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KINEMATIC ANALYSIS OF COMPLEX MECHANISMS

BY THE METHOD OF MULTIBODY SYSTEMS

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ABSTRACT

This paper presents a procedure for automated kinematic solution of constrained multibody systems such as complex mechanisms possessing low or high degree of complexity. The method utilizes the properties of constrained mechanisms which relate the kinematic characterestics of the individual links to those of the respective input link. The procedure developed is non iterative in nature and non graphical therefore can be conveniently programmed and executed on a minicomputer.
An example illustrating the concepts of the method is presented. The acceleration analysis is executed on the basis of the auxiliary acceleration pattern corresponding to constant angular speed of the alternative input link. The results are compared with those obtained by using two different graphical methods.

INTRODUCTION

The traditional approach for the kinematic solution of plane or space mechanisms involves the sequential application of the relative velocity and relative acceleration equations. However, a mechanism which can not be solved by this method directly is referred as a complex mechanism [1]. A characterestic feature of complex mechanism is the existence of a multipaired floating link with at least three moving links as shown in Fig.1.

Moreover, the complex mechanism is classified as a mechanism with low degree of complexity if only one radius of curvature of its terminal points is not known, otherwise the mechanism has high degree of complexity.

There are many graphical methods for the kinematic solution of complex linkage such as the technique presented by Rosenauer and Willis [2]. Hall [3] utilized the concept of auxiliary points for both velocity and acceleration diagrams. Goodman [4] showed that the principle of kinematic inversion can be used for the

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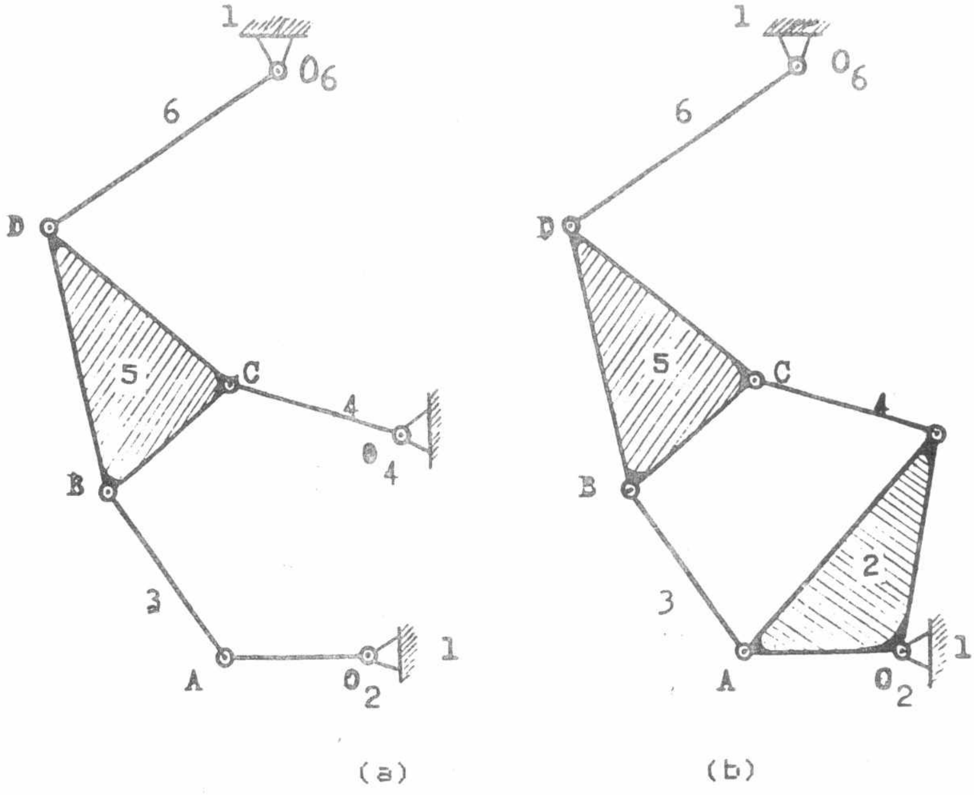


Fig.1 Complex Mechanisms
a) low degree of complexity
b) high degree of complexity

graphical solution of complex systems, however his approach fails when no inversion of a given system converts it to a simple one. Rakesh and others [5] presented a graphical technique which is iterative in character and yields the solution within several iterations. Other graphical methods include the three-line construction [6], Carter's method [7] and the method of normal acceleration [8].

Only few analytical methods had been offered for the solution of such mechanisms. Suh and Radcliffe [9] resolvd the analysis of complex systems into a superpositions of solutions of two or more simpler systems. Gray and Chang [10] developed the former technique by incorporating dual slider, slider crank and inverted slider crank modules, permitting mechanisms to be analyzed when a slider input, e.g. hydraulic or pneumatic cylinder is involved. It should be mentioned that the majority of these methods are suitable essentially for mechanisms having low degree of complexity.

This paper presents an analytical method for kinematic solution of mechanisms possessing low or high degree of complexity. The kinematic characterestics of the individual bodies are determined by using the method of multibody system. These characterestics may be obtained in both local and global coordinate systems.

METHOD OF MULTIBODY SYSTEMS

Multibody systems are characterized by rigid or elastic bodies with inertia as well as springs, dampers and actively controlled servomotors. The method of multibody systems is used to simulate and design such large scale systems that undergo large relative translational and rotational displacements.

Position Analysis

A general rigid body is shown in Fig.2, in two frames of reference. They are the fixed (global or inertial) X_1 X_2 X_3 and the moving (local, rotating or body fixed) $\bar{X}_1$ $\bar{X}_2$ $\bar{X}_3$ frames.

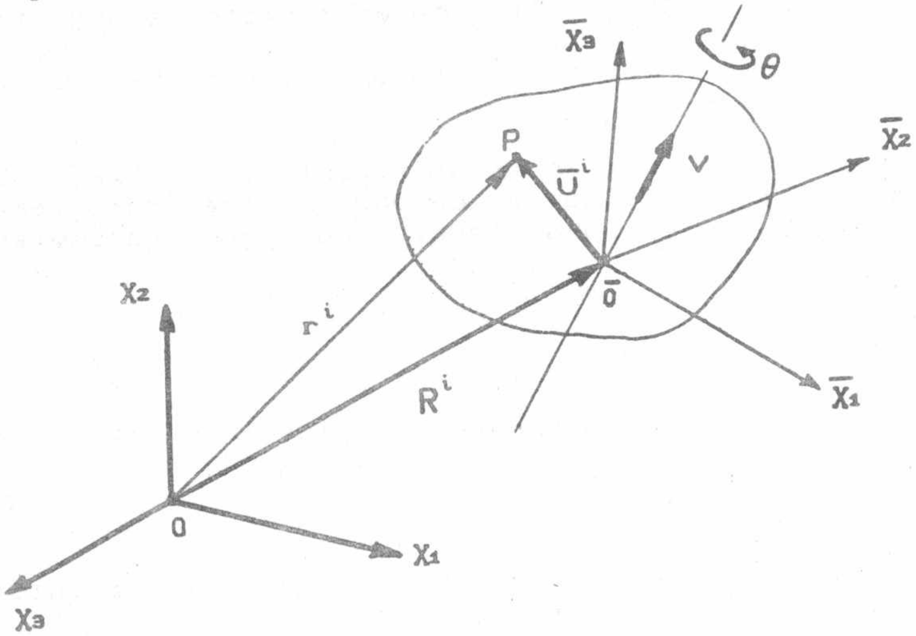


Fig.2 Global Position of a Point on the rigid body

The position and orientation of the moving frame with respect to the fixed one are uniquely defined by six variables [11]. The global position vector of an arbitrary point P on the i^{th} body can be expressed, in terms of the translation and rotation of the body, by the vector r^i given by:

$$r^i = R^i + A^i\bar{U}^i \tag{1}$$

where R^i is the position vector of the origin $\bar{O}$ of the body reference.
$\bar{U}^i$ is the position vector of the point P in the body fixed frame.

The superscript i refers to body i in the multibody system. The orientation of this body with respect to the inertial coordinate system $X_1 X_2 X_3$ is defined by the 3 X 3 transformation (or rotation) matrix A^i which takes the form:

$$A^i = I + \tilde{V} \sin(\vartheta) + 2\,\tilde{V}^2 \sin^2(\vartheta/2) \tag{2}$$

where I is the unit matrix,
$\tilde{V}$ is 3X3 skew symmetric matrix given by:

$$\tilde{V} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$$

where $V = [\, v_1\ v_2\ v_3]^T$ is the unit vector along the axis of rotation.
(ϑ) is the angle of rotation of the body about the axis of rotation.

The rotation matrix can be written in different forms [11]. In each case the elements of the matrix depend on the rotational coordinates which may be Euler parameters, Rodridgues parameters and Euler angles.

Velocity Analysis

Differentiating equation (1) with respect to time leads to:

$$\dot{r} = \dot{R} + A\,\dot{\bar{U}} + \dot{A}\,\bar{U} \tag{3}$$

in which $\dot{r}$ and $\dot{R}$ are the absolute velocities of the points P and $\bar{O}$, respectively.

If ω and $\bar{\omega}$ are the angular velocity vectors in global and moving coordinate systems, respectively, then $\dot{r}$ can be written in terms of the components of ω and $\bar{\omega}$ as follows:

$$\dot{r} = \dot{R} + A\,\dot{\bar{U}} + \tilde{\omega}\,A\,\bar{U} \tag{4a}$$

$$\dot{r} = \dot{R} + A\,\dot{\bar{U}} + A\,\tilde{\bar{\omega}}\,\bar{U} \tag{4b}$$

where $\tilde{\omega}$ and $\tilde{\bar{\omega}}$ are 3X3 skew symmetric matrices given by:

$$\tilde{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \qquad \tilde{\bar{\omega}} = \begin{bmatrix} 0 & -\bar{\omega}_3 & \bar{\omega}_2 \\ \bar{\omega}_3 & 0 & -\bar{\omega}_1 \\ -\bar{\omega}_2 & \bar{\omega}_1 & 0 \end{bmatrix}$$

Furthermore, the angular velocity vectors ω and $\tilde{\omega}$ can be written as:

$$\omega = H\,\dot{Q} \tag{5a}$$

$$\bar{\omega} = \bar{H}\,\dot{Q} \tag{5b}$$

in which the time derivatives of rotational coordinates are isolated. The form of equation (5) is general and can be developed irrespective of the set of rotational coordinates used. This form is commonly used to develop the dynamic equations of motion of rigid and deformable bodies in multibody systems.

Acceleration Analysis

Differentiating equation (4) with respect to time leads to:

$$\ddot{r} = \ddot{R} + A\,\ddot{\bar{U}} + 2\,\tilde{\omega}\,\dot{U}_g + \dot{\tilde{\omega}}\,A\,\bar{U} + \tilde{\omega}\,\tilde{\omega}\,A\,\bar{U} \tag{6}$$

where $\dot{U}_g = A\,\dot{\bar{U}}$ is the time derivative of $\bar{U}$, defined in global coordinate system.
Using the notation:

$$\alpha = \dot{\omega}$$
$$= [\,\alpha_1 \quad \alpha_2 \quad \alpha_3\,]^T$$

where α is the angular acceleration vector, equation (6) reduces to:

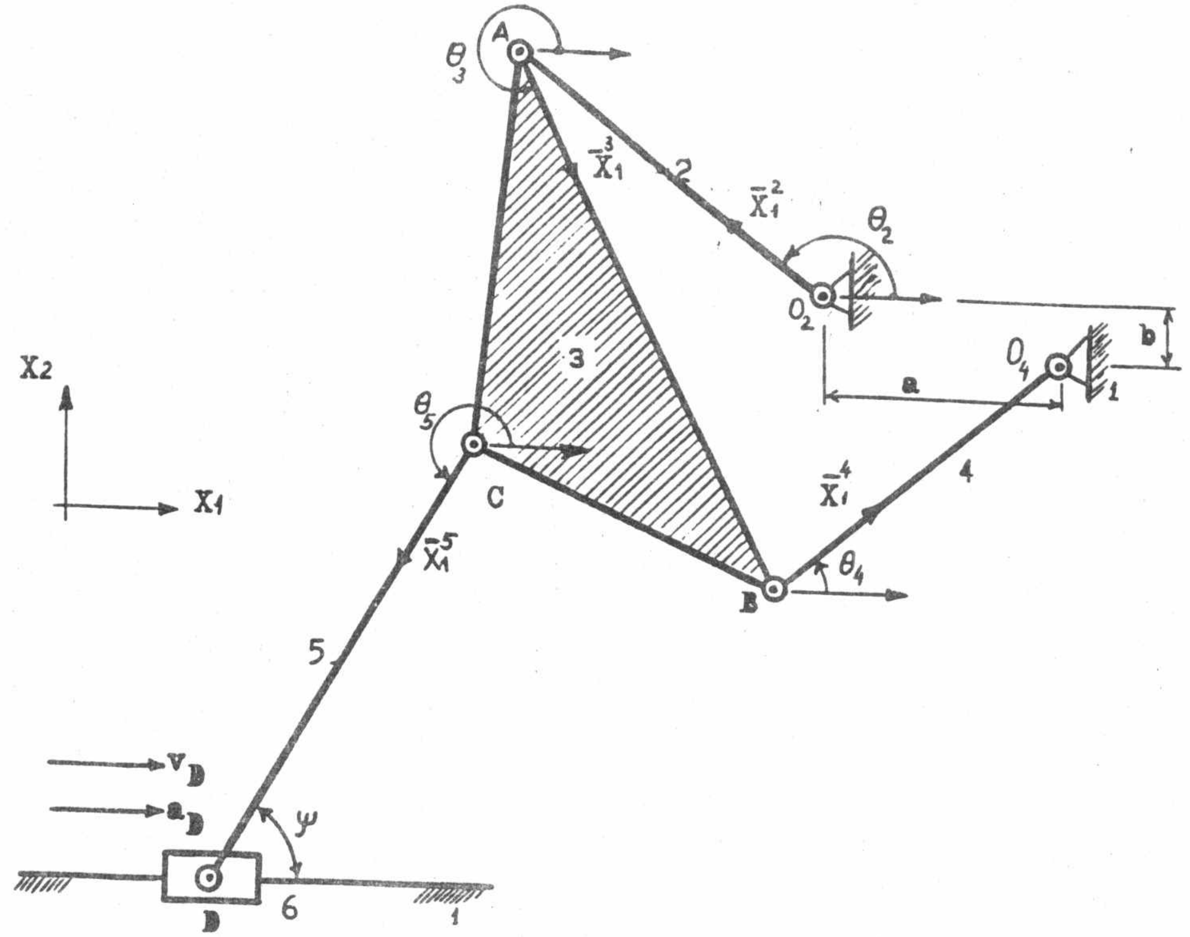
$$\ddot{r} = \ddot{R} + A\,\ddot{\bar{U}} + 2\,\tilde{\omega}\,\dot{U}_g + \tilde{\alpha}\,A\,\bar{U} + \tilde{\omega}\,\tilde{\omega}\,A\,\bar{U} \tag{7}$$

APPLICATION

The described theoritical approach is applied to a plane complex system, that is the ATKINSON engine mechanism shown in Fig.3.
The starting point in analyzing such complex system is to apply the kinematic inversion to convert it to a simple one. In this case, the input quantities for the inverted mechanism are assumed and the analysis is conducted using the presented theoritical approach. Then, the true kinematic values for the members of the actual mechanism can be calculated on the basis of the following important relations, derived by Goodman [1], concerning the constrained plane linkage.

1) The angular velocities and accelerations of links are linear functions of the respective "input" quantities:

The anular velocity of link l may be expressed in terms of the angular velocity of the input link i , as follows:



Dimensions:

$a = 22$ cm
$b = 10$ cm
$AO_2 = 50$ cm
$BO_4 = 40$ cm
$AB = 70$ cm
$AC = 45.8$ cm
$BC = 31$ cm
$CD = 70$ cm

Input Values:

$\psi = 60^\circ$
$v_D = 200$ cm/s
$a_D = 12000$ cm/s^2

Fig. 3 ATKINSON Engine

$$\omega_l = \frac{d\varphi_l}{dt} = \frac{d\varphi_l}{d\varphi_i}\,\frac{d\varphi_i}{dt} = \frac{d\varphi_l}{d\varphi_i}\,\omega_i$$

$$= K_l\ \omega_i \tag{8}$$

where the coefficient K_l is a purely geometrical property of the configuration, or phase, of the mechanism. Therefore, the velocities constructed with different input quantities are similar for the same phase of the system.

The angular acceleration of link l is given by:

$$\alpha_l = K_l^{\grave{}}\ \omega_i^2 + \alpha_i\ \omega_l/\omega_i \tag{9}$$

where $K_l^{\grave{}}$, too, is a geometrical property of the configuration of the linkage.
Equation (9) can be written in the form:

$$\alpha_l = {}^{\circ}\alpha_l + \alpha_i\ \omega_l/\omega_i \tag{10}$$

where the superscript ∘ is used to indicate that the particular quantities has been determined by means of an auxiliary acceleration pattern, constructed on the basis of the actual velocities but with zero input acceleration.

2) The relative angular velocities and accelerations of links are not affected by a direct kinematic inversion of the linkage.

Equation (10) can be written as:

$$\alpha_{lp} = {}^{\circ}\alpha_{lp} + \alpha_{ip}\ \omega_{lp}/\omega_{ip} \tag{11}$$

because the term absolute is relative to the stationary frame p. In this form, equation (11) is applicable to any direct inversion of the mechanism. Consequently, the symbol p is no longer restricted to the fixed link and the symbol i indicates the alternative input link with zero acceleration.

Kinematic Solution of the System

The configuration of the shown mechanism is determined by using equations (1) and (2) in conjunction with the method given by Suh and Radcliffe [9], where the complex mechanism under study is decoupled into two simple linkages of four bar and slider crank. The configuration of the system is, thus, given by:

$$\vartheta_2 = 135^\circ \quad , \quad \vartheta_3 = 290^\circ$$

$$\vartheta_4 = 30^\circ \quad , \quad \vartheta_5 = 240^\circ$$

The alternative input link is assumed to be the crank 2 which rotates with an angular velocity $\omega_2^* = -10$ rad/s . The negative sign indicates clockwise direction.

The angular velocities of the individual links are calculated by successive application of equation (4). Then,

$$\omega_3^* = -7.003 \quad \text{rad/s}$$

$$\omega_4^* = -5.365 \quad \text{rad/s}$$

$$\omega_5^* = 10.1 \quad \text{rad/s}$$

Moreover, the corresponding velocity of the slider 6 is:

$$V_D^* = 64.502 \quad \text{cm/s}$$

Consequently, the true angular velocity of link 2 is determined by using Goodman's relations. So,

$$\omega_2 = -31.007 \quad \text{rad/s}$$

$$\omega_3 = -21.714 \quad \text{rad/s}$$

$$\omega_4 = -6.635 \quad \text{rad/s}$$

$$\omega_5 = 31.317 \quad \text{rad/s}$$

Auxiliary Acceleration Pattern

The angular accelerations of the links are calculated assuming constant input angular velocity for the alternative input link $\omega_2 = -31.007$ rad/s

Equation (7) is used in successive manner and the results are:

$$^\circ\alpha_3 = -103.083 \quad \text{rad/s}^2$$

$$^\circ\alpha_4 = 316.528 \quad \text{rad/s}^2$$

$$^\circ\alpha_5 = 1344.607 \quad \text{rad/s}^2$$

$$^\circ a_D = 14510.224 \quad \text{cm/s}^2$$

Application of equation (10) leads to the determination of the true values of α_j :

$$\alpha_2 = 389.138 \quad rad/s^2$$

$$\alpha_3 = 169.428 \quad rad/s^2$$

$$\alpha_4 = 525.297 \quad rad/s^2$$

$$\alpha_5 = 951.579 \quad rad/s^2$$

The presented technique is simple and more accurate than the available graphical techniques. For the purpose of comparison, the same mechanism is solved by using two different methods. The first one is based on Goodman's relations in which the auxiliary acceleration diagram is drawn. The solution of the system by this method is shown in Fig.4. The second method is the graphical iteration technique. The procedure of solution using this method is shown in Fig.5.
The results obtained by using the three methods, for the same phase of motion, are given in Table 1.

	Auxiliary diagram	Graphical iteration	Proposed technique
ω_2	-30.6	-31.5	-31.007
ω_3	-21.4	-22.1	-21.714
ω_4	-16.4	-16.8	-16.635
ω_5	31.2	31.4	31.317
V_D	200	200	200
α_2	389.2	380	389.138
α_3	175.7	178.6	169.428
α_4	512.5	525	525.297
α_5	978.6	931	951.579
a_D	12050	12000	12000

Table 1 Results of Kinematic Solutions

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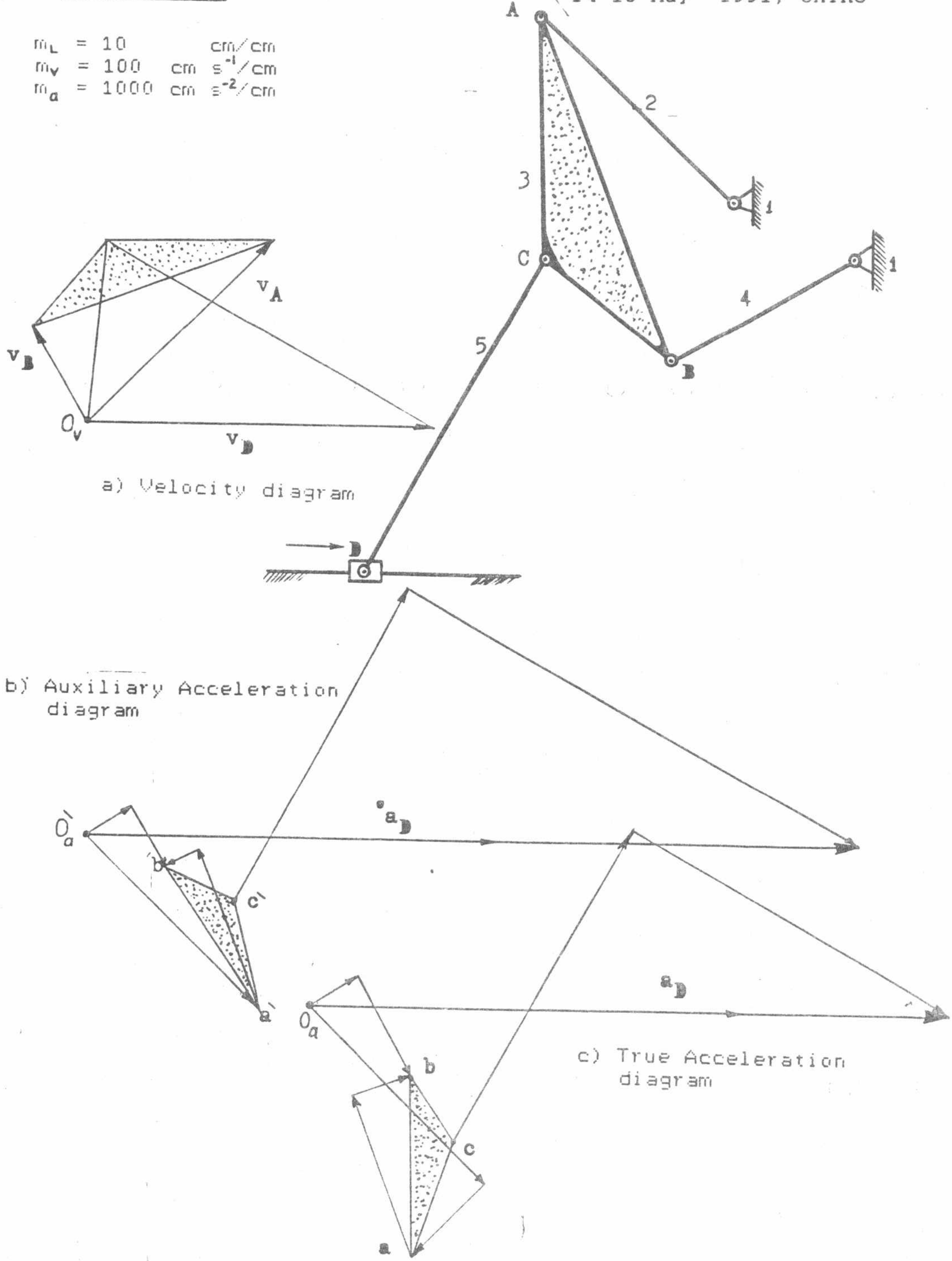

Fig. 4 Solution by Using Auxiliary Acceleration Diagram

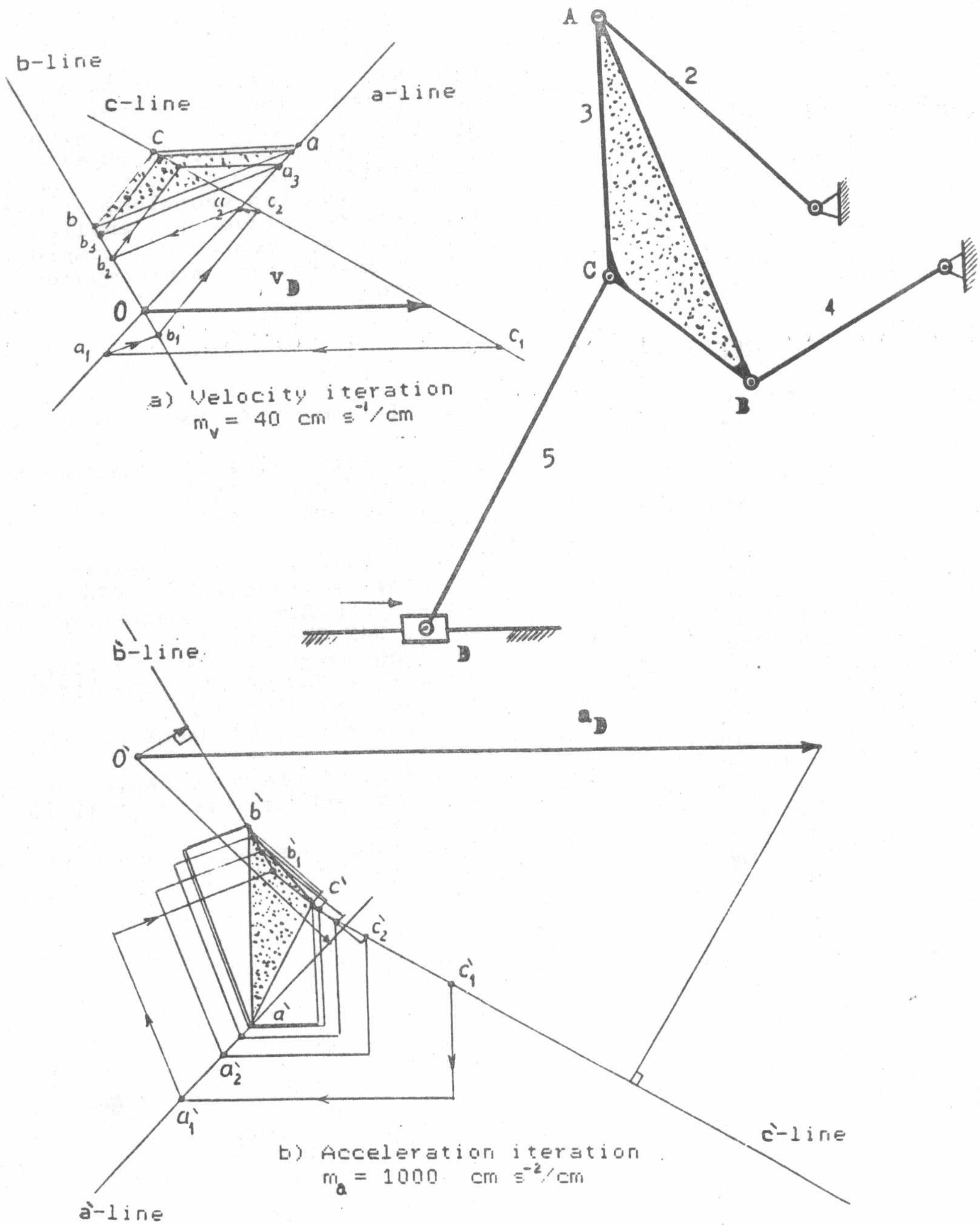

Fig. 5 Solution by Using Graphical Iteration

CONCLUSION

General expressions for the kinematic analysis of plane or space multibody systems have been presented. The equations can be applied to machines, mechanisms, robots and all kinds of vehicles. The position, angular velocity and angular acceleration of any body in the system can be determined by simple matrix multiplications. Consequently, the governing equations of motion can be constructed. The proposed technique is applied for kinematic analysis of mechanisms with any degree of complexity. The technique is simple, easy implemented on minicomputer and yields to accurate and rapid results.

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