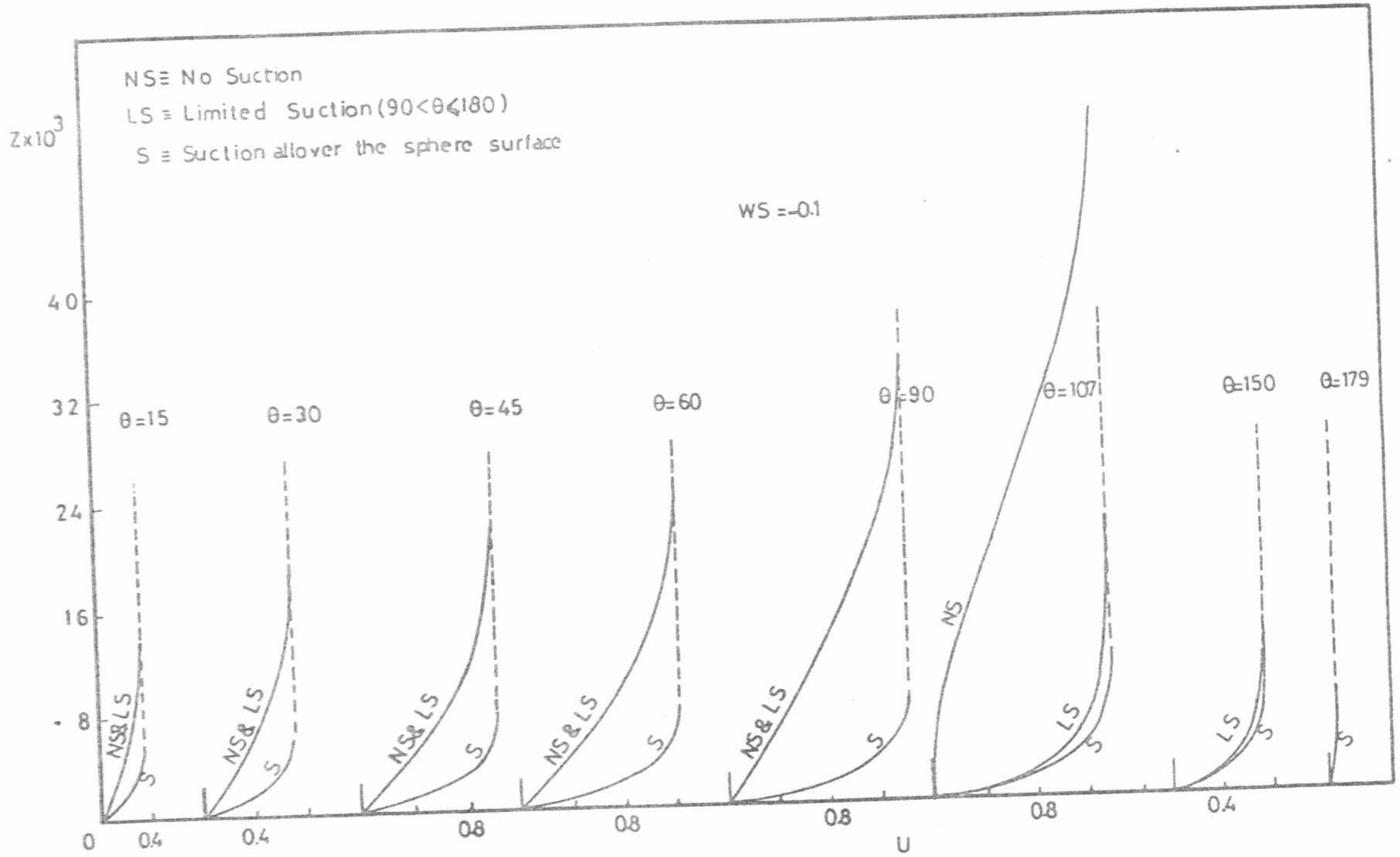

Fig. 4: Region of Suction Effect on the Development of Meridional Velocity Component (WS = - 0.1).

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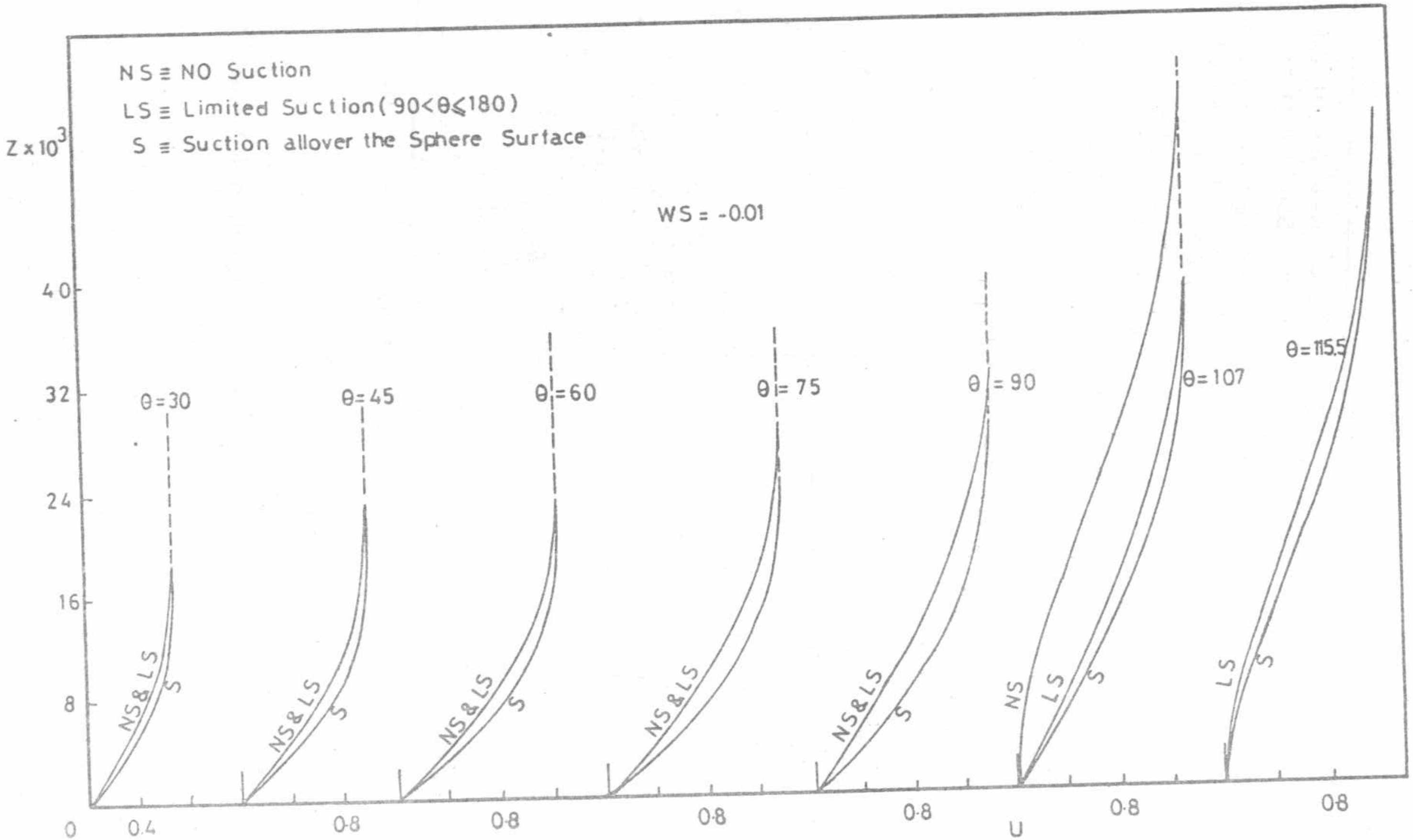

Fig. 5: Region of Suction Effect on the Development of Meridional Velocity Component, (WS = - 0.01).

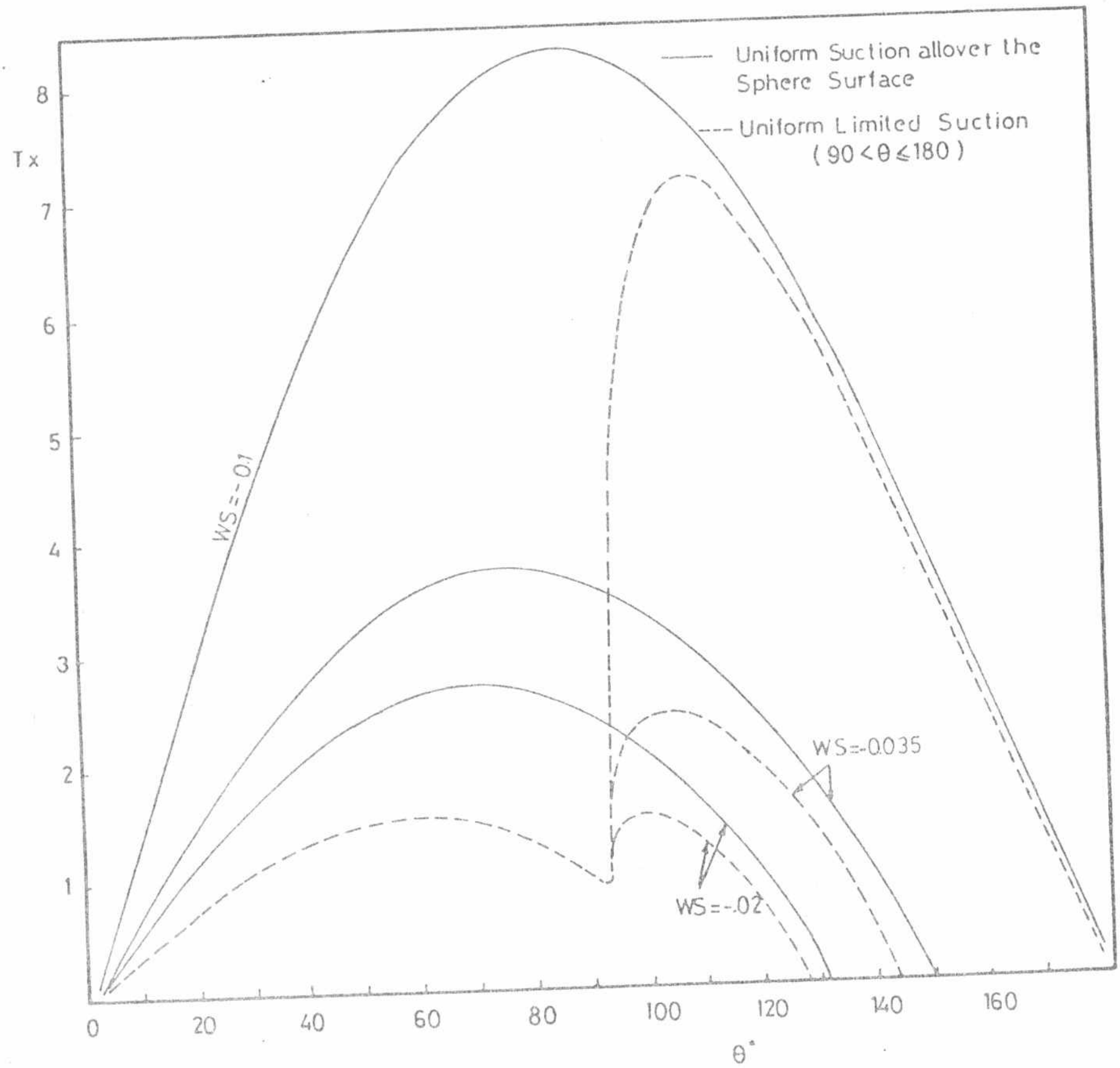

Fig. 6: Region of Suction Effect on the Meridional Shear Stress (T_x).

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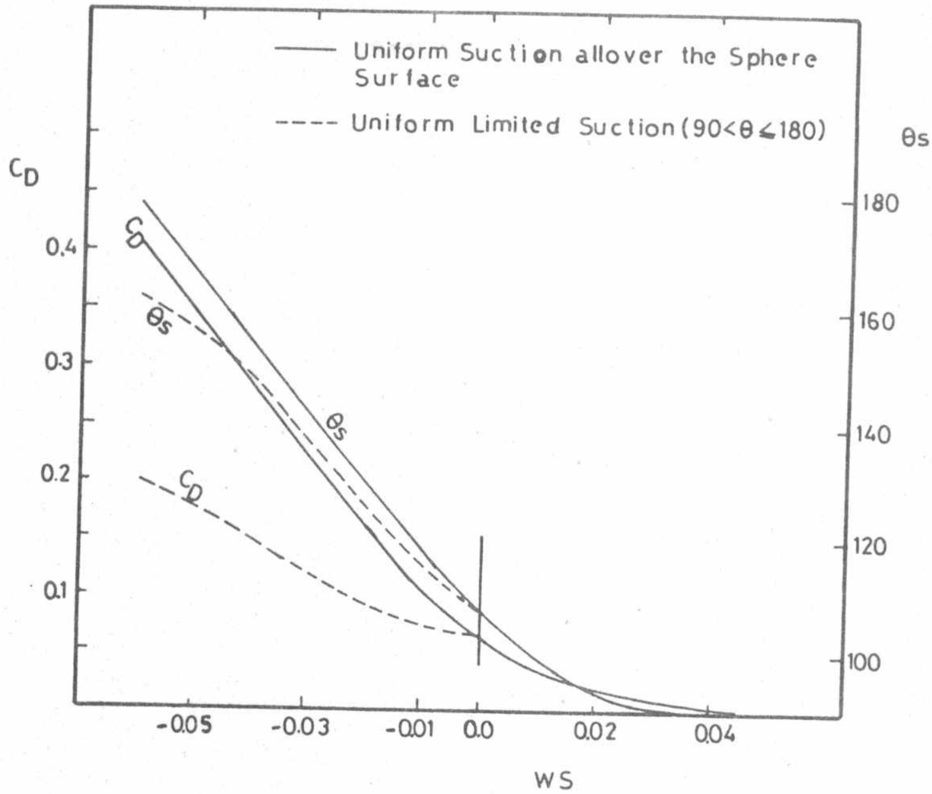

Fig. 7: Region of Suction Effect on Drag Coefficient and Angle of Separation.