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Military Technical College
CAIRO - EGYPT

A CHEBYSHEV METHOD FOR THE SOLUTION OF BOUNDARY VALUE PROBLEMS

By

H.NASR*, I.A.HASSANIEN**, H.M.EL-HAWARY**

ABSTRACT

An expansion procedure using the Chebyshev polynomials is proposed by using El-Gendi method [1], which yields more accurate results than those computed by D.Hatziavrmidis [2] as indicated from solving the Orr-Sommerfeld equation for both the plane poiseuille flow and the Blasius velocity profile. The present results are also more accurate results than those computed by A.R.Wadia & F.R.Payne [3] as indicated from solving the Falkner-Skan equation, which uses a boundary value technique. This method is accomplished by starting with Chebyshev approximation for the highest-order derivative and generating approximations to the lower-order derivatives through integration of the highest-order derivative.

* Military Technical College, Kobbri El-Kobba, Cairo, Egypt.

** Faculty of Science, Assiut University, Assiut, Egypt.

. INTRODUCTION:

The use of chebyshev polynomial approximations for the solution of boundary value problem in fluid mechanics has been advocated and developed by Orszag [4]. These approximations which include the Galerkin and Tau spectral methods pseudo spectral collocation method, are discussed in detail by Gottlieb and Orszag [3].

In section II we solve the Orr-Sommerfeld equation. We also show how this method may be applied to solve the Blasius equation which is reduced to a non-linear algebraic system of equations in the highest derivative and we solve by Newton's method. Comparison of the eigenvalues for either of the plane poiseuille flow or the Blasius velocity profile with those computed by D.Hatzivarmids [2] are presented.

In section III the problems of laminar or turbulent boundary layer resulting from the flow of an incompressible fluid past a semi-infinite wedge is of considerable practical and theoretical interest. Non-linear problems with semi-infinite domains are frequently encountered in the study of laminar and turbulent boundary layer. Due to the appearance of irregular boundaries, shock waves, boundary layers, derivative boundary conditions, etc., the solutions so obtained have in many cases been unsatisfactory because of poor resolution, spurious oscillations, and excessive computer time or storage.

For the past two decades finite difference numerical methods have been used extensively for the evaluation of flow and high speed, viscous and inviscid problem in fluid mechanics. Recently the similarity solutions are well studied for the testing finite difference methods. Wadie & Payne [6], Rosenhead [5] and White [7] have been discussed different finite difference schemes for the solutions of the Falkner-Skan equations or Blasuis equation.

In any case the farfield boundary condition is a problem, where the correct value of the unknown shear stress f''_w at the wall must be found which ensures an asymptotic approach of the velocity values f' at infinity, to unity , the well known matching condition of viscous solution (near-wall) to the inviscid solutions. Such techniques are termed as "shooting" method. In this paper we introduce a new procedure for the numerical solution based on the Chebyshev approximation using the EL-Gendi method [1] for the numerical solution of the Falkner-Skan equation. Our method defines the unknown wall shear stress f''_w directly without need any correction interpolation method. We compare our results with values obtained by Wadie & Payne [6], Rosenhead [5] and White [7] are shown.

II. THE ORR-SOMMERFELD EQUATION.

We choose to apply our method to the solution of the Orr-Sommerfeld equation, which governs the stability of laminar boundary layers in the parallel flow approximation, for plane poiseuille and Blasius boundary layers flows.

Consider the Orr-Sommerfeld equation

$$\psi^{(4)} - 2 a^2 \psi^{(2)} + a^4 \psi - iaR[(u-\lambda)(\psi^{(2)} - a^2\psi) - u^{(2)}\psi] = 0, \tag{1}$$

with

$$\psi(\pm 1) = \psi^{(1)}(\pm 1) = 0$$

where $\psi(x)e^{ia(y-\lambda t)}$ is the disturbance stream function in usual normal mode analysis , $u(x)$, is the basic velocity distribution, R is the Reynolds number based on the boundary layer thickness, a and λ are the wave number and wave speed, respectively and super-scripts in parenthesis indicate derivatives with respect to x.

To solve equation (1) , we put $\psi^{(4)}(x) = \phi(x)$, then

$$\psi^{(3)}(x) = \int_{-1}^{x} \phi(x)dx + C_1,$$

$$\psi^{(2)}(x) = \int_{-1}^{x}\int_{-1}^{x} \phi(x)\, dxdx + (x+1)C_1 + C_2,$$

$$\psi^{(1)}(x) = \int_{-1}^{x}\int_{-1}^{x}\int_{-1}^{x} \phi(x)\, dxdxdx + \frac{(x+1)^2}{} C_1 + (x+1)C_2 + C_3, \quad (2)$$

$$\psi(x) = \int_{-1}^{x}\int_{-1}^{x}\int_{-1}^{x}\int_{-1}^{x} \phi(x)\, dxdxdxdx + \frac{(x+1)^3}{6} C_1 + \frac{(x+1)^2}{2} C_2 + (x+1)\, C_3 + C_4.$$

From the boundary conditions, we get

$$C_3 = C_4 = 0$$

$$C_2 = \frac{3}{2}\left[\int_{-1}^{1}\int_{-1}^{x}\int_{-1}^{x}\int_{-1}^{x} \phi\, dxdxdxdx - \int_{-1}^{1}\int_{-1}^{x}\int_{-1}^{x} \phi\, dxdxdx \right], \quad (3)$$

and

$$C_1 = \frac{1}{2}\left[2 \int_{-1}^{1}\int_{-1}^{x}\int_{-1}^{x} \phi\, dxdxdx - 3 \int_{-1}^{1}\int_{-1}^{x}\int_{-1}^{x}\int_{-1}^{x} \phi\, dxdxdxdx\right].$$

Here, we can give an approximation to equation (2) as follows

$$\psi_1^{(3)} = \sum_{j=0}^{N} \ell_{ij}^{(3)} \phi_j.,$$

$$\psi_i^{(2)} = \sum_{j=0}^{N} \ell_{ij}^{(2)} \phi_j.,$$

$$\psi_i^{(1)} = \sum_{j=0}^{N} \ell_{ij}^{(1)} \phi_j.,$$

$$\psi_i = \sum_{j=0}^{N} \ell_{ij} \phi_j.,$$

where

$$\phi_j = \phi(x_j) \;, \quad \psi_i^{(r)} = \psi^{(r)}(x_i), \quad r = 0(1)4,$$

$$x_i = -\cos(i\frac{\pi}{N}) \quad ; \; i = 0(1)N \;,$$

$$\ell_{ij} = b_{ij}^{(4)} + \frac{(x_i+1)^3}{12}(2b_{Nj}^{(3)} - 3b_{Nj}^{(4)}) + \frac{3}{4}(x_i+1)^2(b_{Nj}^{(4)} - b_{Nj}^{(3)}),$$

$$\ell_{ij}^{(1)} = b_{ij}^{(3)} + \frac{(x_i+1)^2}{4}(2b_{Nj}^{(3)} - 3b_{Nj}^{(4)}) + \frac{3}{2}(x_i+1)(b_{Nj}^{(4)} - b_{Nj}^{(3)}),$$

$$\ell_{ij}^{(2)} = b_{ij}^{(2)} + \frac{(x_i+1)}{2}(2b_{Nj}^{(3)} - 3b_{Nj}^{(4)}) + \frac{3}{2}(b_{Nj}^{(4)} - b_{Nj}^{(3)}),$$

$$\ell_{ij}^{(3)} = b_{ij} + \frac{1}{2}(2b_{Nj}^{(3)} - 3b_{Nj}^{(4)}),$$

and

$$b_{ij}^{(r)} = \frac{(x_i - x_j)^{r-1}}{(r-1)!} b_{ij} \;, \; r = 2(1)4 \quad , \; i,j = 0(1)N$$

where b_{ij} are the elements of the matrix B, as mentioned in [1]

Substituting equation (4) in equation (1), we arrive at the following algebraic eigenvalue problem

$$(A - \lambda D)\phi = 0 \tag{5}$$

where $\Phi = [\phi_o \;\; \phi_1 \;\; . \; . \; . \;\; \phi_N]$.

In the equation (5) A and D are $(N+1)\times(N+1)$ matrices with the elements

$$a_{mn} = \delta_{mn} - (2a^2 + iaRu)\ell_{mn}^{(2)} + (a^4 + ia^3Ru + iaRu^{(2)})\ell_{mn}$$

and

$$d_{mn} = -iaR\ell_{mn}^{(2)} + ia^2R\ell_{mn}$$

Numerical solution of the matrix eigenvalue problem (5) is obtained using the QZ method with NAGLIBRARY routine F02GJF.

II.1. Plane Poiseuille Flow

Here $u(x)= 1-x^2$, $x \in [-1, 1]$. For the plane Poiseuille flow, the eigenfunctions are either symmetric or asymmetric about $x = 0$. Then for N even the (N+1) equations in (5) are reduced into the following (M+1) equations where M= N/2.

$$(A^{(1)} - \lambda D^{(1)})\ \Phi = 0 \tag{6}$$

where

$$a^{(1)}_{ij} = a_{ij} + a_{i(N-j)} \quad , \ j = 0(1)M-1, \quad i = 0(1)M,$$

$$a^{(1)}_{iM} = a_{iM} \quad , \ i = 0(1)M,$$

$$d^{(1)}_{ij} = d_{ij} + d_{i(N-j)} \quad , \ j = 0(1)M-1, \quad i = 0(1)M,$$

and

$$d^{(1)}_{iM} = d_{iM} \quad , \ i = 0(1)M,$$

In Table 1 we show the most unstable eigenvalue (which corresponds to a symmetric eigenfunction) computed here and in [2] for a=1 and R=10000 where the exact value is 0.23752649 + 0.00373967i [4]. In table 2 we show the most unstable eigenvalue for the critical Reynolds number and wave number, R=5772.22 and a_c = 1.02056. The values generated by our method compares well with the value obtained by Orszag [4], $0.26400174 + 5.9\times10^{-10}i$.

II.2. Blasius Velocity Profile

The Blasius boundary-layer flow for which the basic flow u is given in terms of a function $f(\eta)$ which satisfies

$$f''' + ff'' = 0 \quad , \ \eta \in [0, \infty), \tag{7}$$

$$f(0) = f'(0) = 0 \quad \text{and} \quad f'(\eta \longrightarrow \infty) \sim 1$$

where super-scripts denote derivatives with respect to η. The infinity condition will be invoked at a finite value of $\eta = \eta_\infty$.

Using the algebraic mapping.

$$x = 2\,\frac{\eta}{\eta_\infty} - 1 \tag{8}$$

The unbounded region $[0, \infty)$ is mapped into the finite domain $[-1, 1]$ and the problem expressed by equation (7) is transformed into

$$f''' + \frac{\eta_\infty}{2} f f'' = 0, \tag{9}$$

$$f(-1) = f'(-1) = 0 \quad , \quad f'(1) = \frac{\eta_\infty}{2}$$

where superscripts denote derivatives with respect to x.

The basic flow for the Blasius problem is obtained as

$$u(x) = f'(x). \tag{10}$$

To solve equation (9), we put $f'''(x) = \phi(x)$, then by using our method we can give approximations to the function $f(x)$ and its derivatives as follows:

$$f_i = \sum_{j=0}^{N} \ell_{ij}\, \phi_j + d_i ,$$

$$f'_i = \sum_{j=0}^{N} \ell^{(1)}_{ij}\, \phi_j + d^{(1)}_i , \tag{11}$$

$$f''_i = \sum_{j=0}^{N} \ell^{(2)}_{ij}\, \phi_j + d^{(2)}_i ,$$

where

$$\phi_i = \phi(x_i),\ f_i = f(x_i),\ f'_i = f'(x_i),\ f''_i = f''(x_i),$$

and for $i, j = 0(1)N$

$$\ell_{ij} = b^{(3)}_{ij} - \frac{(x_i+1)^2}{4}\, b^{(2)}_{Nj} , \qquad d_i = (x_i+1)^2\, \frac{\eta_\infty}{8}$$

$$\ell^{(1)}_{ij} = b^{(2)}_{ij} + \frac{(x_i+1)}{2}\, b^{(2)}_{Nj} , \qquad d^{(1)}_i = (x_i+1)\, \frac{\eta_\infty}{4} ,$$

$$\ell^{(2)}_{ij} = b_{ij} - \frac{1}{2} b^{(2)}_{ij} , \qquad d^{(2)}_{i} = \frac{\eta_\infty}{4}$$

where

$$b^{(3)}_{ij} = \frac{(x_i - x_j)^2}{2} b_{ij} , \quad b^{(2)}_{ij} = (x_i - x_j) b_{ij}$$

and b_{ij} are the elements of the matrix B defined in [1].

By using equation (11) equation (9) is transformed into the following system of non-linear equations.

$$\phi_i + \frac{\eta_\infty}{2} \left(\sum_{j=0}^{N} \ell_{ij}\phi_j + d_i \right) \left(\sum_{j=0}^{N} \ell^{(2)}_{ij} \phi_j + d^{(2)}_{i} \right) = 0, \qquad (12)$$

$$i = 0(1)N$$

In Table 3 we show the most unstable eigenvalue for the Blasius flow, obtained by our method for a=0.179 and R=580. For comparison, values obtained by D.Hatziavramidis [2] are shown where the exact value is 0.36412286 + 0.00795972i [8]

III.METHOD OF SOLUTION

Consider the Falkner-Skan equation for stagnation flows with the similarity property [5, 7].

$$f'''(\eta) + \alpha f(\eta) f''(\eta) + \beta[1-f'^2(\eta)] = 0 \qquad (13)$$

together with the boundary conditions:

$$f(0) = f'(0) = 0 \qquad \text{and} \qquad f'(\eta \to \infty) = 1. \qquad (14)$$

Here α is assumed constant, β is a measure of the pressure gradient. The prime denotes differentiation with respect to η. The special case of the Blasius similarity relation for incompressible viscous flow along a flat plate results when β=1 and β=0.

The domain $0 \le \eta \le \eta_\infty$;where η_∞ is one end of the user specified computational domain. Using the algebraic mapping

$$x = \frac{2\eta}{\eta_\infty} - 1 \tag{15}$$

The unbounded region $[0, \infty)$ is mapped into the finite domain $[-1, 1]$ and the problem expressed by equation (13),(14) is transformed into

$$f''' + \alpha \frac{\eta_\infty}{2} ff'' + \beta[(\frac{\eta_\infty}{2})^3 - (\frac{\eta_\infty}{2}) f'^2] \tag{16}$$

together with the boundary conditions

$$f(-1) = f'(-1) = 0 \qquad \text{and} \qquad f'(1) = \frac{\eta_\infty}{2} \tag{17}$$

where the prime denotes differentiation with respect to x

Our technique is accomplished by starting with Chebyshev approximation for the highest-order derivative, f''', and generating approximation to the lower-order derivatives, f'', f' and f, through integration of the approximation of the highest-order derivative as follows:

Setting $\phi(x) = f'''(x)$ then, by integration, we get

$$f''(x) = \int_{-1}^{x} \phi(x)dx + C_1$$

$$f'(x) = \int_{-1}^{x}\int_{-1}^{x} \phi(x)\, dxdx + (x+1)C_1 + C_2 \tag{18}$$

$$\text{and} \quad f(x) = \int_{-1}^{x}\int_{-1}^{x}\int_{-1}^{x} \phi(x)\, dxdxdx + \frac{(x+1)^2}{2} C_1 + (x+1)C_2 + C_3.$$

From the boundary conditions (17), we get

$$C_2 = C_3 = 0,$$

and

$$C_1 = \frac{\eta_\infty}{4} - \frac{1}{2} \int_{-1}^{1}\int_{-1}^{x} \phi(x)\, dxdx$$

Therefore, we can give an approximation to equation (18) as follows:

$$f_i = f(x_i) = \sum_{j=0}^{N} \ell_{ij}\phi_j + d_i ,$$

$$f'_i = f'(x_i) = \sum_{j=0}^{N} \ell^{(1)}_{ij} \phi_j + d^{(1)}_i , \qquad (19)$$

and

$$f''_i = f''(x_i) = \sum_{j=0}^{N} \ell^{(2)}_{ij} \phi_j + d^{(2)}_i$$

for all $i = 0(1)N$, where

$$\ell_{ij} = b^{(3)}_{ij} - \frac{(x_i+1)^2}{4} b^{(2)}_{Nj} , \qquad d_i = (x_i+1)^2 \frac{\eta_\infty}{8}$$

$$\ell^{(1)}_{ij} = b^{(2)}_{ij} + \frac{(x_i+1)}{2} b^{(2)}_{Nj} , \qquad d^{(1)}_i = (x_i+1) \frac{\eta_\infty}{4} ,$$

$$\ell^{(2)}_{ij} = b_{ij} - \frac{1}{2} b^{(2)}_{ij} , \qquad d^{(2)}_i = \frac{\eta_\infty}{4} ,$$

where

$$b^{(3)}_{ij} = \frac{(x_i - x_j)^2}{2} b_{ij} ,$$

$$b^{(2)}_{ij} = (x_i - x_j) b_{ij} , \qquad i,j = 0(1)N$$

and b_{ij} are the elements of the matrix B, as mentioned in [1].

By using equation (19) the equation (16) transformed to the following system of non-linear equations

$$\phi_i + \alpha\left(\frac{\eta_\infty}{2}\right)\left(\sum_{j=0}^{N} \ell_{ij}\phi_j + d_i\right)\left(\sum_{j=0}^{N} \ell^{(2)}_{ij} \phi_j + d^{(2)}_i\right)$$

$$+ \beta\left[\left(\frac{\eta_\infty}{2}\right)^3 - \left(\frac{\eta_\infty}{2}\right)\left(\sum_{j=0}^{N} \ell^{(1)}_{ij} \phi_j + d^{(1)}_i\right)^2\right] = 0, \qquad (20)$$

$$i = 0(1)N$$

in the highest derivative and we solve it by Newton's method.

IV. NUMERICAL RESULTS:

The method described here-in has been used to calculate the flow for several different combination of α and β. Table 4 shows comparison of the wall shear stress f''_w for N=20 and different η_∞. Table 5 illustrates the effect of the degree of approximation for $\alpha=1$ and $\beta=0.5$ (Homann axisymmetric stagnation flow), $\alpha= 1$ and $\beta=1$ (Hiemenz flow) All values of f', f''_w are in good agreement with those reported by Rosenhead [5] and white [7]. Decreasing the degree of approximation, the values of the wall shear stress f''_w and the fluid velocity f' do not change after fixed degree of approximation. Table [6] lists the value of f''_w, the wall shear stress, for $\eta_\infty = \eta_{\infty 1}$, and $\eta_\infty = 2\ \eta_{\infty 1}$, for two sample cases of Homann and Hiemenz flows and $\alpha=1$, $\beta=10$ (a flow with a strongly favorable pressure gradient) for a fixed degree of approximation N=20.

The computations were carried out on a VME2955, ICLcomputer.

Table (1): The Most Unstable Eigenvalue of Plane Poiseuille Flow for a = 1, R = 10000

M+1	Present Method	D.Hatziavramidis [2]
14	0.236981433 + 0.00321181517i	0.237235137 + 0.0041940510i
15	0.237237495 + 0.003665072 i	0.237711145 + 0.0038379230i
16	0.237530489 + 0.0034384176i	0.237512114 + 0.0038506654i
17	0.237630658 + 0.0037645283i	0.237519937 + 0.0037373238i
18	0.237515439 + 0.0037577974i	0.237542640 + 0.0037998328i
19	0.237531318 + 0.0037542197i	0.237526382 + 0.0037769869i
20	0.237519668 + 0.0037484922i	0.237535288 + 0.0037894188i
21	0.237529756 + 0.0037451062i	0.237529654 + 0.0037744669i
22	0.237529122 + 0.0037496205i	0.237526039 + 0.0037518324i
23	0.237527908 + 0.0037493184i	0.237533259 + 0.0037893048i
24	0.237527620 + 0.0037488123i	0.237529593 + 0.0037705812i
25	0.237527969 + 0.0037485690i	0.237528078 + 0.0037631389i
26	0.237528070 + 0.0037487614i	0.237533319 + 0.0037892652i
27	0.237527987 + 0.0037488076i	0.237536556 + 0.0038054500i
28	0.237527963 + 0.0037487717i	0.237529579 + 0.0037706030i
29	0.237527981 + 0.0037487605i	0.237538549 + 0.0038154119i

Table (2): The most Unstable Eigenvalue for THe Critical Reynolds.Number R_c = 5772.22 and Wave number a_c = 1.02056.

M+1	Present Method	D.Hatziavramidis [12]
14	$0.263746649 + 4.77294095\times10^{-5}i$	$0.264157871 + 1.78540477\times10^{-4}i$
15	$0.264033131 + 2.25185862\times10^{-4}i$	$0.263974543 + 1.90346143\times10^{-4}i$
16	$0.264054957 + 4.38697795\times10^{-5}i$	$0.264010755 + 8.70870695\times10^{-7}i$
17	$0.263994745 + 6.56876409\times10^{-6}i$	$0.263999880 + 2.65828567\times10^{-5}i$
18	$0.263998427 + 1.16036057\times10^{-5}i$	$0.264002880 + 2.88167435\times10^{-5}i$
19	$0.263999550 + 1.45006128\times10^{-6}i$	$0.264002108 + 2.56037714\times10^{-5}i$
20	$0.264004096 + 5.80368212\times10^{-6}i$	$0.264002649 + 3.09871639\times10^{-5}i$
21	$0.264002190 + 6.66335930\times10^{-6}i$	$0.264001857 + 2.18119503\times10^{-5}i$
22	$0.264001755 + 6.11743756\times10^{-6}i$	$0.264000685 + 7.60825237\times10^{-6}i$
23	$0.264002024 + 5.82002144\times10^{-6}i$	$0.264002645 + 3.11928323\times10^{-5}i$
24	$0.264002143 + 5.97293879\times10^{-6}i$	$0.264001665 + 1.94065400\times10^{-5}i$
25	$0.264002075 + 6.01441200\times10^{-6}i$	$0.264001272 + 1.46951159\times10^{-5}i$
26	$0.264002060 + 5.98590778\times10^{-6}i$	$0.264002646 + 3.11933774\times10^{-5}i$
27	$0.264002073 + 5.98201964\times10^{-6}i$	$0.264003496 + 4.14084377\times10^{-5}i$
28	$0.264002074 + 5.98817384\times10^{-6}i$	$0.264001665 + 1.94110248\times10^{-5}i$
29	$0.264002072 + 5.98876646\times10^{-6}i$	$0.264004018 + 4.76961341\times10^{-5}i$

Table (3): The Most Unstable Eigenvalue for The Blasius Flow for a = 0.179 , R = 580

η_∞	N-1	Present Method	D.Hatziavramidis [2]
10	36	0.367592246 + 0.0064084024i	0.367592887 + 0.00643824879i
10	44	0.367588206 + 0.00642506277i	0.367595570 + 0.00644268539i
20	36	0.363916305 + 0.00674679702i	0.363996006 + 0.00767898122i
20	44	0.364106627 + 0.00784644693i	0.364154863 + 0.00797282057i
20	46	0.364175339 + 0.00793035553i	0.364119456 + 0.00796321134i
30	40	0.362303015 + 0.00315191622i	0.363403437 + 0.00908507014i
30	42	0.364116688 + 0.00641033755i	0.363341298 + 0.00824699194i
30	44	0.363602568 + 0.00690295526i	0.363851749 + 0.0078842921 i
30	46	0.364255651 + 0.00663699103i	0.364156290 + 0.00803873966i

The computations were carried out on VME 2955, ICL computer.

Table (4): Comparison of the Wall Shear Stress f''_w.

η_∞	α	β	Present Method	Wadia & Payne [2]	Rosenhead [3]	White [4]
8.0	1	-0.15	-0.1334299	-0.131999	-0.132	-
8.0	1	-0.18	-0.0976934	-0.097	-0.097	-
5.6	1	-0.18	0.1289902	0.1285	0.1285	0.12864
5.2	1	-0.15	0.2167541	0.216101	0.2161	-
6.9	1	0.0	0.469600	0.469604	0.4696	0.4696
4.4	1	0.3	0.774783	0.774764	-	0.77476
3.7	1	0.5	0.927805	0.925999	0.928	-
3.5	1	1	1.232617	1.23247	1.233	1.23259
3.1	1	2	1.687226	1.6866	1.687	1.68722
4.6	0	1	1.154712	1.1547	1.1547	-
3.6	-1	4	2.272784	2.1602	2.273	-
10.0	-1	1	1.086362	1.0628	1.086	-
2.0	1	10	3.675234	3.33	-	-
2.0	1	15	4.491487	3.93167	-	-
2.0	1	20	5.180718	4.38311	-	-

Table (5): The Effect of The Degree of Approximation on The Fluid Velocity f', $\eta_\infty = 4$.

α, β	α $\beta = 0.5$			$\alpha = 1$ $\beta = 1$		
N	10	20	40	10	20	40
η	f'	f'	f'	f'	f'	f'
0.097886	0.088414	0.088416	0.088416	0.115869	0.115869	0.115869
0.381966	0.317920	0.317912	0.317912	0.399046	0.399047	0.399047
0.824429	0.596248	0.596262	0.596262	0.698565	0.698563	0.698563
2.000000	0.942310	0.942331	0.942331	0.973229	0.973226	0.973226
2.618034	0.986080	0.986060	0.986060	0.994922	0.994925	0.994925
3.618034	0.999436	0.999428	0.999428	0.999848	0.999849	0.999849
4.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
f''_w	0.9277187	0.9277190	0.927719	1.232591	1.232591	1,232591

Table (6): The Wall Shear Strees f''_w for Different Values of η_∞.

α	1	1	1	1	1	1
β	1.0	1.0	0.5	0.5	10.0	10.0
$\eta\infty$	3.5	7.0	3.7	7.4	2.0	4.0
Present Method	1.232617	1.232588	0.9278054	0.9276800	3.675234	3.675234
Wadia & Payne[2]	1.232173	1.23137	0.929779	0.927965	3.5548	3.55426
White[4]	1.232588		0.927680			

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