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OPTIMAL MODIFIED COMBINED SYSTEM FOR VIBRATION CONTROL

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ABSTRACT

This paper presents the design considerations and performance capabilities of a modified series combination of passive and active elements as a system for vibration control. The investigated primary model consists of a main mass and a randomly excited support. The performance measures, which considered to be minimal, are the main mass acceleration and the relative displacement between the mass and the system support. The operation of the modified system depends essentially on an implemented ideal filtration process in the proposed combination. This process is realized in two different ways: (1) by the use of an ideal low pass filter (LPF), (2) by the use of an ideal high pass filter (HPF). Additionally, this process controls the flow of the command signal of the combined system which operates the active element. Thereby, the modified system has the property of operating either as a combined system or as a classical passive system. The stochastic optimal control theory in addition to a graphical criterion are well posed together in order to determine the effective cut-off frequency for any of the implemented ideal filters. The pertinent results are compared with those of the combined system. In contrast, they are found to be more encouraging, superior, and compatible with the requirements of many actual engineering applications.

INTRODUCTION

Basically, the passive vibration isolation systems offer simplicity, reliability, stability, and low cost [1]. In contrast, the fully active systems are costly, complex, and require an external power supply. While, the semi-active systems are significantly simpler and less costly than the fully active ones. The original concept of semi-activity has been presented by Karnopp et al [3]. This concept depends essentially on the implementation of an active damper such that its associated power is always dissipative. In addition, it requires only signal processing and low power supplies. Moreover, the semi-active systems offer the reliability and cost effectiveness

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comparable to passive systems enhanced by performance close to that of active ones [4]. Alternative schemes of semi-activity have been presented by Rakheja and Sankar [5], and Alanoly and Sankar [6] for the realization of a more simplified system hardware. The active systems are intended to be used in combination with classical passive elements in order to increase their reliability and to reduce, if possible, the supplied power. In such situation, the full performance potential of active systems can not be achieved due to the control policy used and the combination suggested [3,4,7]. The full performance potential has been proven by Guntur and Sankar [8] for various parallel combinations in case of harmonically excited single-DOF system. In essence, The parallel combination complicates, to a large degree, tunability of the active element for varying operating conditions, since the passive force is always a part of the total control force. While in case of a series combination the active force is independent of values of the passive elements [2].
In this work, a series combination of a one active element and two passive elements (linear spring and viscous damper) is considered. Such combination provides the fail-safe feature for vibration control, i.e., the system works as a passive one in case of failure of the active element. Moreover, it allows the adaptation of the system for varying operating conditions. On basis of many frequency response predictions, a quite different methodology from that of Karnopp is investigated for the realization of a semi-active vibration control.

MODIFIED COMBINED SYSTEM CONFIGURATION

The schematic diagram of the modified system is shown in Fig.(1). The system states are measured and combined properly by the aid of a signal conditioner and a microcomputer supervisor to perform the electrical signal which equivalent to the desired control law (force) of the combined system u_c. This signal is fed into an ideal filtration process to give the modified combined control law u_m. Then, the signal u_m can be amplified and fed into a force generator which generates a force proportional to the controlled state variables. On basis of a prescribed criterion for determining the effective cut-off frequency, the ideal filtration process determines whether the force generator (actuator) will work or not. In the time where the actuator becomes inactive, it is regarded as a rigid connection between the main mass and the massless plate, and the system seems to be a classical passive single-DOF system. Additionally, the mass of the actuator is neglected in comparison with the main mass of the primary system. But, when the actuator operates it is regarded as an ideal force generator, i.e., no limitations to the frequency band or the peak force capability [5,6,8,9], and the system seems to be a combined vibration control system of two-DOF. In practice, the actuator can be realized as a pneumatic servovalve, electrohydrualic servomotor, etc. Also, it is assumed here that the system state variables are all measurable as well as the excitation.

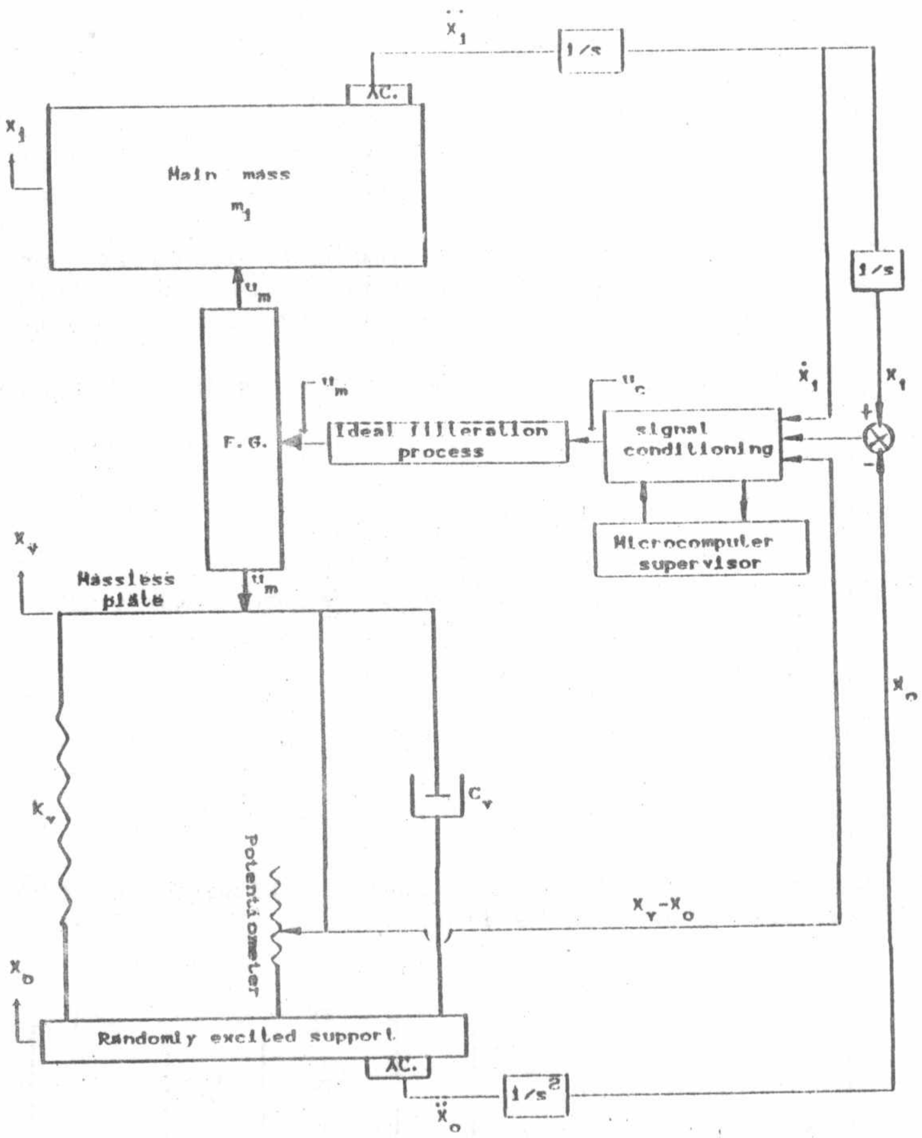


Fig.1 Schematic diagram of the modified combined system

SYSTEM ANALYSIS

A band-limited stationary random excitation X_o is imparted at the support of the investigated system shown in Fig.1. The power spectral density function of this excitation is [3,9].

$$S_o(\omega) = \frac{V_o}{\omega^2} = \frac{V_o}{-s^2} \tag{1}$$

It is common practice in literature [9,10] to treat the velocity of such excitation as a white noise process such that :

$$\dot{X}_o = w(t) \tag{2}$$

A white noise excitations satisfies :

$$E[w(t)] = 0 \tag{3}$$

$$E[w(t)\ w(\tau)] = 2\,\pi\, V_o\ \delta(t-\tau) \tag{4}$$

It is assumed here that w(t) has a Guassian probability distribution and that Eqn.(3) implies a zero mean value, while Eqn(4) indicates the independence of the mean square value of time.
From Fig.1 the equations of motion of the modified system are.

$$m_1 \ddot{X}_1 = u_m \tag{5}$$

$$K_v (X_v - X_o) + C_v (\dot{X}_v - \dot{X}_o) = -u_m \tag{6}$$

And the system state variables are selected such that,

$$Y_1 = X_1 - X_o$$

$$Y_2 = X_v - X_o$$

$$Y_3 = \dot{X}_1$$

Differentiating these states and substituting Eqns. (2), (5) and (6) to get on the state space form:

$$\begin{bmatrix} \dot{Y}_1 \\ \dot{Y}_2 \\ \dot{Y}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -\frac{K_v}{C_v} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{C_v} \\ \frac{1}{m_1} \end{bmatrix} u_m + \begin{bmatrix} -w(t) \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

With controlled outputs.

$$[Z] = \begin{bmatrix} Z \\ Z_v \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} \tag{8}$$

The modified control law can be related to the desired combined as follows.

$$u_m = f_c\ u_c \tag{9}$$

where, u_c according to the signal processing in Fig.1 is given by.

$$u_c = G_1 (X_1 - X_o) + G_2 (X_v - X_o) + G_3 \dot{X}_1 \tag{10}$$

and f_c is a constant determines the operation of the ideal filtration process, and can be defined in case of operation of a LPF as follows.

$$f_c = \begin{cases} 1 & \text{if} \quad \omega < \omega_c \\ 0 & \text{if} \quad \omega \geq \omega_c \end{cases} \tag{11}$$

While in case of operation of a HPF it can be defined as :

$$f_c = \begin{cases} 1 & \text{if} \quad \omega > \omega_c \\ 0 & \text{if} \quad \omega \leq \omega_c \end{cases} \tag{12}$$

Therefore, when f_c equals 1 u_c replaces u_m in the second term of Eqn.(7), and the modified system behaves exactly like a combined system of two-DOF. Conversely, when f_c equals 0 u_m in Eqn.(7) vanishes, and the actuator becomes a rigid connection between the main mass and the massless plate, and the modified system behaves exactly like a classical passive single-DOF system.

The performance measures either in case of operation as a combined system or as a passive system can be calculated by substituting Eqns.(9) and (10) into Eqns.(5) and (6) to yield :

$$\begin{aligned} & m_1\ddot{X}_1 - f_c\,[G_1(X_1-X_o) + G_2(X_v-X_o) + G_3\,\dot{X}_1] = 0.0 \\ & K_v(X_v-X_o) + C_v(\dot{X}_v-\dot{X}_o) + f_c\,[G_1(X_1-X_o) + G_2(X_v-X_o) + G_3\dot{X}_1] = 0.0 \end{aligned} \tag{13}$$

After performing some algebraic manipulations the set (13) can be rewritten in a complex matrix form as follows.

$$\left[\begin{bmatrix} -m_1\omega^2-f_cG_1 & -f_cG_2 \\ f_cG_1 & K_v+f_cG_2 \end{bmatrix} + j\begin{bmatrix} -f_cG_3\omega & 0 \\ f_cG_3\omega & C_v\omega \end{bmatrix}\right]\begin{bmatrix} X_1 \\ X_v \end{bmatrix} = \left[\begin{bmatrix} -f_c(G_1+G_2) \\ K_vf_c(G_1+G_2) \end{bmatrix} + j\begin{bmatrix} 0 \\ C_v\,\omega \end{bmatrix}\right] X_o \tag{14}$$

Without loss of generality X_o in the last complex matrix equation is equated to unity. Hence, the outcomes of this equation will be :

$|X_1|$ represents the transfer function of the main mass displacement.

$|X_1 - 1|$ represents the transfer function of the relative displacement between this mass and the system support.
Since the excitation is a stationary random process, the mean square values of the acceleration and the relative displacement can be conveniently written as [12] :

$$E[\ddot{X}_1^2] = \int_{\omega_l}^{\omega_u} |X_1|^2 \frac{V_o}{\omega^2} \omega^4 \, d\omega.$$

$$E[Z^2] = \int_{\omega_l}^{\omega_u} |X_1 - 1|^2 \frac{V_o}{\omega^2} \, d\omega.$$

OPTIMIZATION PROBLEM

It should be kept in mind that the optimization problem of the modified system is considered only when it works as a combined system at all times of operation over the specified frequency band. While in the time of operation as a passive system, there is no need for optimizing the passive elements, since their optimality may deteriorate the potency of the filtration process. Thus any set of passive elements by which the maximum exploitation of the filtration process can be achieved is said to be optimum.
The state space form of Eqns.(7) and (8) can be rewritten in a more general form as follows.

$$[\dot{Y}] = [A]\,[Y] + [B]\,[u] + [w] \qquad (15)$$

$$[Z] = [V]\,[Y] \qquad (16)$$

The statement of the regulator problem here is to find out the optimum steady state control law u_c that minimizes the following performance index :

$$P.I. = .5 \lim_{t \to \infty} E\left[[Z]^T[Q]\,[Z] + [u]^T[R]\,[u]\right] \qquad (17)$$

subject to the system dynamics in the specified state space form of Eqns.(15) and (16)

$$\text{Where, } R = R_1, \qquad Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \qquad (18)$$

The interpretation of the P.I. in Eqn.(17) is that the term $E[Z^T Q Z]$ minimizes the controlled output (Z and Z_v), while the term $E[u^T R u]$ minimizes the control law which, in some sense, proportional to the main mass acceleration. The general solution to this problem can be found in Appendix A. For Further details about the behaviour of the combined system in comparison with the fully active system you can see Ref.[2].

IMPLEMENTATION

The numerical calculations are done in the region of fixed parameters $m_1 = 100$ kg. and $v_o = .001\ m^2.s$. During the solution of the regulator problem the weighs R_1 and Q_2 are equated to the unity, while the weight Q_1 is varied between 1×10^6 to 3×10^9 with step equals 1×10^6 to obtain a considerable range of trade-off solutions. Then Simpson's rule is used for calculating the r.m.s. values of the specified performance measures over a frequency band ranges between $\omega = 1$ and $\omega = 100$ rad./s. The DIAGONALIZATION method [11] is used to solve matrix Riccati equation (A3)

RESULTS

In the sense of the resulted set of optimum trade-off solutions, three solutions are selected such that one of them is medium and the two others are extremals. Hence, the frequency response of the combined system at these three solutions are plotted versus that of the passive system. Figs.2,3,--7 show many frequency response predictions at various combination of ξ_v and ω_v. In general, the effort in the present work is posed for treating the problem of determining ω_c of any of the suggested filtration processes in a general sense. In other words, it is very significant from the practical point of view to relate the determination of ω_c to either fixed or variable parameters in the system. Of course, this will save, to a considerable extent, many sophisticated frequency response predictions before deciding the effective value of ω_c.

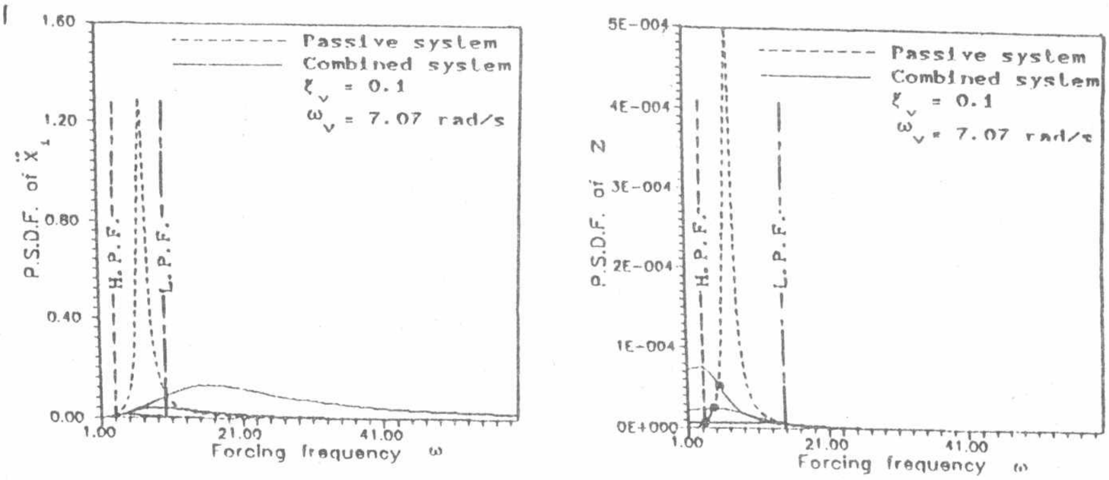
In this manner, the points of intersection between the vertical centre lines and the horizontal axes in Figs.(2-a), (3-a), and (4-a) indicate that ω_c of a LPF can be approximately related to ω_v as follows.

$$\omega_c \cong \sqrt{2}\ \omega_v \tag{19}$$

The points of intersection between the centre lines and the horizontal axes in Figs.(2-b), (3-b), and (4-b) indicate that ω_c of a LPF can be related to ω_v by use of the following approximate relation.

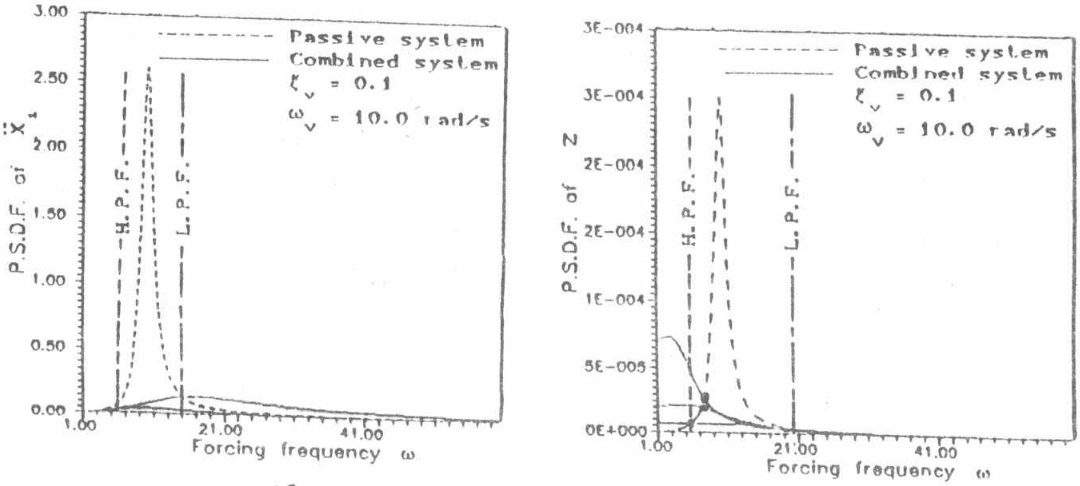
$$\omega_c \cong 2.0\ \omega_v \tag{20}$$

In Eqn.(19) the articulation is given to $\ddot{X}_1$ to have the maximum exploitation in case of using a LPF, while in Eqn.(20) the articulation is given to Z. However, both Eqns.(19) and (20) are not applicable when ξ_v becomes either moderate or large. The last cautionary remark can be verified by predicting the points of intersection between the centre lines and the horizontal axes in Figs.5, 6, and 7



-a-　　　-b-

Fig. 2



-a-　　　-b-

Fig. 3

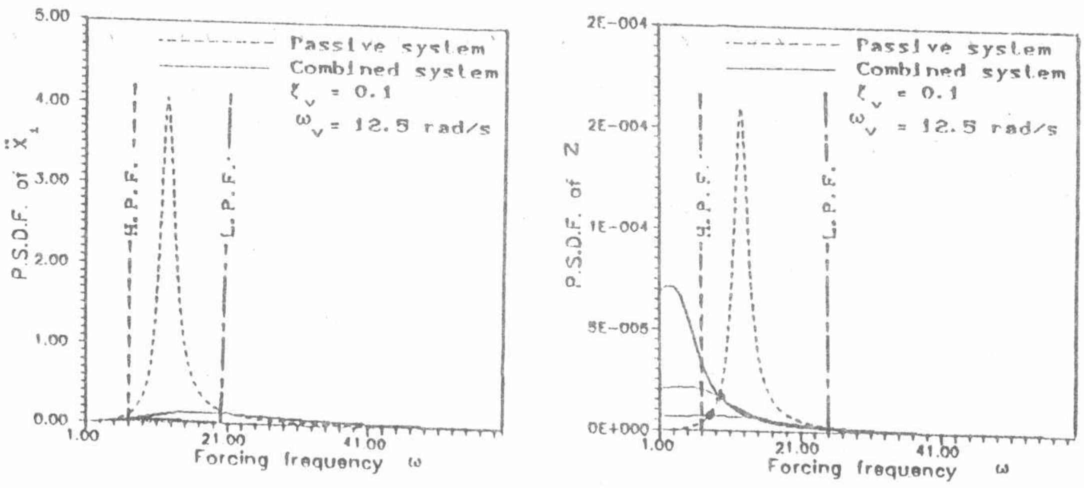


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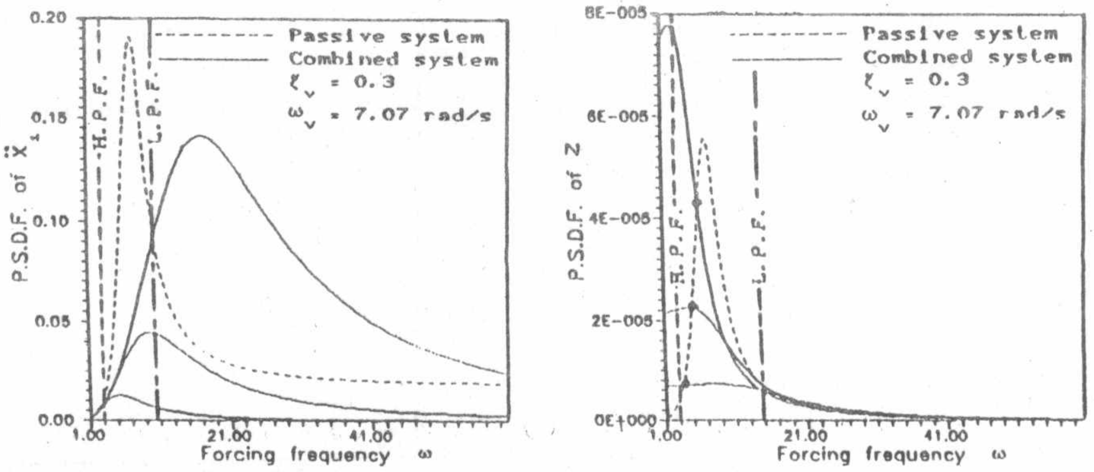
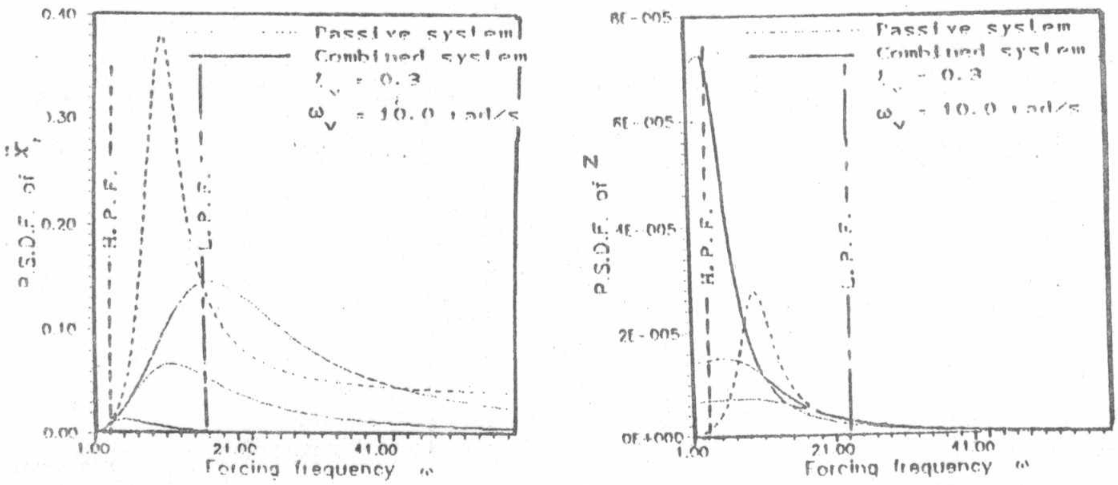
Fig. 4

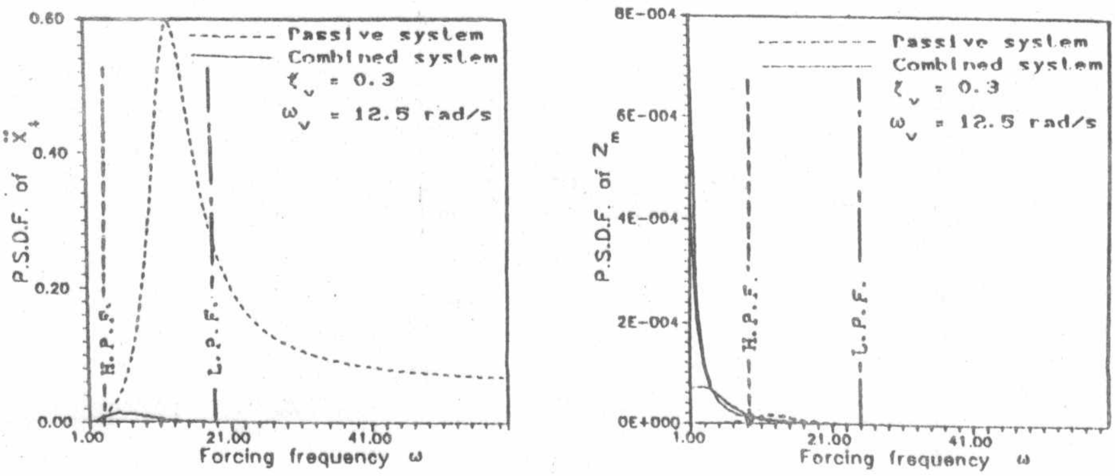
(Frequency response curves of the combined system at three different optimum solutions versus that of the passive system)



-a- Fig.5 -b-



-a- Fig.6 -b-



-a- Fig.7 -b-

(Frequency response curves of the combined system at three different optimum solutions versus that of the passive system)

The first method to determine ω_c of a HPF is to point out the frequency at which the coincidence, that begins at the lower frequency limit, between the frequency response curves of $\ddot{X}_1$ of both the combined and the passive systems begins to vanish. Thereby, a considerable part from the eminent frequency response of Z can be omitted without any loss in the frequency response of $\ddot{X}_1$. In this manner, the points of intersection between the dashed lines and the horizontal axes in Figs.(2-a), (3-a), and (4-a) indicate that ω_c of a HPF can be approximately related to ω_v as follows.

$$\omega_c \cong 0.5\ \omega_v \qquad (21)$$

Also, it is shown in Figs.(5-a), (6-a), and (7-a) that a damping ratio ξ_v greater than 0.1 violates, to a great extent, Eqn.(21). The above mentioned figures indicate that at $\xi_v = 0.3$, and whatever the value of ω_v, ω_c becomes a constant value equals 3 rad./s.

An alternative approach would be to use a variable value of ω_c of a HPF. What is meant by variable ω_c is to alter its value everytime the trade-off solution changes. For example, from the point of view of the frequency response of Z, the black points in each of Figs.(2-b), (3-b), and (4-b) determine the correct positions of ω_c by which the maximum exploitation from the operation of a HPF can be taken. Since, whatever the value of the desired trade-off

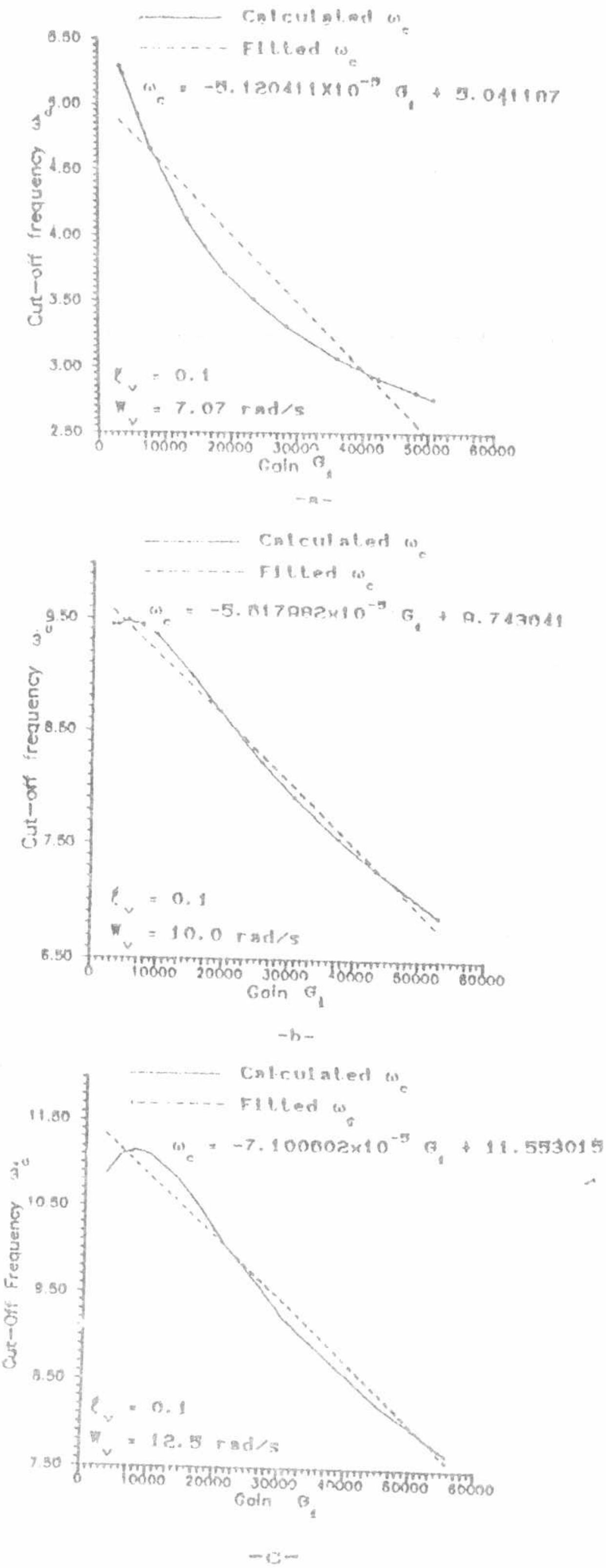


Fig.8 Variable ω_c as a function in the feedback gain G_1

solution, values of ξ_v and ω_v remain unchanged, the logic here is to relate the variable ω_c to any of the optimum feedback gains, because they change every time the solution changes as shown in Table 1. A FORTRAN code is written by us to determine, with a considerable accuracy, the positions of the mentioned black points along the obtained set of optimum trade-off solutions. The solid lines in Fig.8 show the output of the FORTRAN code, while the dashed lines show a linearly fitted relation between ω_c and the feedback gain G_1. The maximum deviation between the fitted and the actual curves occurs at relatively small values of ξ_v and ω_v as shown in Fig.(8-a). It is also obvious in Fig.(7-b) that at relatively large values of ξ_v and ω_v all the frequency response curves of the combined system intersect that of the passive system at a unique point at ω = 10 rad/s. Therefore, there is no need for doing approximate relation or to use variable ω_c, since ω_c can be taken as 10 rad/s. However, for the sake of exploring the performance of the modified system with the two suggested ideal filters, the problem is solved at three different combinations of ξ_v and ω_v. The results are shown in Figs.9, 10, and 11.

DISCUSSION OF RESULTS

It should be stressed from the very beginning that, from practical point of view, the LPF seems to be better than The HPF. A LPF requires an actuator to operate at low exciting frequencies. Therefore, it reduces merely the cost of the system hardware. By the aid of Fig.9 it can be concluded that the LPF reaches its maximum potency at relatively small values of ξ_v and ω_v. While the maximum exploitation can be taken from the operation of a HPF at relatively large values of ξ_v and ω_v as shown in Figs.10 and 11.

It is worth noting that the resulted sets of trade-off solutions by the use of a LPF in Fig.(9-a) seem to be very significant, since they are approximately horizontal, i.e., whatever the desired r.m.s. value of Z, the r.m.s. value of $\ddot{X}_1$ remains unchanged. Of course, such trend of solutions may meet the requirements of many actual applications. In addition, as shown in Fig.(9-b) a LPF with articulation to be given to the relative displacement (ω_c=15) achieves a considerable save in the r.m.s. actuated force rather than that when the articulation is given to the acceleration (ω_c=10). At relatively large values of ξ_v and

Q_1	G_1	G_2	G_3	U_1	r.m.s. $\ddot{X}_{1m}$	r.m.s. Z_m	r.m.s. Z_{vm}
5x10^6	-2236.0	428.0	-682.0	98.0	.3245	.0248	.0030
3x10^7	-5477.0	398.0	-1067.0	198.0	.6297	.0202	.0050
1x10^8	-10000.0	371.0	-1440.0	315.0	.9789	.0175	.0088
2x10^8	-14142.0	352.0	-1711.0	410.0	1.2595	.0161	.0112
4x10^8	-20000.0	332.0	-2032.0	534.0	1.6175	.0147	.0141
6x10^8	-24494.0	319.0	-2248.0	623.0	1.8704	.0140	.0161
9x10^8	-30000.0	306.0	-2486.0	726.0	2.1609	.0133	.0183
2x10^9	-4472.0	277.0	-3031.0	980.0	2.8619	.0120	.0266
3x10^9	-5477.0	262.0	-3352.0	1141.0	3.2942	.0113	.0266

Table (1) Some numerical examples declare the optimum feedback gains along with the optimized performance measures for the combined system w.r.t. the first fotmulation.
$R_1 = 0$, $Q_2 = 10^7$, $v_o = 0.001\ m^2/s$.
$\xi_v = 0.1$, $\omega_v = 10.0$ rad/s.

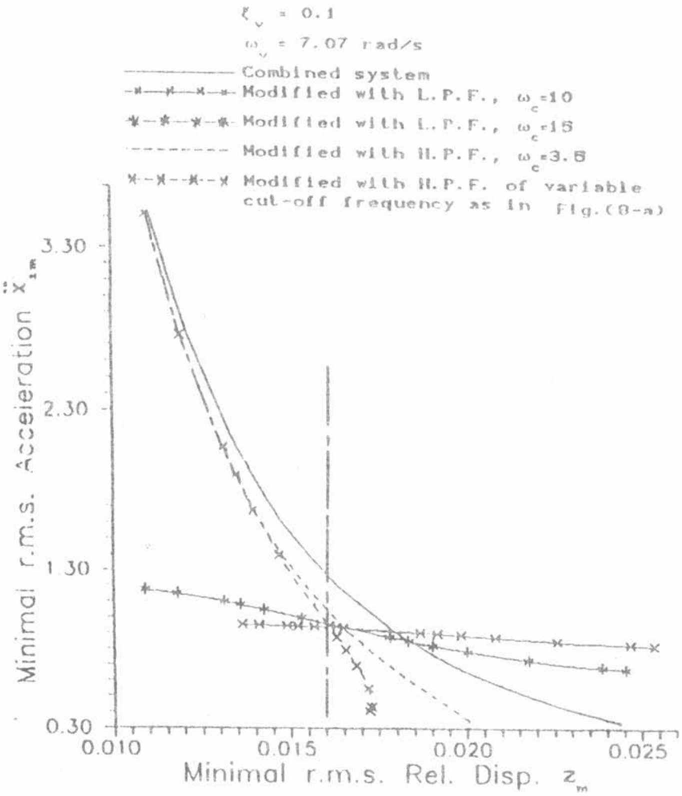
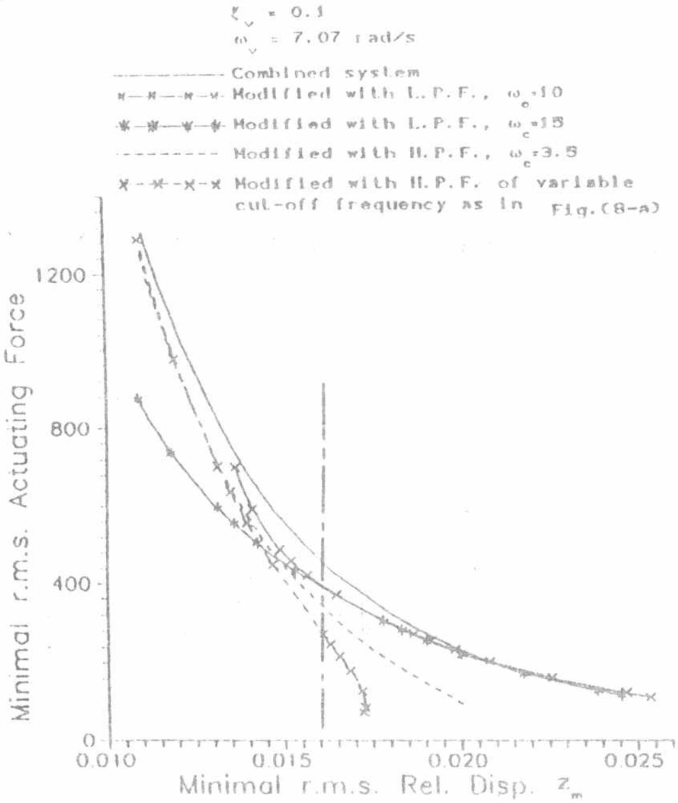


Fig.9 -a- Comparison of trade-off solutios of various suggested schemes for the operation of the modified system along with the combined system



-b- Comparison of r.m.s. actuated forces of various suggested schemes for the operation of the modified system along with that of the combined system

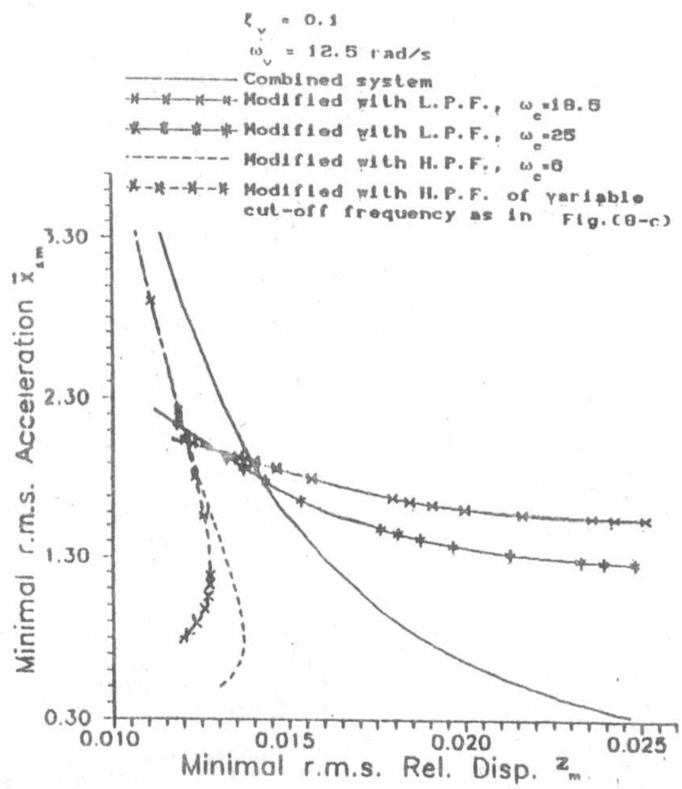
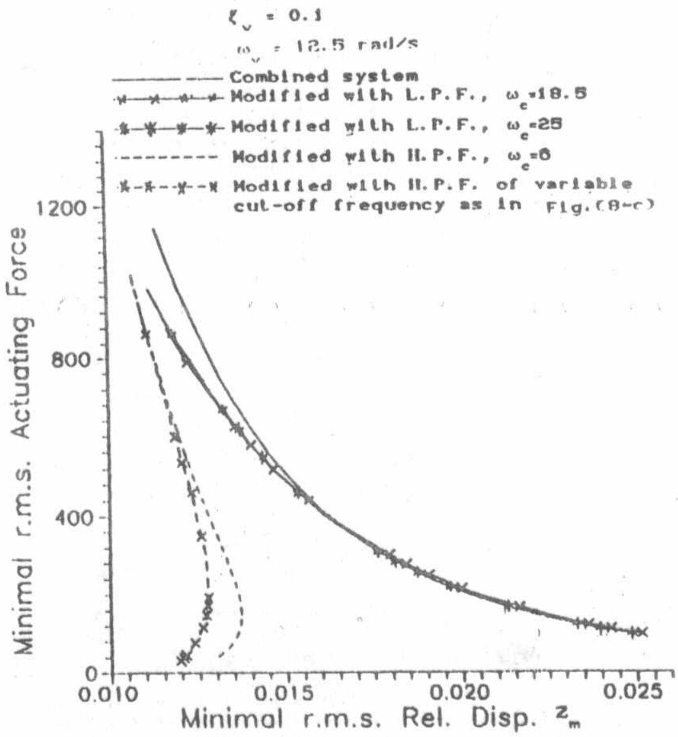


Fig.10 -a- Comparison of trade-off solutios of various suggested schemes for the operation of the modified system along with the combined system



-b- Comparison of r.m.s. actuated forces of various suggested schemes for the operation of the modified system along with that of the combined system

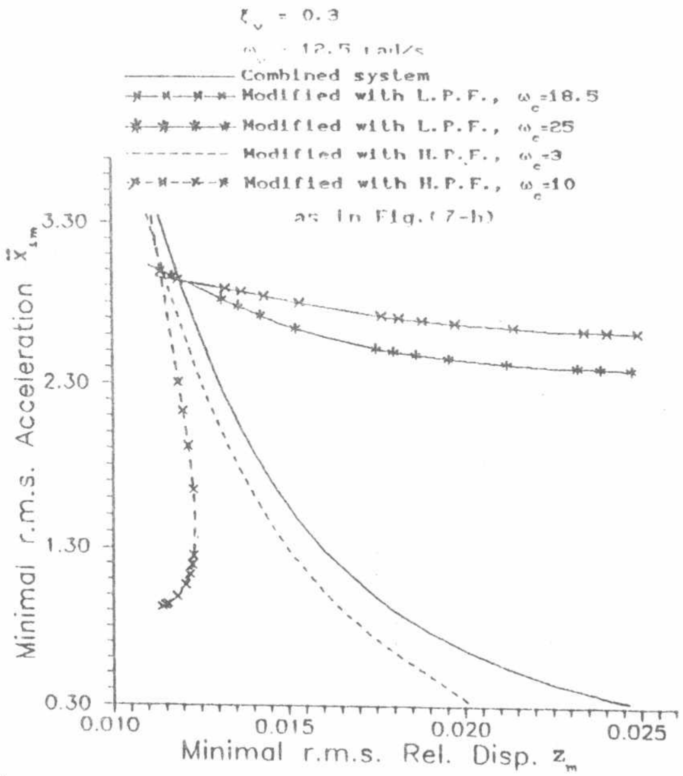
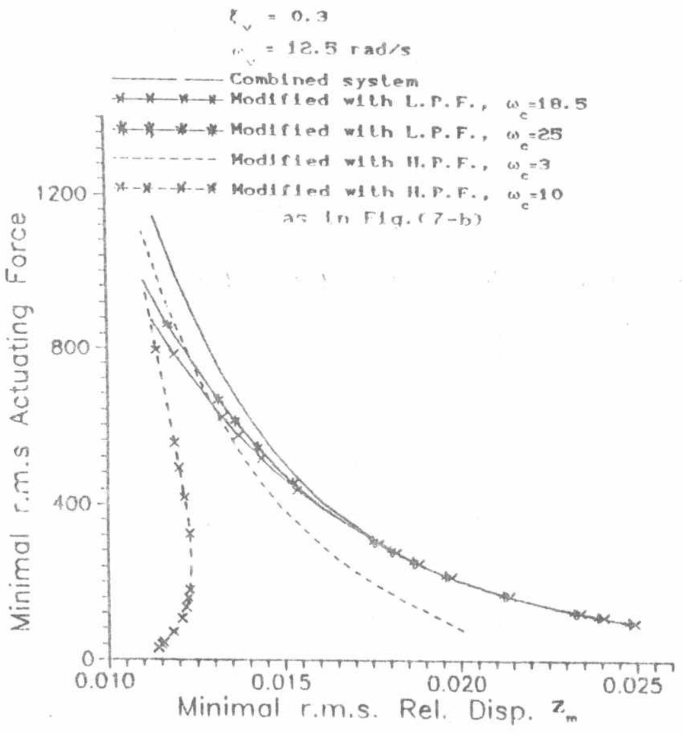


Fig.11 -a- Comparison of trade-off solutios of various suggested schemes for the operation of the modified system along with the combined system



-b- Comparison of r. m.s. actuated forces of various suggested schemes for the operation of the modified system along with that of the combined system

ω_v as shown in Fig.11 it is clear that no actual benefit can be taken from the use of a LPF. This is in the time where the HPF with variable ω_c reaches its maximum potency. The offered trend of trade-off solutions by use of a HPF with ω_c equals 10 rad/s in Fig.(11-a) seems to be very significant, since various values of $\ddot{X}_1$ can be taken with little change in the r.m.s. value of Z well as in the r.m.s. value of the actuated force as shown in Fig.(11-b). Also, this trend of solutions is needed by many practical applications.

CONCLUSIONS

1- The modified combined system is capable of providing a performance which can not be achieved by the combined system. In addition to its superior performance, it can provide a considerable save in the r.m.s. actuated force.

2-Although the low fixed cost and the attainable reliability which can be achieved by implementing a LPF, the potency of such filter is not effective except at low passive elements. In other words, a LPF loses quite its potency at relatively large passive elements.

3- The maximum exploitation by implementing a HPF in the modified system can be taken at relatively large values of the passive elements, but it remains effective at either low or moderate values of these elements.

4-The fixed passive elements can be properly selected and a LPF is implemented to achieve trade-off solutions in which the r.m.s. value of the accelerations seems to be approximately constant whatever the value of the relative displacement.

5- Also the fixed passive elements can be properly selected and a HPF can be used to obtain trade-off solutions in which the r.m.s. acceleration can be changed without effective change in the r.m.s. relative displacement as well as the r.m.s. actuated force.

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APPENDIX (A)

Since the excitation is a stationary random process, the performance index in Eqn.(17) can be conveniently redefined by.

$$P.I. = 0.5 \int_0^{\infty} (Z^T Q\, Z + u^T R\, u)\, dt \qquad (A1)$$

Note that the matrix suffix is omitted for simplicity.
Moreover, the solution of the problem will be equivalent to that of a deterministic regulator problem in which [11].

$$u(t) = - R^{-1} B^T P\, Y \qquad (A2)$$

And requires the solution of the following matrix Riccati equation to determine the matrix P.

$$A^T P + P A^T + C^T Q C - P B R^{-1} B^T P = 0 \qquad (A3)$$

The main advantage of using the matrix riccati equation is that it provides a unique solution which ensures the stability of the system. The solution of this matrix equation is feasible when the the pair A and B of the state space form (15) and (16) constitutes a completely controllable system such that :

$$\text{rank}\left[B\ ,\ A\,B\ ,\ A^2\,B, \text{--------------}, A^{n-1}\,B \right] = n \qquad (A4)$$

and the pair A and C ensure a completely observable system such that :

$$\text{rank}\left[V^T,\ A^T\,V^T,\ (A^T)^2\,V^T, \text{----}, (A^T)^{n-1}\,V^T \right] = n \qquad (A5)$$

It is worth noting that the state form of Eqns.(7) and (8) satisfies (A4) and (A5).

NOMENCLATURE

[A] : matrix of state space form.
AC. : accelerometer.

[B] : matrix of state space form.
C_v : classical passive damping factor.
E[.]: stands for variance.
F.G.: force generator.
G_1, G_2, G_3 : feedback gains
[P] : Riccati matrix
P.I. : performance index.
P.S.D.F.: power spectral density function
[Q] : weighting matrix.
Q_1, Q_2 : weighting factors.
[R] : weighting matrix.
R_1 : weighting factor.
[V] : output matrix
X_1 : displacement of the main mass
X_o : Guassian random input.
X_v : displacement of massless plate
$\ddot{X}_{1m}$: minimal main mass acceleration
Y_1, Y_2, Y_3 :system state variables.
Z : relative displacement, X_1-X_o.
Z_m : minimal relative displacement
Z_v : relative displacement, X_v-X_o.
[Z] : vector of controlled outputs .
f_c : constant determines operation of a filtration process
m_1 : primary mass.
n : order of state space form
s : Laplace operator.
t : time instant.
u_c : control law of a combined system
u_m : control law of a modified system
v_o : excitation constant.
w(t): white noise excitations.
$\delta(t)$: Dirac Delta Function.
ξ_v : damping ratio of classical passive system, $C_v \backslash 2 m_1 \omega_v$.
τ : time instant.
ω : forcing frequency.
ω_c : cut-off frequency.
ω_u : upper frequency limit.
ω_l : lower frequency limit.
ω_v : natural frequency of passive system, $\sqrt{K_v / m_1}$.