1- # DIFFUSION PROCESS IN A TRANSISTOR USING THE BOUNDARY ELEMENT

2- # METHOD

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4- ## ABSTRACT

The diffusion process in a transistor is simulated by two partial differential equations: *Laplace's* and *Poisson's*. The proposed solution consists in an iterative scheme for a determination of the interface between these two equations. The first one consists in the use of an adjoint potential for the reduction of the domain to the boundary integral . The second method consists in a an analytical reduction of the domain integral and a numerical evaluation of the boundary integral using Simpson's rule and Gauss quadrature scheme. The two methods are applied to a free boundary value problem : A Junction Gate Field Effect Transistor. The results obtained are analysed and compared.

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## INTRODUCTION

The determination of the saturation zone in a PNP transistor is according to Tanaka [1] a free boundary value problem . This problem was successively solved by Liggett [2], Elliot *et al.*[3], using a finite difference scheme, by Crank [4] using both finite difference and finite element methods,by Liu *et al.*[5] and Bruch [6] using the boundary element method (BEM). Lately the same problem was approximated by Nochetto *et al.*[7] using a parabolic interpolating function. Free boundary value problems apply to different engineering fields ranging from diffusion in a porous media to plasma diffusion in a container as studied by Guelter [8].Here the diffusion process in a PNP transistor, is simulated by a dual solution of Laplace's and Poisson's equations. The interface between the depletion P region and N channel is preliminary fixed, and its final position determined through and iterative procedure.At each iteration steps , the voltage and flux at the interface are evaluated using two variants of the boundary element method. The first variant consists in a reduction of the domain integral to the boundary by using an additional variable: the adjoint potential [9]. The second variant consists in a *quasi-analytical* reduction of the domain integral. Both methods are used to solve a Junction-Gate Field Effect Transistor (JFET) and the results are analysed.

## PROCEDURE

### Dual Boundary Element Method of Solution of the Problem.

The distribution of the space charge in the depletion region is illustrated in Fig.1. and is regulated by Poisson's equation:

$$\nabla^2 V = -\frac{\rho}{\varepsilon} \qquad (1)$$

where V is the voltage, $\rho$ is the space charge density and $\varepsilon$ is the semi conductor permissivity. While the potential distribution in the N channel is regulated by Laplace's equation:

$$\nabla^2 V = 0 \qquad (2)$$

These two equations are simultaneously solved using the boundary integral. In this case Poisson's and Laplace equations are weighted by the fundamental solution of the problem , and inversely integrated giving:

$$[G]\,\underline{q} - [H]\,\underline{V} = \underline{B} \qquad (3)$$

Where [H] and [G] are the system matrices respectively vector written as :

$$\underline{B} = - \int_{\Omega} \beta \, V^* \, d\Omega \tag{4}$$

$V^*$ is the Green's function, $\Omega$ is the domain, and $\beta = \frac{\rho}{\varepsilon}$ This integral is reduced to the boundary, by assuming that the *adjoint potential* $\phi$, satisfies an equation of the form [9]:

$$\nabla^2 \phi = V^* \tag{5}$$

This equation is used to reduce equation (4) to a boundary integral form:

$$\underline{B} = -\int_{\Omega} \beta \, V^* \, d\Omega \quad = - \int_{\Omega} (\beta \, \nabla^2 \phi - \nabla^2 \beta \, \phi) \, d\,\Omega \tag{6}$$

In this equation : $\nabla^2 \beta = 0$ , as the space charge distribution $\varepsilon$ is constant over the depletion region. Applying Green's theorem, this equation reduces to:

$$\underline{B} = - \int_{\Gamma} (\beta \, \nabla\phi - \phi \, \nabla\beta \,) \, d\Gamma \tag{7}$$

as $\nabla\beta = 0$. Hence:

$$\underline{B} = - \int_{\Gamma} \beta . \nabla\phi \, d\Gamma \tag{8}$$

This equation reduces the vector $\underline{B}$ to a boundary integral with a term $\nabla\phi$, that will be deduced in the following lines. $\nabla\phi$ is obtained by integrating equation (5). For axisymmetric problems, this equation is written as:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \, \frac{\partial \phi}{\partial r} \right) = V^* \tag{9}$$

Integrating once gives:

$$r \, \frac{\partial \phi}{\partial r} = \int r \, V^* \, \partial r \tag{10}$$

Where $V^*$ for a two dimensional potential problem is equal to: $-\ln r$. Hence get :

$$\frac{\partial \phi}{\partial r} = \frac{r}{2}\left[1-\ln r\right] \qquad (11)$$

For a non axisymmetric problem the derivative $\nabla\phi$ is written in a polar form as: $\frac{\partial \phi}{\partial r} \cdot \frac{\partial r}{\partial n} = \frac{\partial \phi}{\partial r} \cdot \cos\theta$ ,where $\theta$ is the normal angle between two boundary segments. Replacing for $\nabla\phi$ in equation (8) gives:

$$\underline{B} = \int_{\Gamma} \beta \frac{r}{2}\left[\ln r-1\right] \cos\theta \, \partial\Gamma \qquad (12)$$

Replacing for $\underline{B}$ in equation (3) and replacing for [H] and [G] by their integral form, gives:

$$C_i V_i = \int_{\Gamma} (V^*.\nabla V - \nabla V^*.V)\, d\Gamma + \int_{\Gamma} \beta \frac{r}{2}\left[1-\ln r\right] \cos\theta \, d\Gamma \qquad (13)$$

Where $C_i$ is a weighting factor to the voltage $V_i$. This is Poisson's integral equation , while Laplace equation is written as :

$$C_i V_i = \int_{\Gamma} (V^*.\nabla V - \nabla V^* \, V)\, d\Gamma \qquad (14)$$

Equations (13) and (14) are simultaneously solved using Gauss quadrature method. The iteration steps to determine the free boundary are described in the flow chart depicted in Fig.2.

## Quasi-Analytical Solution.

The same problem is solved here by an analytical reduction of the domain to boundary integral. The details of the reduction are given for an initial elliptic free boundary. The term to be reduced is :

$$\underline{B} = \int_{\Omega} \beta \ln \vec{r} \, d\Omega \qquad (15)$$

Where $\vec{r}$ is the radial distance between two boundary segments, and $-\ln r$ is the fundamental solution of the problem. Replacing for $\Omega$ by a set of cartesian coordinates equation (15) is rewritten as:

$$B_i = \beta \int_{-b}^{b} \int_{0}^{\xi} \ln \sqrt{(x_i-x)^2 + (y_i-y)^2}\, dx\, dy \qquad (16)$$

Where $x_i, y_i$ are the coordinates of the boundary node "i", $B_i$ is evaluated over an elliptic domain, with major and minor radii "a" and "b" and $\xi = \frac{a}{b}\sqrt{b^2-y^2}$ .Equation (16) is rewritten as:

$$B_i = \beta \int_0^b \int_0^\xi \ln\left[(x_i-x)^2+(y_i-y)^2\right] dx\, dy. \tag{17}$$

This equation is first integrated by part with respect to x, giving:

$$B_i = \beta \int_0^b \left\{ \xi \ln\left[(x_i-\xi)^2+(y_i-y)^2\right] - \int_0^\xi \frac{2x\,(x_i-x)}{(x_i-x)^2+(y_i-y)^2} dx \right\} dy. \tag{18}$$

The last term is reduced by partial fraction giving:

$$I = \int_0^\xi 2 + \frac{2x_i\,(x-x_i)-2(y_i-y)^2}{(x_i-x)^2+(y_i-y)^2} dx \tag{19}$$

or

$$I = \left[ 2x+x_i \ln\left[(x_i-x)^2+(y_i-y)^2\right] - 2(y_i-y)\tan^{-1}\left[\frac{x_i-x}{y_i-y}\right] \right]_0^\xi \tag{20}$$

Inserting equation (20), in equation (18) gives:

$$B_i = \beta \int_0^{-b} \left\{ \xi \ln\left[(x_i-\xi)^2+(y_i-y)^2\right] + 2\xi + x_i \left\{ \ln\left[(x_i-\xi)^2+(y_i-y)^2 - \ln(x_i^2+(y_i-y)^2)\right\} - 2(y_i-y)\left\{ \tan^{-1}\frac{x_i-\xi}{y_i-y} - \tan^{-1}\frac{\xi}{y_i-y} \right\}\right\} dy \tag{21}$$

$B_i$ is numerically integrated using Simpson's rule and its value is inserted in equation (3). The *quasi-analytical method*, saves half of the computational effort required in the dual BEM and this because only one instead of two equations needs to be solved .

## APPLICATIONS

The two proposed methods are used to solve a free boundary problem, in the form of a JFET transistor. The symmetry of the PNP junction is used, and only half of the domain is considered, as illustrated in Fig. 3. An initial elliptic shape is adopted. The boundaries are discretized to 52 nodes and the interface to 20 nodes. The gate voltage $V_g$ is considered equal to 8 volts, the drain voltage $V_D$ is taken equal to -5 volts and the source voltage $V_s$ is equal to zero.

## DISCUSSION OF RESULTS

### Using the Dual Boundary Element Method

The steps depicted in the flow chart are followed. These steps consists in a stepwise increase of the minor radius "b" keeping the major radius "a" fixed . The flux deviation from zero , at the interface is evaluated as :

$$\|e\| = \left[ \sum_{i=1}^{n} (flux)_i^2 \right]^{1/2} / n \qquad (22)$$

Where n is the number of nodes on the interface. $\|e\|$ is plotted versus b for different values of "a" in Fig.4,5,6. In Fig.4: three curves appears. The first one is plotted for a = 2.5 cm and b is varying from 0.5 to 3cm. Similarly the two other curves are respectively plotted for a=3,a=4 .From these curves it is found that $\|e\|$ is minimum at b=3cm. Another test is run. This time the gate length is reduced and the dimensions of a and b are varied in the same sequence. The results are plotted in Figs.5 and 6 where it appears that $\|e\|$ decrease for large a. This work is followed by a plot of the voltages at the interface for different gate lengths $L_g$. It appears in Figs.7-8-9-10. that interface voltage increases with the gate length. This is followed by a test on the various system parameters, plotted in Fig.11. It appears from this figure that the potential at the interface increases with the increase of gate voltage $V_g$. Finally the problem is solved for different levels of gate and drain voltages . The results are plotted in Fig.12.where it appears that for a large difference between $V_g$ and $V_D$ , the voltage at the interface increases.

### Using the Quasi-Analytical Method.

The same problem, with different dimensions, is solved using the *quasi-analytical* method of solution. the error,$\|e\|$ is plotted for 7 iteration steps in Figs.13,14 .It is found that $\|e\|$ is minimal for a= 3 , b = 3,5 cm.The voltage at the interface are plotted in Figs.15,16,17. It appears in these figures that the voltage at the interface spreads in a form similar to the voltage distribution previously obtained. A comparison of both methods, regarding the computational effort shows that the *quasi-analytical* method of solution extremely reduces the computation effort.

## CONCLUSION

The determination of the saturation surface in a JFET transistor is resolved here using two variants of the BEM. The *Dual* and the *quasi-analytical* BEM. In the first method Laplace's and Poissn's equation are dually solved, with a reduction in the non homogenous term through the use of an adjoint variable. While in the second method Poisson's non homogenous term is analytically reduced from domain to boundary integral. The integral equation is solved using Simpson's and Gauss-quadrature scheme. Both methods are used to simulate the response of the JFET transistor to different system parameters. The advantage of the proposed scheme appears in the reduction of the mesh size and computational effort. This reduction is larger when using the *quasi-analytical* boundary element method of solution.

## FIGURES

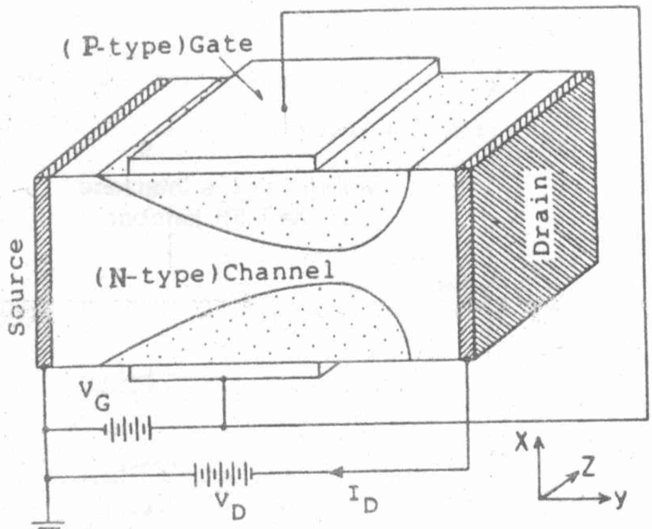
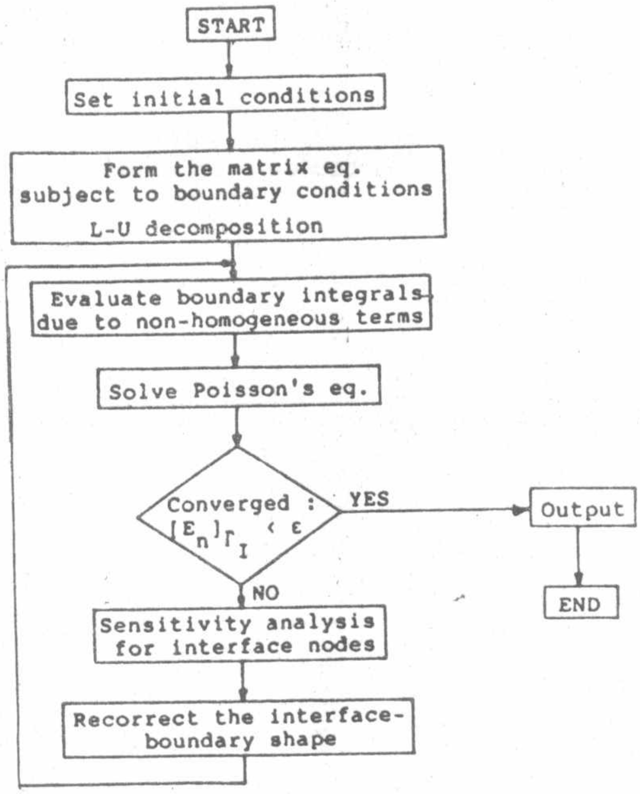



Fig 1. Diagram of a junction-gate field effect transistor.



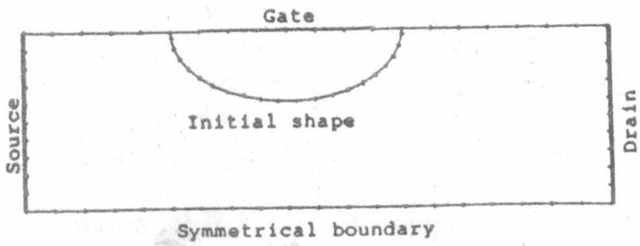
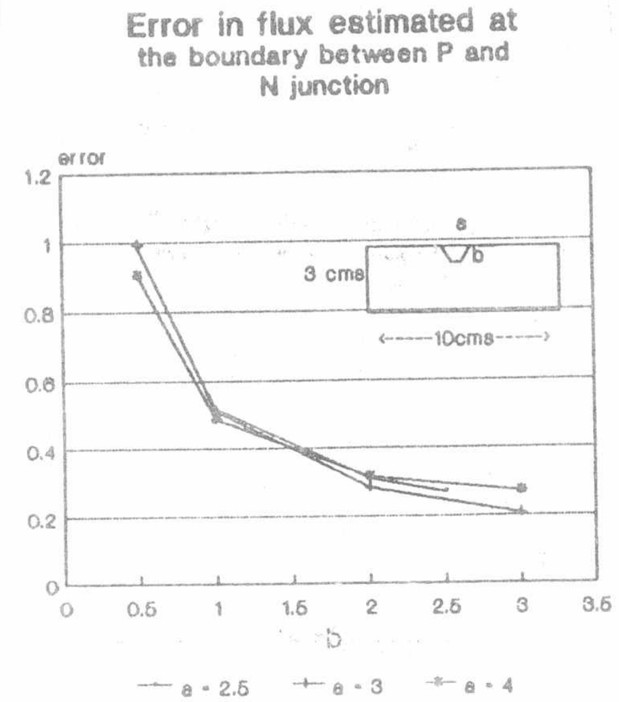



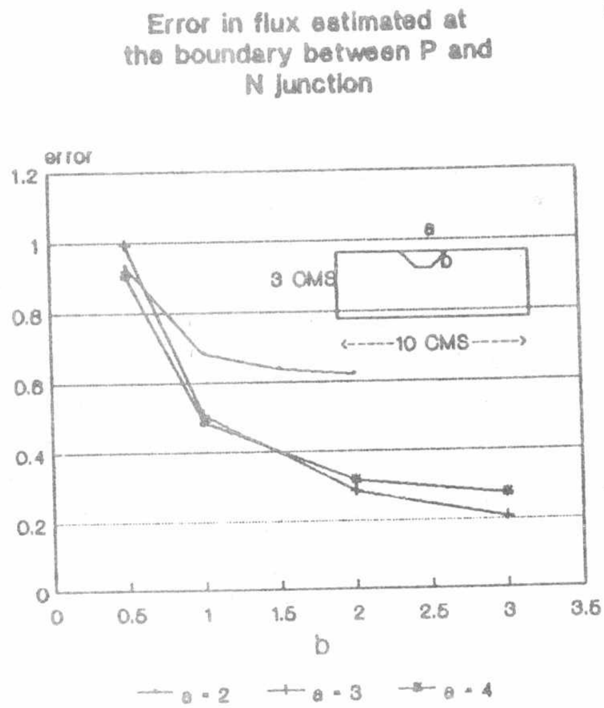

Fig.3 Discretization of the JFET transistor

Fig 2. Dual solution of Laplace's and Poisson's equations



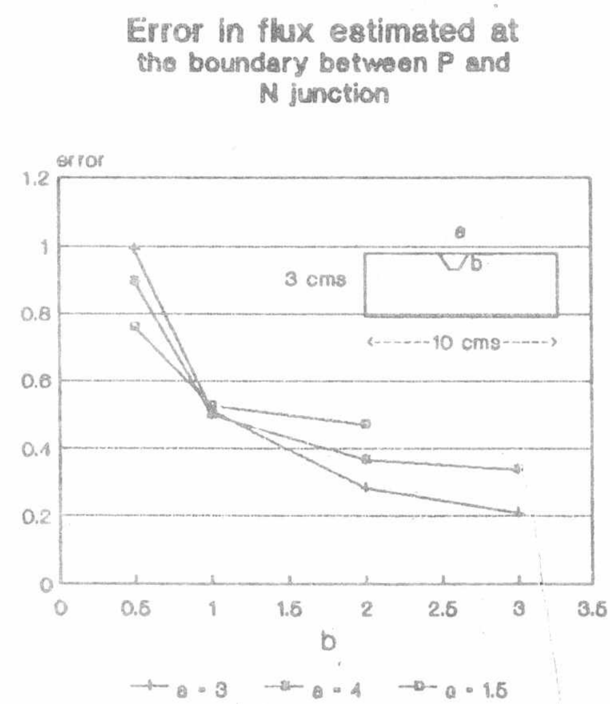

Lg = 4cms , Vg = 8 volts

Fig.4 Error in flux at the free boundary for $L_g$ = 4 cms



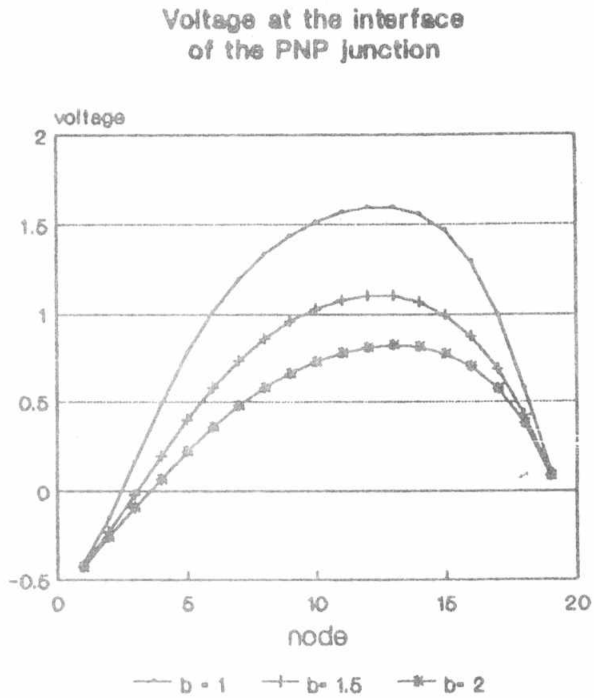

Lg = 3 cms , Vg = 8 volts

Fig.5 Error in flux at the free boundary for $L_g$ = 3 cms



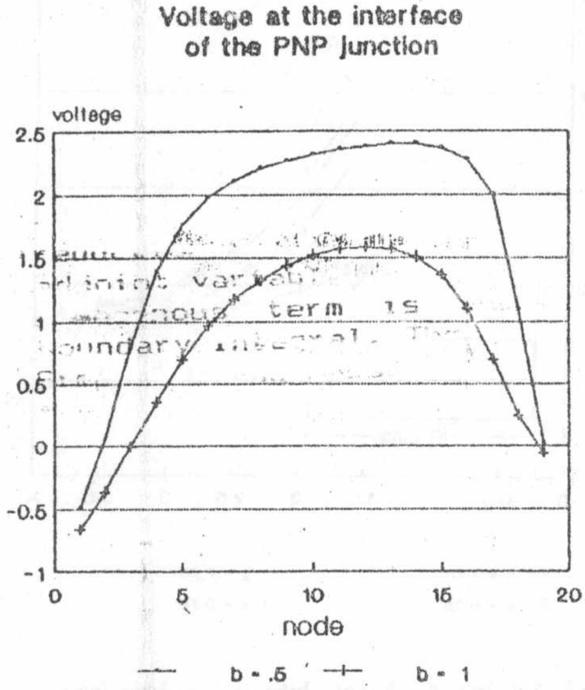

Lg = 2cms , Vg = 8 volts

Fig.6 Error in flux at the free boundary for $L_g$ = 2 cms



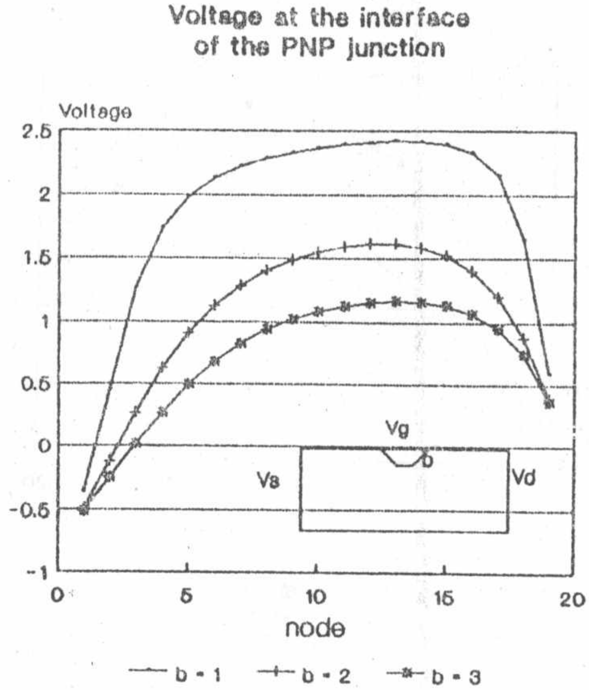

Gate voltage = 8 volts , Lg= 2 , a=2cms
Drain voltage = -5 voltage
Source volage = 0 volts v822.cht

Fig.7. Voltage at the free boundary for a = 2,$L_g$ = 2 cms



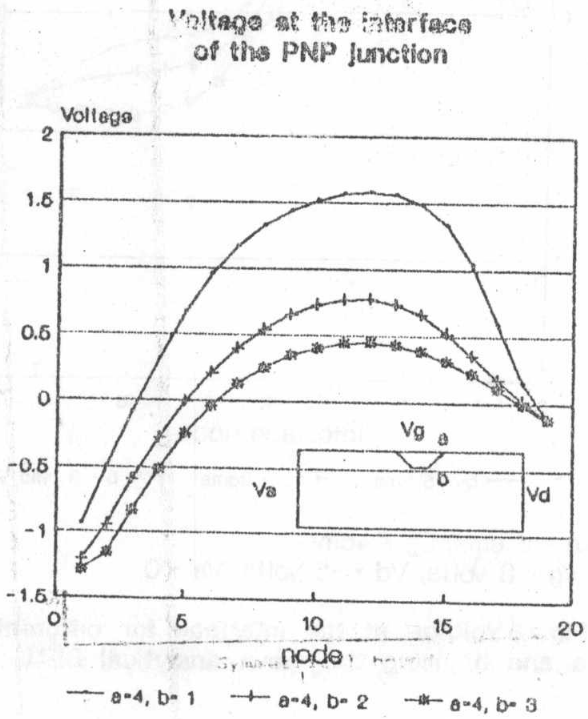

Gate voltage = 8 volts, Lg= 2 , a=3 cms
Drain voltage = -5 volts
Source voltage = 0 volt

Fig.8. Voltage at the free boundary for a=3, $L_g$ =2 cms



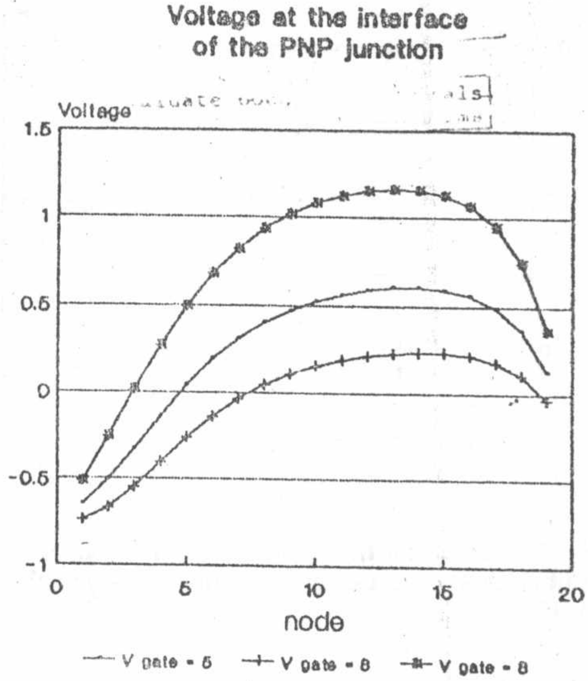

Vg= 8 volts , Vd = - 5 volts, Vs=0
Lg = 4 cms , a = 3 cms

Fig.9. Voltage at the free boundary for a = 4, $L_g$ = 2cms




Vg= 8 volts , Vd= -5 , Vs=0
Lg = 2 cms , a = 4 cms

Fig.10. Voltage at the free boundary for a = 3 , $L_g$ = 4 cms




Vd = - 5 volts (Drain voltage)
Gate length = 4 cms
a and b = 3 cms

Fig.11 Voltage for different gate voltages

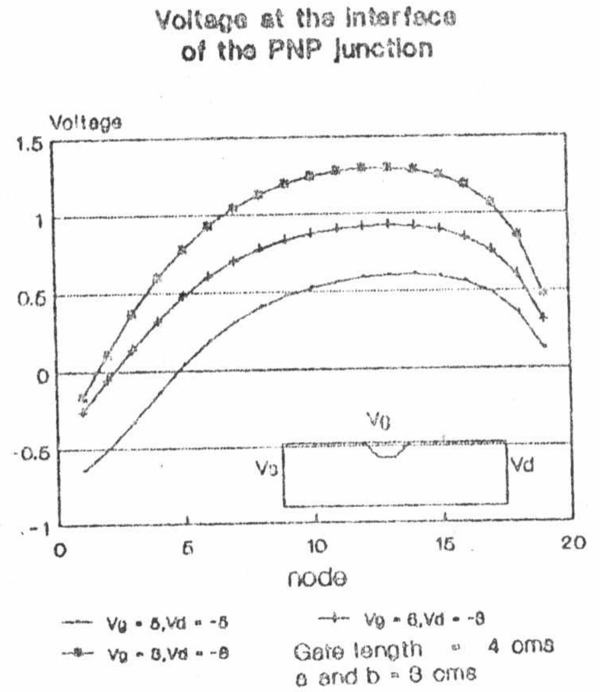



Fig.12 Voltage at the interface for different gate and drain voltages,using the dual BEM

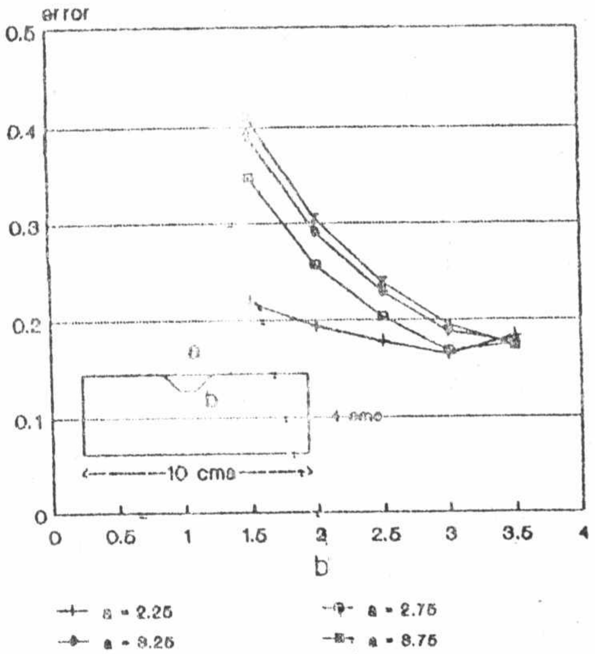



Fig.13 Error at the interface using the quasi-analytical BEM

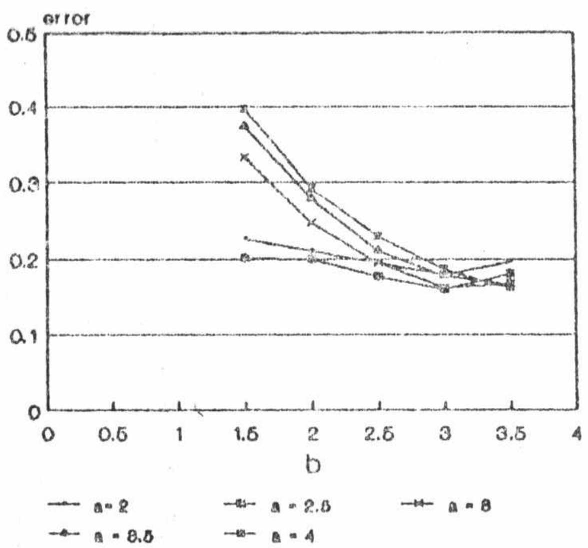
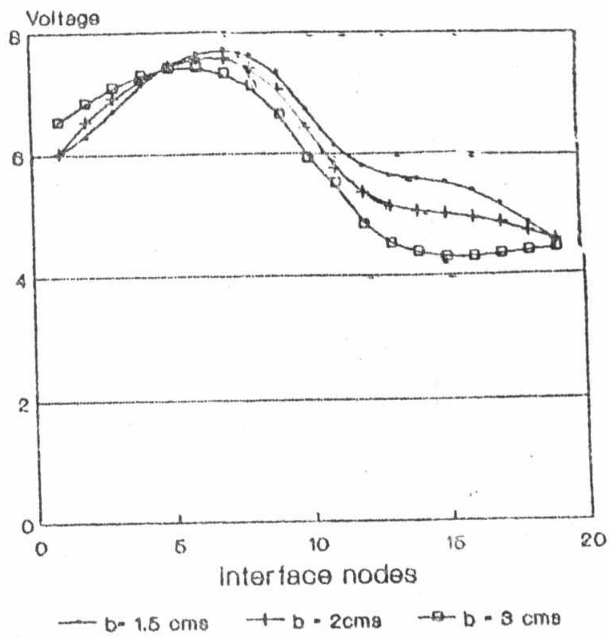
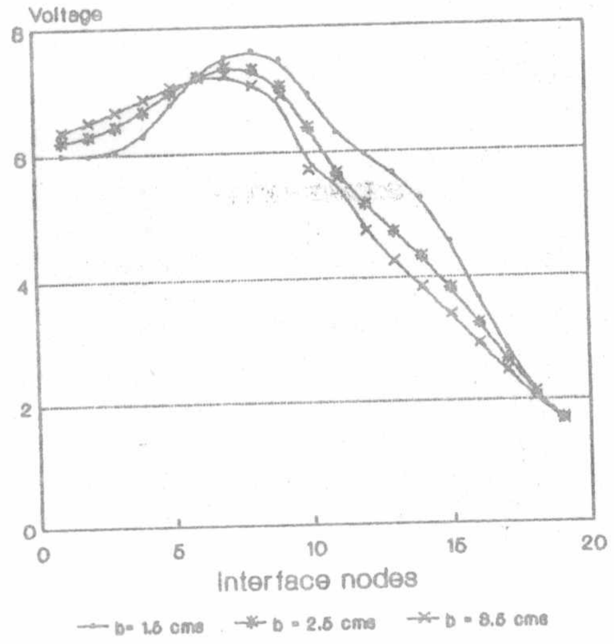



Fig.14. Error in flux estimate , for different a and b , using the quasi-analytical BEM



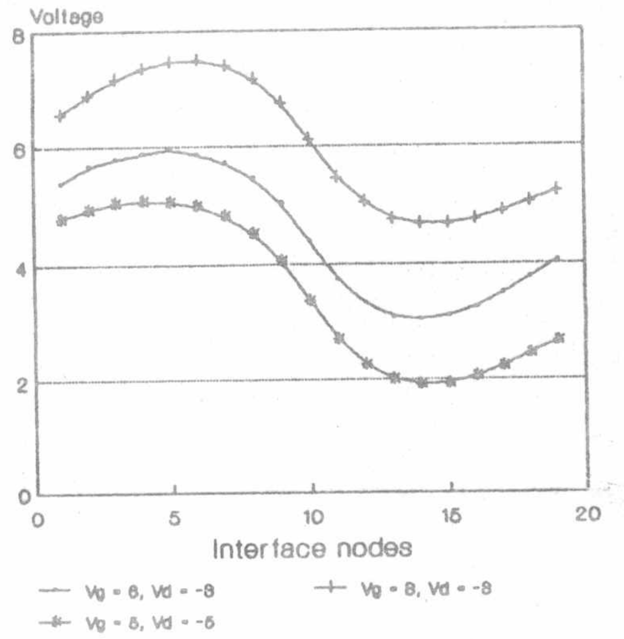

a = 3 cms, Lg = 4cms
Vg = 8 volts, Vd = -5 volts ,Vs = 0

Fig.15 Voltage at the interface for different a and b ,using the quasi-analytical BEM




a = 4 cms, Lg = 4cms
Vg = 8 volts, Vd = -5 volts ,Vs = 0

**Fig.16 Voltage at the interface for different a and b, using the quasi-analytical BEM**




a = 3 cms, b = 3 cms , Lg = 4cms
Vs = 0 volts

**Fig.17 Voltages at the interface for different gate and drain voltages , using the quasi-analytical BEM**

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