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WAVE PROPAGATION INDUCED BY IMPACT OF FAST-FLYING AIRCRAFT UPON A RIGID TARGET USING SPECIAL GAP FINITE ELEMENT

M.M. EL-MADDAH * and N.A. EL-SEBAI *

ABSTRACT

The shock dynamic response of structures subjected to projectile impact loading has been recognized by engineers as long as a quarter of a century ago. Several computer programs have been develped to evaluate the loading history (the reaction versus time curve) for impact projectiles (missiles and aircrafts) impinging against fixed or moving targets. Here, the Finite Element Method with special gap element is used for prediction of reaction vs. time curve as well as the displacement, velocity and acceleration response of different points of the projectile. It is shown that the reaction-time curve at the interface can be reliably predicted. First a problem of elastic impact of a uniform bar has been solved analytically and numerically and the results showed good agreement. Another problem of practical interest is the dynamic impact of Phantom airplane hitting upon a rigid barrier. This problem is solved with three different options, first with linear elastic material, second with nonlinear elastic-plastic material and third with consideration of large displacement, finite strain plasticity and updated Lagrange analysis. The results of FEM are compared to those obtained with the well known Riera and lumped mass approaches.

INTRODUCTION

The safety design of a building (nuclear power plant, dam,... etc) for the case of impact by crashing airplanes, aircraft debris, missiles,etc. requires detailed information of the forces which are exerted upon the building hit by these projectiles. When one assumes both a non deformable projectile and a massive target, then all of projectile's kinetic energy is available to penetrate the target. With a deformable projectile or a non-massive target a portion of projectile's kinetic energy is used to deform the projectile and the target and this reduces the energy available to penetrate the target.

The pioneering paper on airplane impact on nuclear shielding structures was written by Riera [1] . Several authors have refined the simple assumptions of Riera and a comparison with lumped mass approach is given by Wolf et al [2] for the case of Phantom airplane crash.

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* Associate Prof. of Mechanical Engineering, MTC, Cairo, Egypt.

Simplifications concerning the material properties of the projectiles and their structure yield in most cases too high values for the impact forces. Drittler and Gruner [3,4] presented models which allow all relevant influences that affect the load-time-function resulting from impact of a projectile upon a rigid building or a structural part to be incorporated. The computation algorithm is based on a difference method. The projectile model has to be divided into elements along the flight trajectory. Different elastic and plastic material properties for each element can be taken into account. Strongly deformed elements are assumed to become separated from the projectile. In a paper by Bignon and Riera [5] the basic hypothesis introduced in the theoretical solutions of three different programs have been subjected to extensive checks by comparing the results obtained with available experimental data using reduced scale missiles.

In most structural shock dynamic problems, the rate of application of the load is small in comparison with the velocity of stress/strain propagation. Therefore the entire structure responds to the loading condition immediately and wave propagation is not a consideration in the solution. If the load is applied at a high rate (as in case of impact of projectile against rigid targets) the propagation of stress/strain waves must be considered in the problem solution. The major difference between a wave propagation problem and a structural dynamic problem is the number of modes that significantly contribute to the response of the structure. In structural dynamics, only a few lower frequencies are excited. Therefore only these lower modes contribute significantly to the response of the system. In a wave propagation problem, a large number of frequencies are excited and significantly contribute to the structural response. For this reason modal analyses generally do not yield cost effective accurate results in the wave propagation analysis and a direct numerical integration procedure must be utilized [6,7].

The Finite Element Method has been used in elastic wave propagation for a long time [8]. The wave propagation was later extended to include nonlinear material properties [9] and large displacement [10, 11]. For impact problems involving elastic plastic flow with large displacement, the finite difference and finite element formulations have many similarities. Both are Lagrangian methods and both are susceptible to numerical instability. FEM , however , is more advantageous because complex geometries , boundary conditions , and material variations can readily be represented .The limitations of the finite element approximation are studied by different authors [12,13,14]. One of these limitations is the fact that spurious reflection takes place at the interface between any two finite element grids with different sizes. However, this problem can be mitigated by using higher order elements as well as by inserting a transit zone where element sizes changes gradually.

For impact problem involving elastic-plastic (nonlinear) material behaviour the propagation of stress waves is a very complex phenomenon. The plastic stress wave travels at a much lower velocity than the elastic portion of the stress wave and will therefore lag behind it.The solution of the elastic-plastic large deformation problems using total Lagrangian approach [15] shows inadequancies in treating finite strain plasticity problems because the rate equations of plasticity are readily expressed with respect to the current configuration using Cauchy stress as opposed to the initial configuration using Kirchhoff stress. The updated Lagrangian technique as proposed by McMeeking and Rice [16] was developed to overcome this difficulty [17]. This approach permits a correct treatment of plasticity constitutive law because at each instant the reference state is updated to coincide with the current state.

In this work, analysis of stress wave propagation due to impact loading of a deformable projectile and rigid wall using FEM and special Gap element is given. First, a problem of elastic impact of a uniform bar is solved analytically and numerically. Another problem of practical interest is the dynamic impact of fast-flying airplane hitting against rigid barrier. This problem is solved with three different options : first with linear elastic material, second with nonlinear elastic-plastic material, and third with consideration of large displacement, finite strain and updated Lagrange analysis. The results are compared together as well as to results of conventional approaches.

BASIC EQUATIONS

Nonlinear Equations of Motion

The FEM can be used to derive the mechanical equations governing the deformation of a structure under impact loading. The principle of virtual work as applied to dynamic phenomena (inertia terms are included) may serve as the starting point for the development of finite element expressions. In a next step stresses are related to strains via constitutive equations. Under crash conditions , certain materials exhibit inelastic properties while elastic contributions to overall strains are negligible. In such cases,it is often advantageous to disregard entirely elastic effects and to deal with an exclusively inelastic material model. For brevity of presentation, it suffices to quote the final equation governing the impact motion and deformation of the structure discretized by FEM in the form

$$M\ddot{u} + C\dot{u} + S(u) = f \qquad (1)$$

This equation can be linearized for a finite domain of time Δt

$$M\Delta\ddot{u} + C\Delta\dot{u} + K\Delta u = \Delta f \qquad (2)$$

Where M,C,K are the mass, damping, and instantaneous stiffness matrices
S(u) is the vector of internal nodal forces
f is the vector of external nodal forces
u, $\dot{u}$, $\ddot{u}$ are position, velocity, and acceleration vectors

For details, however, interested readers are refered to [18].

Once the discretized equations governing the response of the structure deforming under impact are established , numerical methods are applied for their solution. In this study, the nonlinear equations, together with boundary and initial conditions are solved using the Newmark-β method of implicit time integration [19] with $\gamma = 0.5$ and $\beta = 0.25$. The Newmark-β operator can effectively give solutions for linear as well as nonlinear problems for a wide range of loading types. While this method is unconditionally stable for linear systems , if nonlinearity occurs, instability may develop. That is why this operator may be used with adaptive time step control. By reducing time step and/or adding stiffness damping we can overcome these problems . The generalization of the operator is:

$$u_{n+1} = u_n + \Delta t\, \dot{u}_n + (0.5 - \beta)\, \Delta t^2\, \ddot{u}_n + \beta\, \Delta t^2 \ddot{u}_{n+1} \qquad (3a)$$

$$\dot{u}_{n+1} = \dot{u}_n + (1-\gamma)\, \Delta t\, \ddot{u}_n + \gamma\, \Delta t\, \ddot{u}_{n+1} \qquad (3b)$$

The particular form of the dynamic equations corresponding to the trapezoidal rule ($\gamma = 0.5$ and $\beta = 0.25$) results in :

$$[\, (4/\Delta t^2)\, M + (2/\Delta t)\, C + K \,]\, \Delta u_{n+1} = \Delta f + M\, [\, 2\, \ddot{u}_n + (4/\Delta t) \dot{u}_n] + 2\, C\, \dot{u}_n \qquad (4)$$

Equation (4) allows implicit solution of the system

$$u_{n+1} = u_n + \Delta u \qquad (5)$$

The above operator has effectively converted a nonlinear second order differential equation (1) into a set of incremental algebraic equations linearly approximated at discrete instants (4). Once the solution of the nodal incremental displacement become available, the corresponding stress components in the elements will be computed from the constitutive equations. The total strains and stresses can be evaluated from the corresponding incremental values as:

$$\epsilon_{n+1} = \epsilon_n + \Delta \epsilon \qquad (6a)$$

$$\sigma_{n+1} = \sigma_n + \Delta \sigma \qquad (6b)$$

Maintaining a numerically stable solution for dynamic problems is generally accomplished by using a numerical integration time increment which is sufficiently less than the lowest period of vibration of the system. However, it is usually not computationally feasible to determine the natural frequencies due to many degrees of freedom. Furtheremore because of large strains and displacements, the frequencies are not constant but rather vary as solution progresses. Since an assemblage of elements will never have periods of vibration less than that of individual elements, a lower limit for the lowest period of vibration can be established by obtaining the periods for individual elements. Therefore, the maximum time increment is selected so that the stress wave propagates the distance between element integration points within that time increment.

$$\Delta t_{max} = (L_e / 2)/ c \qquad (7)$$

Where L_e is the length of an element in the direction of wave propagation

c is the velocity of wave propagation.

From experience [20]

$$\Delta t \leq (1 / 3) \Delta t_{max} \qquad (8)$$

yields accurate results.

The velocity of wave propagation in one dimensional elastic medium is given by:

$$c = \sqrt{E / \rho} \qquad (9)$$

However, the velocity of wave propagation in plastic medium is a function of the slope of $\sigma - \epsilon$ curve and is defined by :

$$c = \sqrt{(\partial \sigma / \partial \epsilon) / \rho} \qquad (10)$$

Gap Element Equations

The contact with a fixed surface is one of the nonlinear boundary conditions which occur during impact of projectiles and a rigid target. This problem may be solved in the FEM through the use of so called special Gap (contact) element. The modeling of the gap (contact) element is based on the imposition of kinematic constraints. The minimization of the total potential energy is subjected to these constraints. This necessitate the introduction of Lagrange multiplier and the solution of an expanded system of equations. During evaluation of the stiffness matrix the gap status is based on the estimated strain increment. The gap status is checked again after solution is obtained during recovery of the strains and stresses. Let the system equation in static analysis

$$K\, u = f \qquad (11)$$

be subject to constraint condition

$$C\, u = 0 \qquad (12)$$

Through minimization of the augmented functional

$$\psi = (1/2)\, u^T K u - u^T f + \lambda^T C u \qquad (13)$$

we obtain

$$\begin{bmatrix} K & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} u \\ \lambda \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix} \qquad (14)$$

This equation can be solved simultaneously for both u (displacement) and λ (Lagrangian multiplier). In the gap (contact) element, the values of the Lagrangian multipliers represent the normal and frictional gap forces as well as frictional slippage. The Gap-Element can be used in dynamic as well as static analysis. However, in dynamic impact care should be given to the application of law of conservation of momentum for the calculation of velocities and accelerations of contact nodes in the case of closed gap.

APPLICATIONS

Example 1 : Impact of an Elastic Bar

In order to test the capability of the FEM with Gap element to calculate the reaction force versus time , the case of impact of homogeneous bar with linear elastic material was considered because of availability of analytical solution and existance of nonlinear boundary condition. The bar has the following data:

$L = 10$ m , $A = 0.03$ m^2 , $\rho = 26800$ kg/m^3

$m = 8040$ kg , $E = 63.8$ GPa , $\nu = 0.3$

$v_0 = 215$ m/s , $c = \sqrt{E/\rho} = 1542.9$ m/s

With this data, one obtains analytically [3] the reaction force to be :

$$F = \rho A c v_o = 266.7 \text{ MN}$$

and the duration time :

$$\Delta\tau = 2 L/c = 0.013 \text{ sec} = 13 \text{ ms}$$

The momentum transfered to the rigid wall during elastic impact is :

$$\Delta I = 2 m v_0 = 3.457 \text{ MNs}$$

This ΔI is known to be equal to the impulse $F\Delta\tau$.

Using the FEM , the bar is modelled as a 30 linear (two-node) straight truss (bar) elements and one gap element at the interface with rigid wall. The equations of motion are derived and integrated for period of 20 ms using 200 increments of 0.1 ms each. The reaction force versus time curve is obtained (Fig.1). The agreement of FEM results and analytical solution appears to be satisfactory.

Example 2 : Crash of Phantom Airplane

In this example , we study the calculation of the reaction force resulting from impact of fast-flying airplane upon a rigid wall. The case of Phantom airplane is considered because of availability of data and results with other approaches [2,3,4]. For Phantom airplane it is given (Fig.2) :

1- The mass distribution per unit length amounting to a total of 20 000 kg.
2- The distribution of the supporting cross-section area A.
3- The material behaviour (E = 68 GPa , H = 3 GPa , σ_y= 500 MPa)
4- The initial impact velocity v_0 = 215 m/s.

This problem is solved using simple one-dimensional model together with gap element. The following three cases are employed for the solution:

Case 1 : Linear elastic material model.
Case 2 : Nonlinear elastic-plastic material model with small strain theory and total Lagrangian technique.
Case 3 : Nonlinear elastic-plastic material model with large displacement, finite strain, and updated Lagrangian technique.

The integration of equations of motion is performed for 100 ms using 500 increments of 0.2 ms each. The reaction force versus time curve is given for different cases in Fig.3, and compared to other approaches in Fig.4. The displacement and velocity vs. time curves for point A (front of airplane) and point B (rear of airplane) are given for different cases in Fig. 5,6 .

DISCUSSION OF RESULTS

From the results of example 1 (Impact of Elastic Bar), one may conclude that FEM with gap element can be used for reliable prediction of the reaction force at interface with rigid wall. However,in practical applications with material and/or geometric nonlinearities as in examlpe 2 (Crash of Phantom Airplane), the problem is more complicated. The results of the three cases are different which means that one should properly select the appropriate model. In case 1 (linear elastic material) the maximum value of the reaction force is 671.6 MN which is 4.74 times higher than that of case 2 (nonlinear elastic-plastic material). Of coarse, case 1 assumption is only theoretical and is given to manifest the importance of nonlinear treatment of this type of problems.

In case 2 (elastic-plastic), the maximum value of the reaction force (141.8 MN) as well as the general behaviour of the force versus time curve are in good agreement with the results obtained from Rierra and lumped mass finite difference approaches [2,4] (Fig.4). However, this model is different from real condition of airplane crash where large displacements and large (finite) strains takes place. Using the capabilities of nonlinear FEM a proper combination of large displacement, finite strain, and updated Lagrangian analysis (case 3) gives a more realistic model. The results of this case gave a maximum force of 283.7 MN which is 2 times higher than that of case 2. In a trial to use only large displacement option with elastic-plastic material model, it has been recognized that the solution was susceptible to greater oscillations and instabilities, where convergence was difficult to achieve.

The comparison of displacement and velocity versus time curves of two points on airplane (A: front and B: rear), in case 2, showed that at begining of impact point A is stopped immediately while point B continued to move towards the wall and reached a maximum compression of 7.2 m after approximately 50 ms . After that time the wave reflected and the point B started motion far from the wall. In case 3, however, the point B moved towards the wall and reached a maximum compression of 5.15 m after 30 ms.

CONCLUSIONS

From previous work, we can recapitulate the following conclusions:

1- The reaction force of impact of deformable projectile against a rigid wall can be reliably predicted using the FEM together with gap element. This force can be used as input for dynamic response calculation of target structure.
2- In spite of the fact that we used only simple one-dimensional models, the procedure can be easily extended to the treatment of complex geometries (two- and three-dimensional models).
3- In elastic-plastic problems, the stress wave is not only reduced in intensity but it is also lengthened due to reduced velocity of the plastic portion of the wave.
4- The nonlinear elastic-plastic material model which is equivalent to conventional lumped mass method gave lower value of the maximum force than that obtained by considering large displacement, finite strain and updated Lagrangian analysis.It may be, therefore, recommended to use the force obtained from the combinations of options as in case 3 in order to achieve conservative design.

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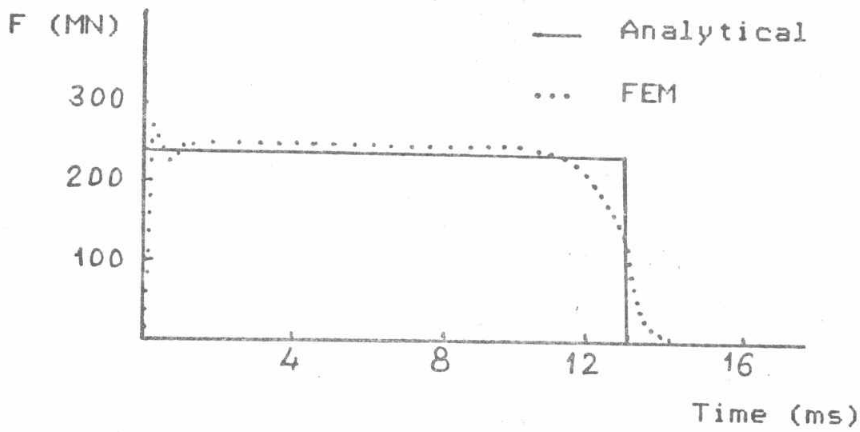
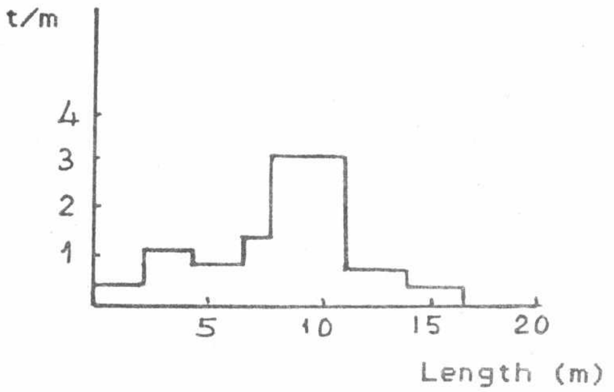
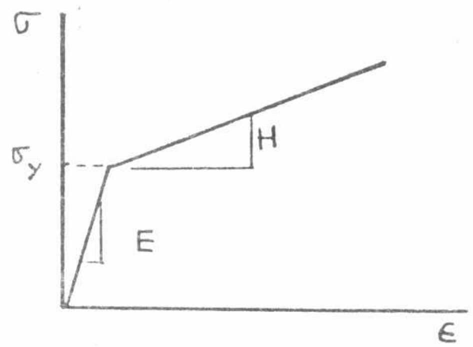


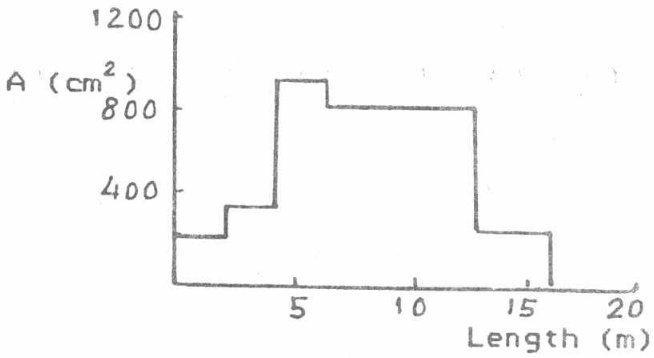
Fig. 1 Reaction force vs. time
(elastic bar)



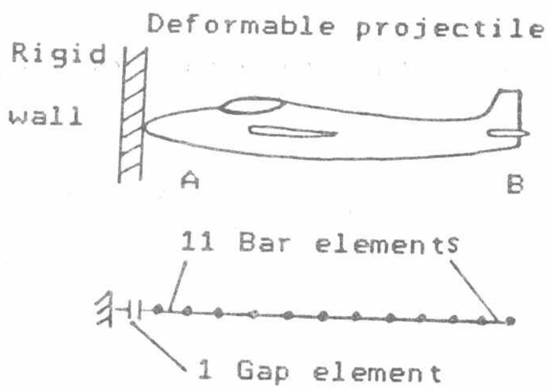
a) Mass distribution



c) Stress strain relation



b) Supporting cross-section
distribution



d) Finite Element Model

Fig. 2 Basic data of Phantom airplane
and Finite Element Model

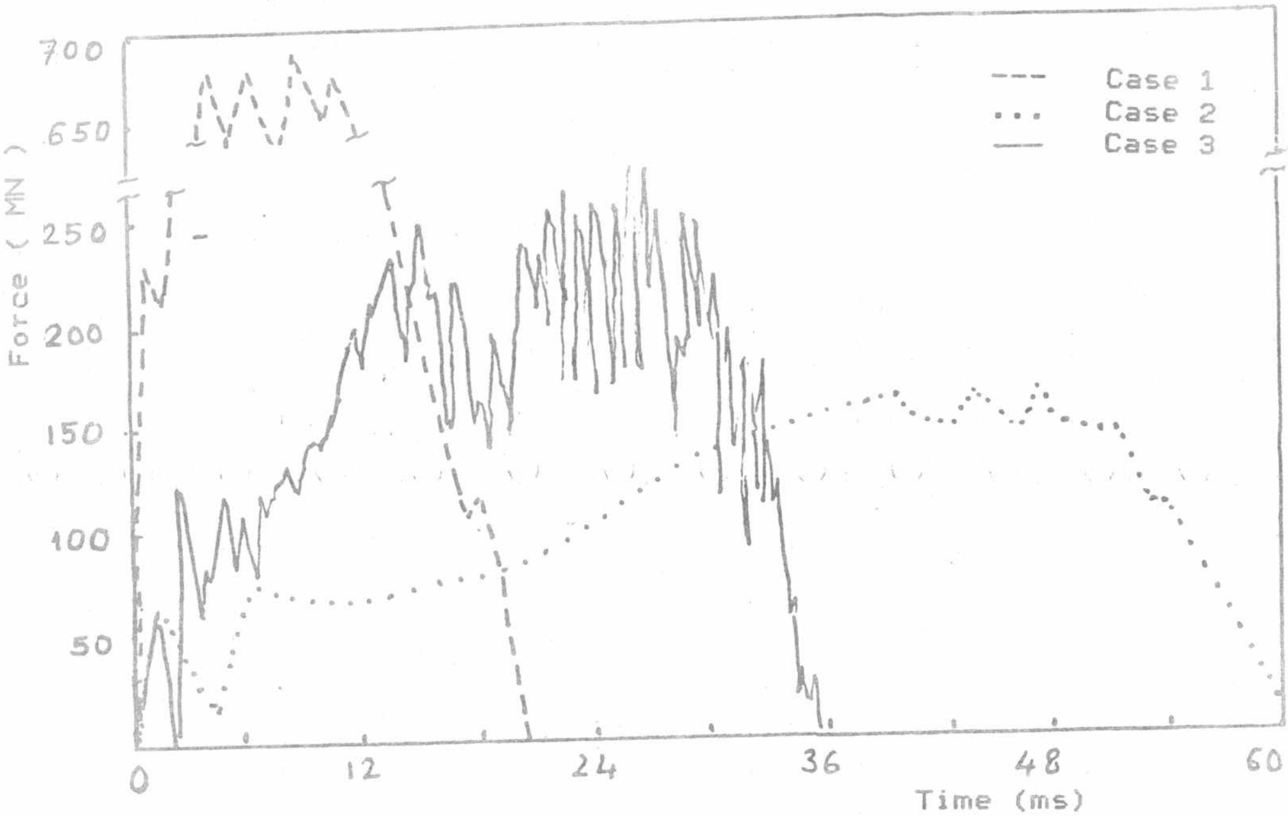


Fig. 3. Reaction force versus time using FEM

(Phantom airplane)

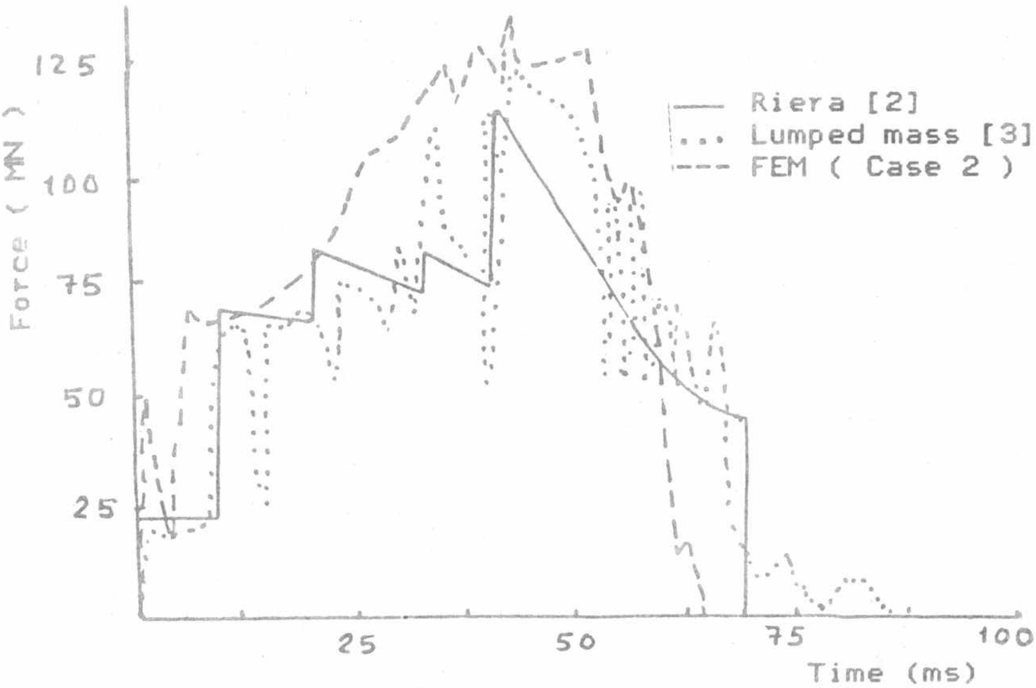
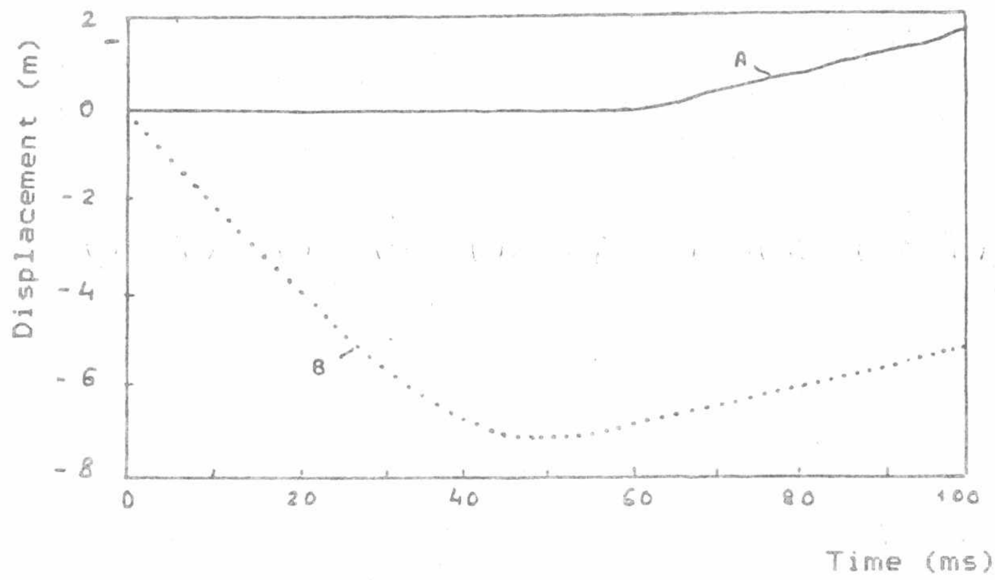
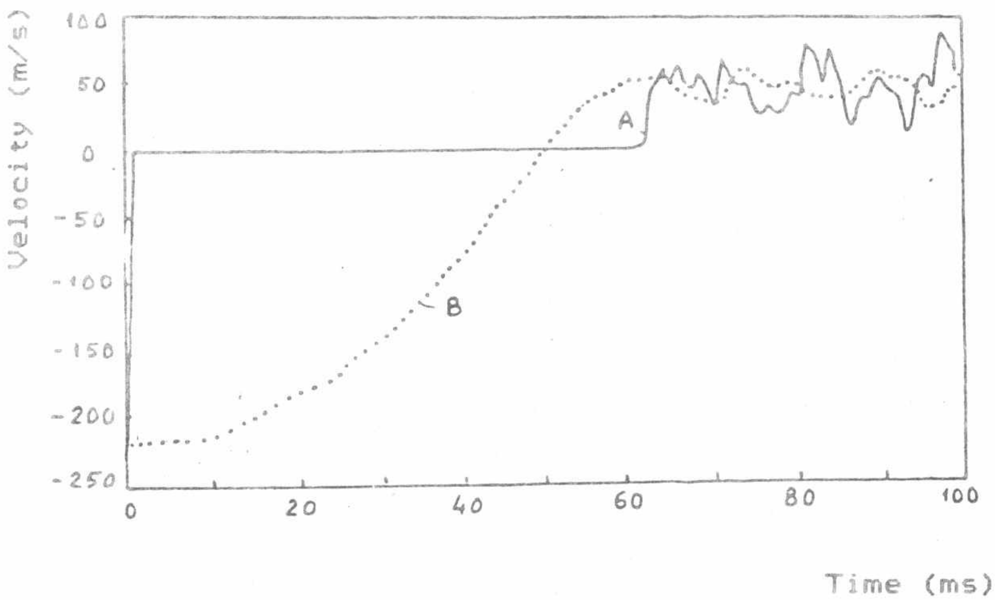


Fig. 4. Force vs. time using different approaches

(Phantom airplane)

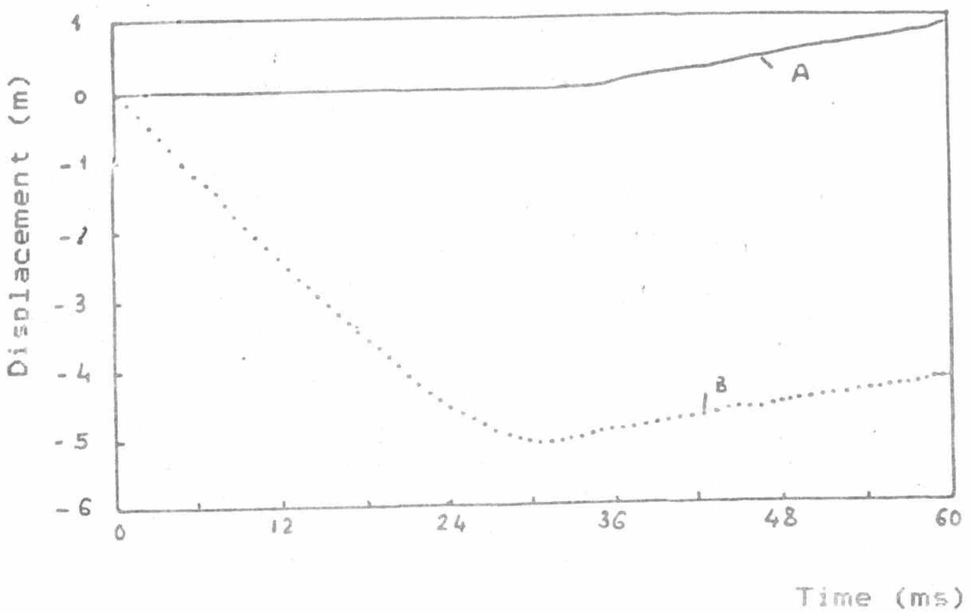


a) Nodal displacements

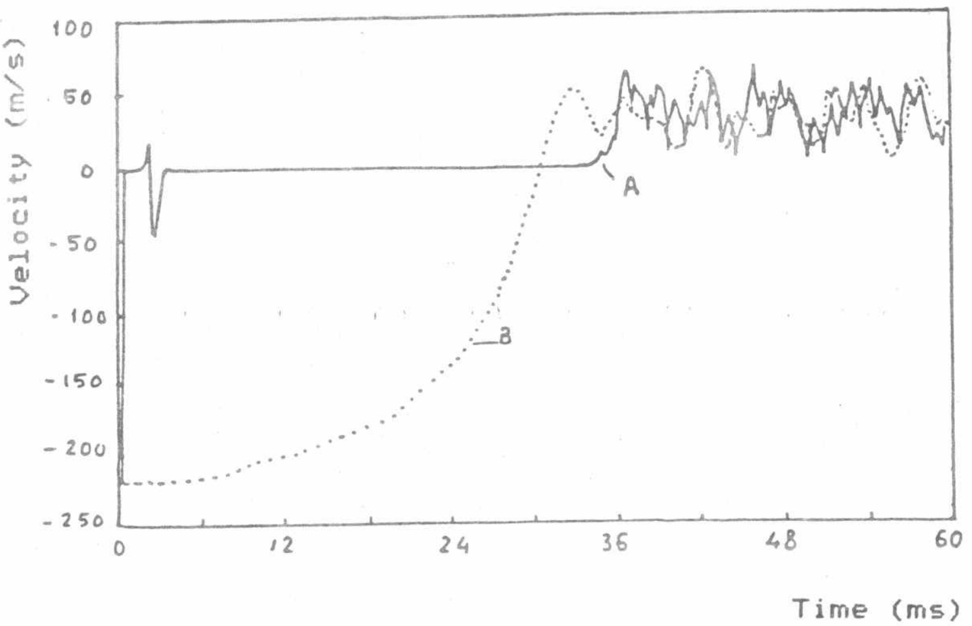


b) Nodal velocities

Fig.5. Nodal displacements and velocities versus time
A : front , B : rear (Case 2)



a) Nodal displacements



b) Nodal velocities

Fig. 6. Nodal displacements and velocities versus time
A : front , B : rear (Case 3)