AN IMPROVED GUIDANCE ALGORITHM FOR SOLID PROPELLENT 
BALLISTIC MISSILES 

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ABSTRACT 

Due to production tolerances and off-nominal environmental conditions, the thrust time profile of solid propellant rocket motors suffers from high uncertainties in both magnitude and burn-out time. This behaviour leads to higher uncertainties in the motion parameters of the missile at the shut-off (burn-out) point. Accordingly, the impact point is highly erroneous. This paper develops a guidance and control strategy for compensating the effects of the above-mentioned uncertainties in such a way as to minimize impact errors.

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1-INTRODUCTION

The performance of ballistic missile systems is measured in three axes: namely: impact accuracy, maximum range, and destruction capability.

Impact accuracy is influenced mainly by:
. Inertial measurement errors
. Computation errors
. Steering and burn out errors
. Gravitational anomalies
. Re-entry errors

Guidance and control strategies are designed to steer the missile on a reference trajectory (corresponding to a specified mission) for a specified state vector at shut-off to achieve suitable impact by minimization of the deviation of the missile from the target point.

Ballistic missile system designers have concentrated on the use of liquid propellant engines recognizing that their advantage is that they can be readily controlled. However, inherent advantage of handling ease has generated increased attention to the use of solid propellants. Due to production tolerances and off-nominal environmental conditions, the thrust time profile of solid propellant rocket motors suffers from high uncertainties in both magnitude and burn-out time. These uncertainties lead to higher uncertainties in the motion parameters of the missile at the shut-off (burn-out) point. Accordingly, the impact point is highly erroneous. This paper develops a guidance and control strategy for compensating the effects of the above mentioned uncertainties in such a way as to minimize impact errors.

2-MODELING AND SIMULATION

2.1-Reference Coordinates and Vector Transformations

The reference coordinate systems used throughout this study are shown in Figure.1.

The transformation of a vector $\mathbf{A}$ in body coordinate system to a vector $\mathbf{A}_e$ in earth coordinate system is carried out through the matrix $[ME]$:

$$[ME] = \begin{bmatrix} I_1 & J_1 & K_1 \\ I_2 & J_2 & K_2 \\ I_3 & J_3 & K_3 \end{bmatrix}$$

$$[ME]^{-1} = [ME]^T = [EM]$$

This transformation contains three intermediate transformations:
- Missile-Fire
- Fire-Local
- Local-Earth

transformations.
2.2- Six degrees of Freedom Motion Equations

The vectorial system of equations describing the ballistic missile motion in space is as follows:

\[ \begin{align*}
R_m &= R_m + \vec{W}_e \times \vec{R}_m \\
\vec{V}_e &= \vec{V}_e + \vec{W}_e \times \vec{V}_e - \vec{A}_e + \vec{G}_e \\
\vec{I}_m &= (\vec{W}_{me} - \vec{W}_e) \times \vec{I}_m \\
\vec{J}_m &= (\vec{W}_{me} - \vec{W}_e) \times \vec{J}_m \\
\vec{H}_a &= \vec{W}_m \times \vec{H}_a + \frac{T_A + T_T}{J_m} \\
\vec{A} &= (\vec{F}_A + \vec{F}_T) / \text{mas} \\
\vec{R}_m &= \vec{I}_m \times \vec{J}_m
\end{align*} \]

2.3- Fire Plane Motion Equations

The fire plane is defined by the launch point, target point, and earth center. Under the assumptions that:
- the launch point is at \((0,0)\)
- the target point is in the north direction
- the missile is roll stabilized such that \(W_{x1} = 0\)
- the missile is of \(X\)-form
- the missile moves in the fire plane with \(W = 0\), \(W_{y1} = 0\)

The equations of motion take the form:

\[ \begin{align*}
X_e &= V_{e1} \\
V_{e1} &= A_1 I_1 + A_2 J_1 + Ge_1 \\
I_1 &= -W_{12} I_2 (I_1 J_2 - I_2 J_1) \\
J_1 &= -W_{12} J_2 (I_1 J_2 - I_2 J_1) \\
W_{12} &= (T_{A3} + T_{T3}) / J_{22}
\end{align*} \]

With initial conditions:

\[ \begin{align*}
x_e(0) &= 0 \\
v_{e1}(0) &= 0 \\
I_1(0) &= \cos \theta_0 \\
J_1(0) &= -\sin \theta_0 \\
W_{12}(0) &= 0
\]
2.4- Autopilot

To achieve adequate stability and reasonable rapid and well damped response with moderate insensitivity to external disturbances a lateral autopilot; Fig.2.; is designed to control the short period dynamics such that:

\[ \delta_z = -K_1 \frac{V}{Z} + K_2 \frac{V}{V_0} + K_3 \delta_z + U_d \]

where \( K_1, K_2, K_3 \) are determined through pole assignment technique.

2.5-Mission (Reference Trajectory)

The assumed mission is described through:

\[ r_0 = \text{constant} = \theta_0 \]

hence \( r_0 = 0 \)

2.6-Attitude Control

The attitude control demand is calculated by augmenting the difference between \( r_0 \) and \( r \) through P.I. compensator.

Fig-2 shows the functional block diagram for the closed loop ballistic missile system illustrating the additive compensators employed for improving both short and long period dynamics behaviour.

This system is simulated under the assumptions:

- launch point is \((0,0)\)
- target point is in the north direction
- \( \theta_0 = \{ 15, 30, 45 , 60 \ldots \} \)

and the results are shown in Figures 3 and 4.

- Fig.3 shows the actual mission for \( \theta_0 = 60^\circ \) and for different rocket motors.
- Fig.4 shows the actual missions for constant burn-out time and different \( \theta_0 \)

It is clear that the attitude errors for different missions have settled to within 2 degrees in a settling time of approximately one-third of the burn-out time.

3-THRUST UNCERTAINTY FORMULATION

The total impulse of the solid propellant rocket motor depends on the chemical compound and the burning rate. The thrust-time profile depends on the form function and the environmental conditions of burning. The burn-out time depends on the form function and the burning rate, so it is also uncertain. Accordingly, the thrust profile may suffer from uncertainties due to production tolerances and off-nominal environmental conditions. These uncertainties can be formulated as randomness in the profile shape parameters.

For the present study the simplified thrust-time curve shown in Fig.5 is considered, where:
- \( F_1 \) (initial thrust value), \( M_1 \) (slope of the segment \( F_1F_2 \)), \( S_1 \) (area under the segment \( F_1F_2 \)), and \( S_2 \) (the remaining area) are considered as Gaussian distributed random variables. Random function generators are used to generate \( F_{li} \), \( M_{li} \), \( S_{li} \), and \( S_{2i} \) where \( i \) is the trial number.

A population of 100 samples is simulated and the limiting curves are shown in Fig.5. The corresponding distribution of \( t_b \) is shown in Fig-6. where the random variations in \( t_b \) are within 2 seconds. The generated thrust-time profiles (100 trials) are sorted w.r.t. \( t_b \) in an ascending order and for each case the impact range is calculated through a 3-dimensional simulation procedure. As shown in Fig.7, it was found that \( r_{limp} \) decreases with increasing \( t_b \) and that the variation in \( r_{limp} \) is within 0.2%.

The determination of \( t_{imp} \) can be carried out through:

1-nominal trajectory off-line simulation
2-statistical means
3-software sensor

For the present case study, the components of the gravitational acceleration \( G_{e1}, G_{e2} \) are nearly constant for a specific mission, i.e. their variation with respect to time is negligible, but they may differ from nominal.

4-GUIDANCE CORRECTION ALGORITHM

4.1-Simplified Free Space Fire Plane Motion Equations

Under the assumption that:
- launch point is at \((0,0)\)
- thus: \( r_1(t) = x_e(t) \)
- \( r_2(t) = y_e(t) - R_e \)
- target point in the north direction
- earth gravity components \( G_{e1} = 0; G_{e2} = -g_0 \).

The free space fire plane motion equations are:

\[
\begin{align*}
r_1 &= V_{e1} \\
r_2 &= V_{e2} \\
W &= 0 \\
I_1 &= 0 \\
I_2 &= 0 \\
J_1 &= 0 \\
J_2 &= 0
\end{align*}
\]

with initial conditions:

\[
\begin{align*}
r_{1}(0) &= r_{1b} \\
r_{2}(0) &= r_{2b} \\
V_{e1}(0) &= V_{e1b} \\
V_{e2}(0) &= V_{e2b} \\
W_{z1}(0) &= 0 \\
I_{1}(0) &= I_{1b} \\
I_{2}(0) &= I_{2b} \\
J_{1}(0) &= J_{1b} \\
J_{2}(0) &= J_{2b}
\end{align*}
\]

Hence; the states affecting the free space ballistic missile motion are \( r_1(t), r_2(t), V_{e1}(t), V_{e2}(t) \).
4.2 - Solution of the equations describing the free space missile motion in the fire plane:

The solution of the above system of equations represents a look ahead predictor for times $t_{\text{imps}}$, $r_{\text{imps}}$ at $t_b$ as follows:

$$
\begin{align*}
  r_1(t) &= r_{1b} + (t-t_b)V_{elb} \\
  r_2(t) &= r_{2b} + (t-t_b)V_{e2b} - \frac{1}{2} g_0(t-t_b)^2 \\
  V_{el}(t) &= V_{elb} \\
  V_{e2}(t) &= V_{e2b} - g_0(t-t_b) \\

t_{\text{imps}} &= t_{\text{imp}} \\
  r_{1\text{imps}} &= r_{1b} + (t_{\text{imps}}-t_b)V_{elb} \\
  r_{2\text{imps}} &= r_{2b} + (t_{\text{imps}}-t_b)V_{e2b} - \frac{1}{2} g_0(t_{\text{imps}}-t_b)^2
\end{align*}
$$

where: $t_1 = t_b + \frac{v_{e2b}}{g_0}$

$$
\begin{align*}
  r_{2}(t_1) &= r_{2b} + (t_1-t_b)V_{e2b} - \frac{1}{2} g_0(t_1-t_b)^2
\end{align*}
$$

4.3 - The Nature of $r_{\text{imps}}, t_{\text{imps}}$

For the specified mission ($\Theta_0=60^\circ$), and through one hundred shootings (for $t_b$ sorted in an ascending order), the performance of the proposed software algorithm is compared with that yielded by the complete time-varying developed mathematical model (actual performance); Figures 7 and 8. It was found that $r_{\text{imps}}$ and $t_{\text{imps}}$ are nearly constant with respect to $t_b$ similar to the actual $r$ and $t_{\text{imp}}$, nevertheless there is some error (nearly constant w.r.t. $t_b$). This error can be minimized by:
- more accurate representation of the gravity model.
- more accurate calculation of $t_{\text{imp}}$ taking into consideration the spherical shape of the earth ($G_{e1}$ and $G_{e2}$).
- making use of perturbation technique.

4.4 - Burn-out impact point transition perturbations:

$$
\begin{align*}
  \delta r_{1\text{imp}} &= \delta r_{1b} + (t_{\text{imp}}-t_b)\delta V_{elb} \\
  \delta r_{2\text{imp}} &= \delta r_{2b} + (t_{\text{imp}}-t_b)\delta V_{e2b}
\end{align*}
$$

4.5 - Guidance Correction Demand Formulation

For annulling the error at impact ($\delta r_{1\text{imp}}=0; \delta r_{2\text{imp}}=0$) then:

$$
\begin{align*}
  0 &= \delta r_{1b} + (t_{\text{imp}}-t_b)\delta V_{elc} \\
  0 &= \delta r_{2b} + (t_{\text{imp}}-t_b)\delta V_{e2c}
\end{align*}
$$

where:

$$
\begin{align*}
  \delta V_{elc}, \delta V_{e2c} \text{ are the command perturbations;}

  \delta V_{elc} &= \frac{-\delta r_{1b}}{t_{\text{imp}}-t_b} \\
  \delta V_{e2c} &= \frac{-\delta r_{1b}}{t_{\text{imp}}-t_b}
\end{align*}
$$
According to Fig. 2 the mission is described by $\Gamma_r$, and the guidance correction demand will be formulated as follows:

$$
\Gamma = \tan^{-1} \left( \frac{v_e}{v_e} \right) ; \quad \Gamma = \frac{v_e}{v_e} - \frac{v_e}{v_e}
$$

$$
\delta \Gamma = \left[ \frac{v_e}{v_e + v_e} - \frac{2v_e(v_e v_e - v_e v_e)}{(v_e + v_e)^2} \right] \delta v + 

\left[ \frac{-v_e}{v_e + v_e} - \frac{2v_e(v_e v_e - v_e v_e)}{2(v_e + v_e)} \right] \delta v - 

\left[ \frac{v_e}{v_e + v_e} \right] \delta v + \left[ \frac{v_e}{v_e + v_e} \right] \delta v
$$

$\delta \Gamma$ is calculated for $t \geq t_b$.  

5-RESULTS

For evaluating the proposed guidance correction algorithm the impact accuracy is used as a criterion. Fig. 9. illustrates the impact error $S_{r_{limp}}$ versus burn-out time, for uncompensated and compensated cases respectively. Fig. 10. shows the impact error distribution for both cases. The corresponding statistical parameters can be summarized as follows:

<table>
<thead>
<tr>
<th>Impact Error</th>
<th>Uncompensated</th>
<th>Compensated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (M)</td>
<td>1.044</td>
<td>0.001</td>
</tr>
<tr>
<td>Dispersion(σ)</td>
<td>0.460</td>
<td>0.160</td>
</tr>
<tr>
<td>CEP</td>
<td>0.306</td>
<td>0.037</td>
</tr>
</tbody>
</table>

6-CONCLUSION

Due to production tolerances and off-nominal environmental conditions, solid propellant thrust time profile has a random character in both magnitude and burn out time. This character leads to uncertainties in the motion parameters near burn out, and consequently the impact point is highly erroneous. The proposed guidance correction algorithm compensates for these uncertainties through transition relations relating the perturbations near burn out to the impact error. The CEP is improved about 10 times and with some sophistications more improvements are attainable.
REFERENCES


NOMENCLATURE

\( Q_x \quad Y_e \quad Z_e \) = Geocentric earth fixed (centered) reference frame

\( L_x \quad Y_1 \quad Z_1 \) = Body fixed frame

\( \hat{1}_m \) = Unit vector in direction of \( x_1 \) with components \( 1_1,1_2,1_3 \)

\( \hat{2}_m \) = Unit vector in direction of \( y_1 \) with components \( 2_1,2_2,2_3 \)

\( \hat{3}_m \) = Unit vector in direction of \( z_1 \) with components \( 3_1,3_2,3_3 \)

\( \theta_0 \) = Initial launch elevation angle

\( \phi_0 \) = Initial launch azimuth angle

\( \vec{R}_m \) = Missile range vector with components \( x_m, y_m, z_m \)

\( R_e \) = Earth equatorial radius (6378165 meters)

\( \vec{V}_e \) = Missile velocity vector with components \( V_{e1}, V_{e2}, V_{e3} \)

\( \vec{A}_e \) = Missile specific force vector of components \( A_{e1}, A_{e2}, A_{e3} \)

\( \omega_0 \) = Earth rotation angular speed = 7.2921*10^-5 rad/sec.

\( \vec{W}_m \) = Missile angular rate vector

\( \vec{W}_{\theta} \) = Missile angular rate vector

\( \vec{V}_m \) = Missile velocity vector

\( \vec{A} \) = Missile specific force vector with components \( A_1, A_2, A_3 \)

\( \vec{G}_e \) = Gravitational field vector with components \( G_{e1}, G_{e2}, G_{e3} \)

\( g_0 \) = Gravitational acceleration (9.8 m/sec^2)
Fig. 1 Coordinate systems configuration
Fig. 2. Ballistic Missile Control Scheme in the Fire Plane

\[ \Gamma \text{ (Degrees)} \]

\[ \begin{array}{c}
\text{1. minimum } \tau_b \\
\text{2. mid. } \tau_b \\
\text{3. maximum } \tau_b \\
\end{array} \]

Time (sec.)

Fig. 3. Flight path response for different \( \tau_b \)
Fig. 4. Flight path response for different missions

Fig. 5. Simulated thrust

Fig. 6. tb Distribution
Fig. 7. $r_{imp}$ and $r_{limps}$ against $tb$

Fig. 8. $t_{imp}$ and $t_{limps}$ against $tb$
Fig. 9. Impact error evaluation

Fig. 10. Impact error distribution