MILITARY TECHNICAL COLLEGE

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ANALYSIS OF TRANSIENT PRESSURES IN TWO-COMPONENT TWO-PHASE FLOW

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ABSTRACT

In this paper, the problem of hydraulic transients resulting from valve closure in a branched pipe carrying a two-component two-phase flow is theoretically investigated. The governing equations were derived on the basis of one dimensional flow. The method of characteristics was used for solving the resulting differential equations. The evaluation of time interval was made in such a way to be suitable for use in a branched pipe system having branches of different lengths.

The analysis of many investigated cases reveals that the maximum pressure rise increases with the increase of the delivery pipeline diameter and with the decrease of valve closure time and void fraction. Moreover the study shows that in all investigated cases, the lowest value of maximum pressure rise occurs when the piping lengths ratio (branched dead end pipe to upstream pipe lengths) approaches unity.

1- INTRODUCTION.

The problem of transient pressures due to valve closure in piping systems has received a remarkable concern due to its critical and detrimental effects in numerous of practical situations.

Streeter [1] introduced methods to control the transient pressures, considering the rate of valve motion. Also, Streeter [2] utilized simplified forms of characteristic equations to form the basis for transient solution of

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piping systems when the system has certain short pipes that restrict the time increment of the computation. Chaudhry and Hussain[3] introduced three second order accurate explicit difference schemes, to solve the partial

differential equations describing water hammer phenomenon in closed conduits.

Gases in oil can cause a great reduction in the bulk modulus of elasticity of the oil [4]. Chaudry et al. [5] solved the governing partial differential equations, representing hydraulic pressure transients in the case of two-component two-phase flow using Second-order explicit finite difference schemes. The one difficulty that met this model was the computation of time interval in the different pipes, where different values of wave speed would result. (This difficulty was overcome in the present work).

Fujii and Akagawa [6] solved the basic partial differential equations of water hammer in two-component two-phase flows, by linearization and iterated Laplace transformation. SAMUEL et al. [7] presented an analytical model for pressure pulse propagation in two-component slug flow.

The main objective of the present work is to elaborate a numerical solution procedure for solving transient flow in the case of two-component two-phase flow and to adapt this procedure where it can be used for the analysis in single and branched piping hydraulic systems. Effects of system physical parameters such as branch length ratio, pipe diameter and liquid phase properties, on pressure transients are to be examined. The effects of variations of valve closure time, pipe diameter, void fraction, liquid properties on the maximum pressure rise and pressure wave propagation are investigated.

The validity of the model is verified by comparing the computed results with the experimental results of Chaudary [5].

2- DIFFERENTIAL EQUATIONS DEFINING UNSTEADY FLOW.

The following assumptions are made in the derivation of the equations of unsteady two-phase flow through circular pipes :

1. The fluid mixture is a homogeneous bubbly two-component two-phase flow. [7].
2. Flow in the pipe is one dimensional.
3. The gas bubbles and liquid posses the same velocity [2].
4. The expansion and contraction of gas bubbles are polytropic with an exponent n = 1.2, i.e between isothermal and adiabatic process [8].
5- The pipe walls and the fluid are linearly elastic, [8].
6- The formulas used for computing the steady state friction losses in pipes are valid during the transient state [2].

Dynamic Equation.

$$\frac{\partial Q}{\partial t} + g.A.\frac{\partial H}{\partial x} + \frac{f}{2DA}.Q|Q| = 0 \qquad (1)$$

Continuity Equation.

$$\frac{a^2}{g.A} . \frac{\partial Q}{\partial x} + \frac{\partial H}{\partial t} = 0 \qquad (2)$$

where

$$a^2 = \frac{Km}{\rho m.(1 + (Km.D)/(e.E))} \qquad (3)$$

$$\rho m = (1-\alpha).\rho l + \alpha.\rho g \qquad (4)$$

$$Km = \frac{1}{(1-\alpha)/Kl + \alpha/Kg} \qquad (5)$$

EQUATION OF STATE.

The equation of state can be written in the form :

$$P . V_g^{n} = P_{atm} . V_{gatm}^{n} \qquad (6)$$

then

$$\alpha = \left(\frac{P_{atm}}{P} \right)^{1/n} . \alpha_{atm} \qquad (7)$$

and

$$\rho_g = \left(\frac{P}{P_{atm}} \right)^{1/n} . \rho_{gatm} \qquad (8)$$

3- SOLUTION OF THE GOVERNING EQUATIONS.

The method of characteristics is used for solving the governing differential equations.

If the conditions at points "A and B" in Fig. (1) are known then the conditions at points "P" can be obtained as follows :

$$Q_p = C_p - C_a . H_p \qquad (9)$$

$$Q_p = C_n + C_a . H_p \qquad (10)$$

Where

$$C_p = Q_A + \frac{gA}{a} . H_A - \frac{f\delta t}{2DA} . Q_A |Q_A| \qquad (11)$$

$$C_n = Q_B - \frac{gA}{a} . H_B - \frac{f\delta t}{2DA} . Q_B |Q_B| \qquad (12)$$

$$C_a = \frac{gA}{a} \qquad (13)$$

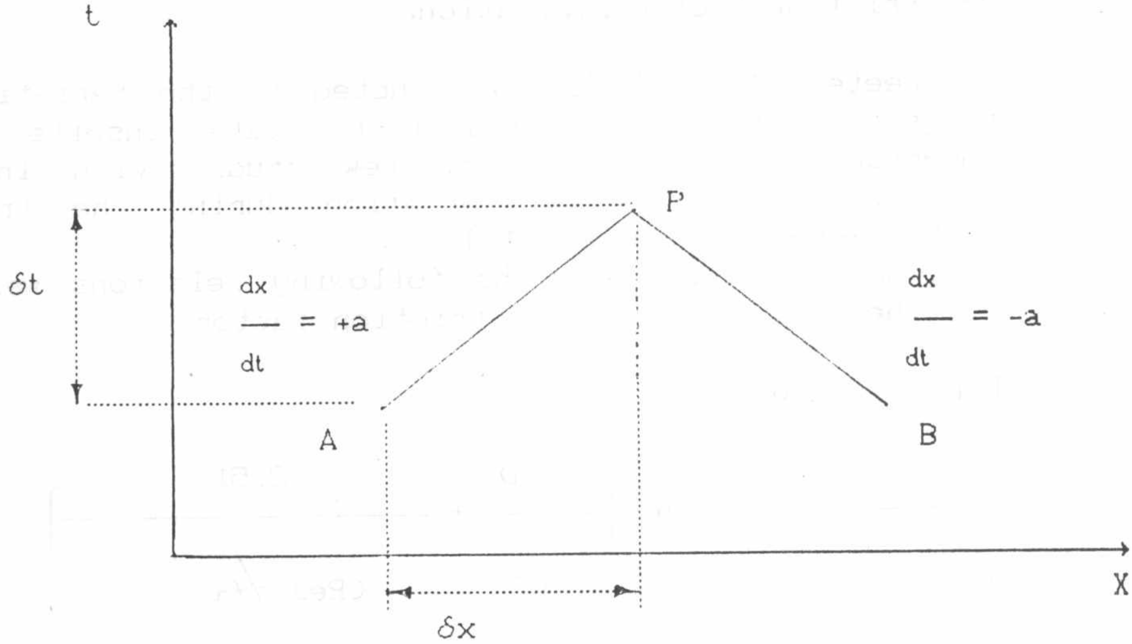

Fig. (1) Characteristic lines in the X-t plane.

3-1 Time Interval Evaluation in a Branched Piping System.

The time interval "δt" depends on the wave speed and the number of pipeline sections used in computation (N). In the case of branched piping systems the equality of time step in all pipes should be verified.

$$\delta t = \frac{L1}{a1 \, . N1} = \frac{L2}{a2. N2} = ... = \frac{Li}{ai. Ni} \qquad (14)$$

where L1, L2, Li expresse pipes lengths and N1, N2, Ni indicate numbers of pipe sections.

In the case of two-component two-phase-flow, the air content has a substantial effect on the values of wave speed, where different values of wave speed are obtained. So it is difficult to verify the equality of time interval for all pipes. To overcome the above mentioned difficulty, it is necessary to use a mean value of the transient pressures along the whole length of pipe at the instant of calculation, where a mean value of wave speed can be obtained at each time step. Thus the equality of time interval in all pipes can be verified.

3-2 Friction Factor Evaluation.

Streeter [9] and Fox [7] noted to the variation in friction factor during transient state. Inspite of the importance of these notes, few studies with inclusion of friction factor variation during the transient state were introduced [10].

According to Fox [7] the following relations were used for the evaluation of the friction factor :

For Re > 2300

$$\frac{1}{\sqrt{f}} = -2.\,\mathrm{Log}_{10}.\left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{(Re).\sqrt{f_a}}\right] \qquad (15)$$

$$f_a = 0.0055.\left\{1 + \left[20{,}000.\frac{\varepsilon}{D} + \frac{10^{6}}{(Re)}\right]^{1/3}\right\} \qquad (16)$$

For Re < 2300

$$f = \frac{64}{Re} \qquad (17)$$

4- BRANCHED PIPING MODEL

A model representing the fluid supply side of a typical hydraulic power system is considered for studying the pressure transients on the basis of the preceding analysis. The supply side would normally have a constant pressure head source feeding a pipeline with a closed dead end branch and ended with a control valve, as shown in Fig. (2). The piping system however includes a main pipe and two branches, or simply three pipes.

Each of the three pipes of the model is divided in the calculations into N sections. This implies that pipe (1) of diameter D1 and length L1, has "N1" sections, pipe (2) of diameter D2 and length L2, has "N2" sections and pipe (3) of diameter D3 and length L3 has "N3" sections.

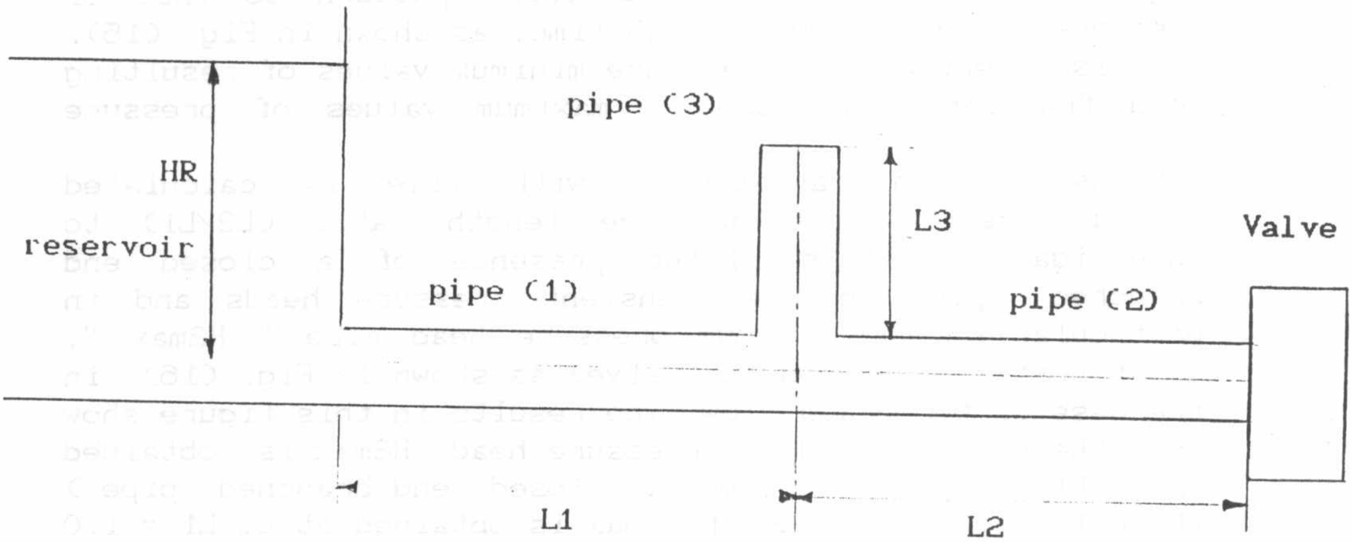

Fig. (2) Branched piping model

5- RESULTS AND DISCUSSION

The validity of the mathematical model in the case of two-component two-phase flow is checked by calculating the pressure head variations during transient state for the same data of the experimental model presented by **Hanif Chaudhry** [5] (Appendix-A2).

Comparison of present computed results and experimental results are shown in Figs. (3) and (4). The study of these figures shows that a satisfactory agreement is obtained between the computed and experimental results.

Figs. (5), (6) and (7) show the variation of fraction accompanied with a delay of the cyclic pressure head response.

In the case of two-component two-phase flow the wave speed, the bulk modulus of elasticity and the viod fraction will have varying values according to pressure changes throughout transients. Figs (12), (13) and (14) show the variation of void fraction, bulk modulus and wave speed with time respectively, for different liquids (water, shell tellus oil, linsed oil). The variation of void fraction takes place in a contrary pattern to that of pressure head variation with time, as shown in Fig. (15). This is essentially true since minimum values of resulting void fraction correspond to maximum values of pressure head.

Pressure head variation with time is calculated for different values of pipe length ratio (L3/L1) to investigate the effect of the presence of a closed end branched pipe on the transient pressure heads and in particular on the maximum pressure head rise " H2max ", (just upstream the control valve) as shown in Fig. (16) in the case of two-phase flow. The results in this figure show that the maximum value of pressure head H2max is obtained at L3/L1 = 0.0 (i.e. without closed end branched pipe) while the minimum value of H2max is obtained at L3/L1 = 1.0

The exact variation of "H2max" with the ratio L3/L1 is checked by calculating more than twenty points, covering the range under consideration, as shown in Fig. (17), for the pipe diameter D = 0.0158 m, valve closure time TC = 0.1 sec., void fraction α_0 = 0.05 and HR =25m with shell tellus oil. The pattern of variation, as plotted in the figure can be distinguished by two regions. The first region is the region where L3/L1 < 1 . Here the variation has a steep dropping characteristics and the value of H2max is increased as L3/L1 is decreased. The second region is where L3/L1 > 1 and this region is of rather slightly rising characteristics. It has lower values of H2max, in comparison with the values in the region L3/L1 < 1 . However, it is quite evident that the minimum value of " H2max " is obtained when L3/L1 approaches unity.

The values of H2max are calculated for different L3/L1 ratio and at different values of pipe diameter, valve closure time and void fraction as shown in Figs. (18), (19) and (20), respectively.

pressure head with time for pipes (1), (2) and (3) respectively, at different pipe sections (X/L = 0.0, 0.5, 1.0). The study of these figures show that for pipes (1) and (2), the values of pressure head peaks decrease as we leave the control valve towards the constant pressure head source and maximum pressure heads are obtained just upstream the control valve (X/L=1.0 in pipe (2) Fig. (6)). In pipe (3), however, as shown in Fig. (7) small pressure head difference is found between the different pipes sections, where no discharge through the closed dead end.

A comparison between the pressure wave forms generated in pipe (2) (at the section X/L = 1.0) for different valve closure times (TC= 0.04,0.1,0.2 s) is demonstrated in Fig. (8). The comparison reveals that the maximum pressure head is obtained at the smallest closure time (TC = 0.04 sec.), corresponding to lower ratio of TC/Tcr (TC/Tcr = 0.2). This is however an expected result. The figure also shows that the longer the closure time is the smaller would be the periodic time of the generated pressure waves (relative to the value of valve closure time) and thus the higher is their frequency.

The comparison between different values of pressure head in pipe (2) (at the section X/L=1.0) for different pipe diameters is presented in Fig. (9). The figure shows that higher peaks and lower valleys of pressure head are resulted with the larger pipe diameter and vice versa.

The pressure head variations with time are calculated for different liquids (water, shell tellus oil and linsed oil), Fig. (10). As can be noticed from Appendix A-1, the major specific difference in properties of these liquids is the distinct values of viscosity for each one of them. The shell tellus oil has a dynamic viscosity of about 20 times that of water and the linsed oil viscosity is as great as 50 times water viscosity. The figure shows that higher peaks of pressure head are obtained with lower viscosity liquids.

The pressure head variations are calculated for different values of admitted void fraction with shell tellus oil. Fig. (11) shows the pressure head variations with time for different void fractions (α_0 = 0.05, 0.1, 0.2), just upstream the control valve (at X/L= 1.0) in pipe (2). The figure shows that great reductions of transient pressure heads are obtained at the higher value of void

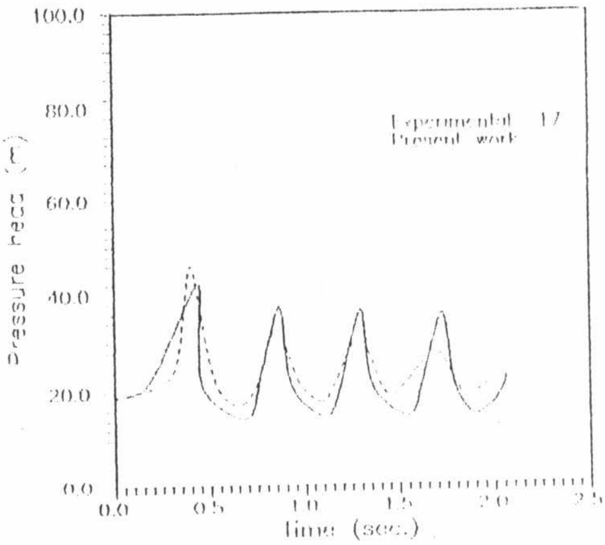

Fig. (3) Comparison between CHAUDARY exerimental results and computed present work (station 1) L=30.6m, D=0.026m, α_o =0.0053, HR=21.7m

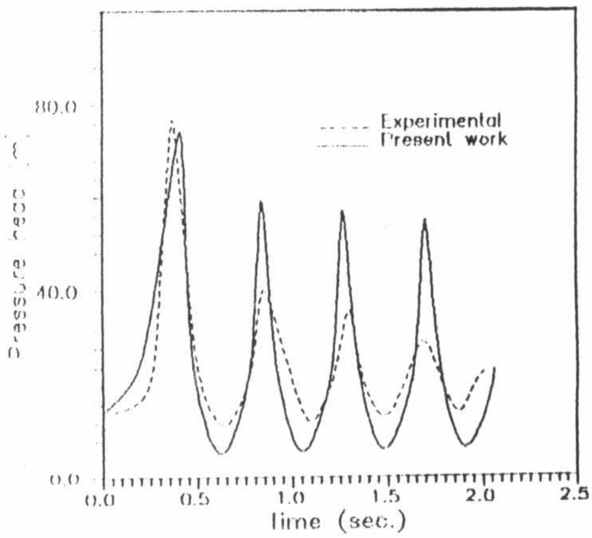

Fig. (4) Comparison between CHAUDARY exerimental results and computed present work (station 2) L=30.6m, D=0.026m, α_o =0.0053, HR=21.7m

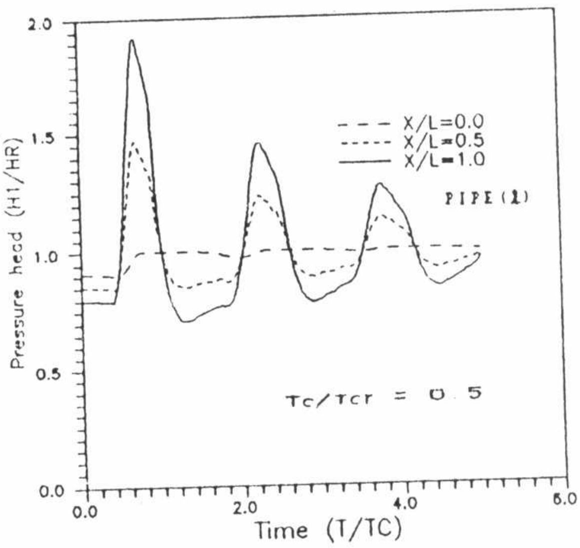

Fig. (5) Pressure head (H1/HR) variation with time (T/TC). pipe (1) shell tellus oil L1=0.64,L2=3.2,L3=0.16m D=0.0158m TC=0.1s α_c=0.05, HR=25.0m.

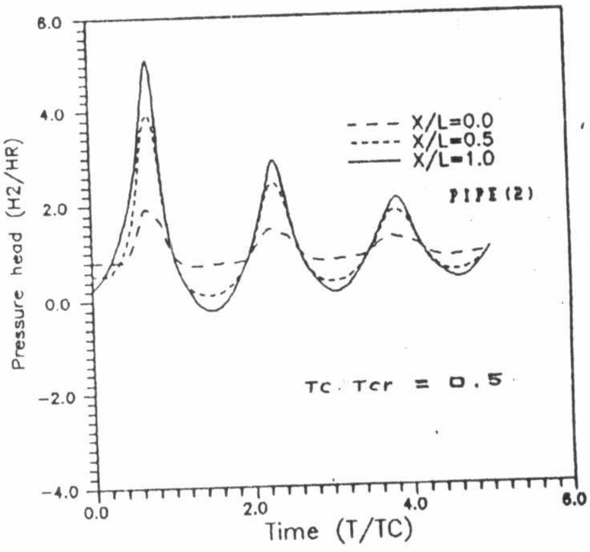

Fig. (6) Pressure head (H2/HR) variation with time (T/TC). pipe (2) shell tellus oil L1=0.64,L2=3.2,L3=0.16m D=0.0158m TC=0.1s α_o=0.05.

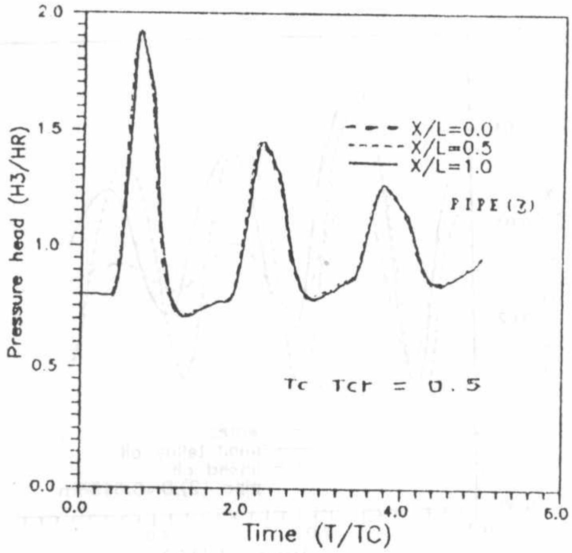

Fig. (7) Pressure head (H3/HR) variation with time (T/TC). pipe (3) shell tellus oil L1=0.64,L2=3.2,L3=0.16m D=0.0158m TC=0.1s α_o=0.05, HR=25.0m.

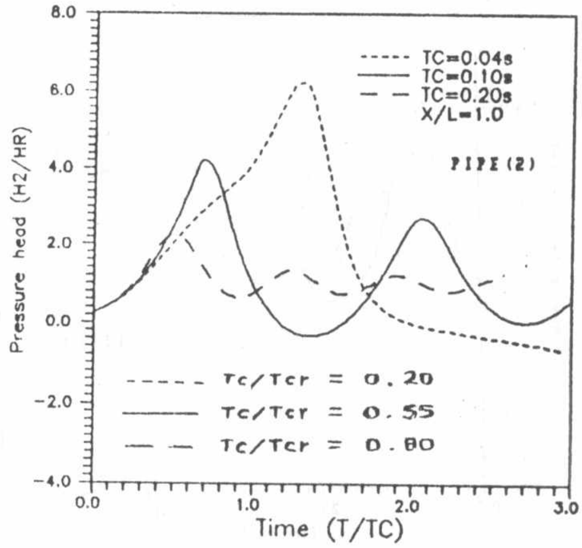

Fig. (8) Pressure head (H2/HR) variation with time (T/TC). pipe (2) shell tellus oil L1=0.16,L2=3.2,L3=0.16m D=0.0158m α_o=0.05, HR=25.0m.

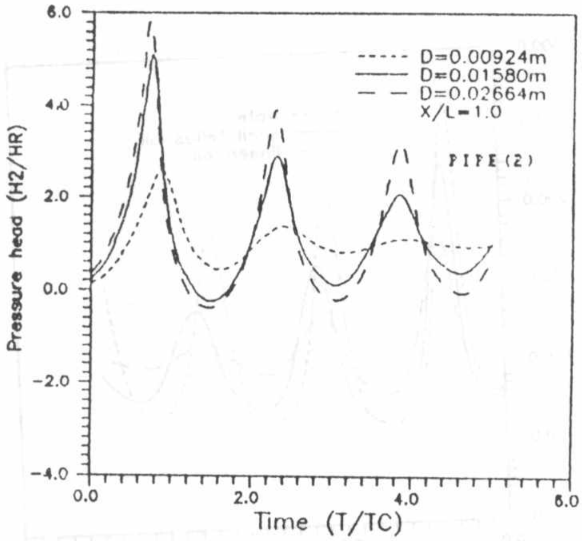

Fig.(9) Pressure head (H2/HR) variation with time (T/TC). pipe (2) shell tellus oil L1=0.64,L2=3.2,L3=0.16m TC=0.1s α_o=0.05,HR=25.0m

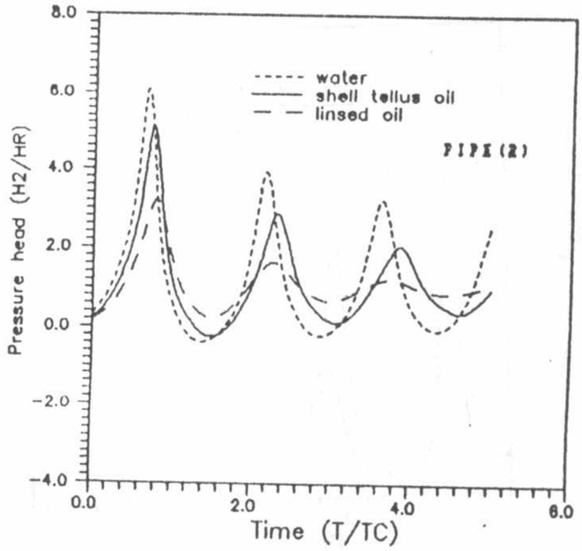

Fig. (10) Pressure head (H2/HR) variation with time (T/TC). pipe (2) , HR=25.0m L1=0.64,L2=3.2,L3=0.16m D=0.0158m TC=0.1s α_o=0.05.

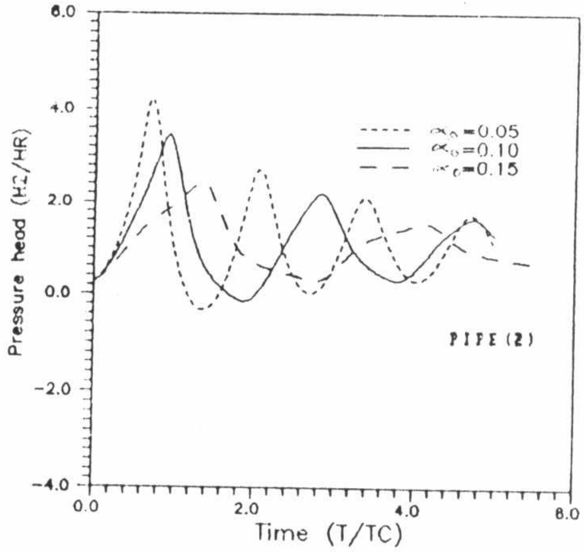

Fig. (11) Pressure head (H2/HR) variation with time (T/TC) pipe (2) shell tellus oil L1=0.16,L2=3.2,L3=0.16m D=0.0158m TC=0.1s, HR=25.0

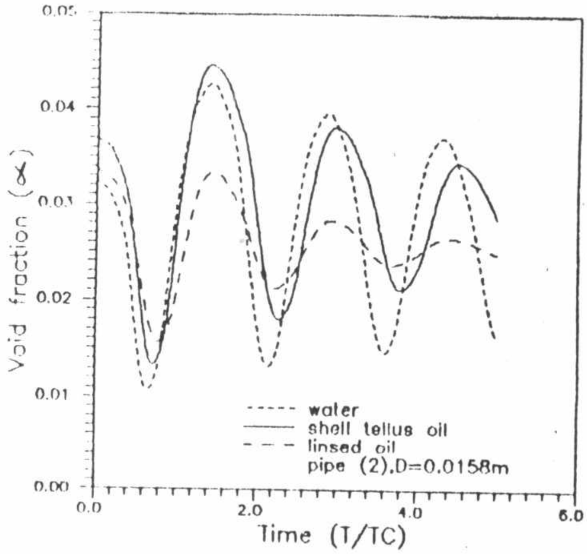

Fig. (12) Variation of void fraction with time for different liquids,TC=0.1s, =0.05 L1=0.64m,L2=3.2,L3=0.16m D=0.0158m

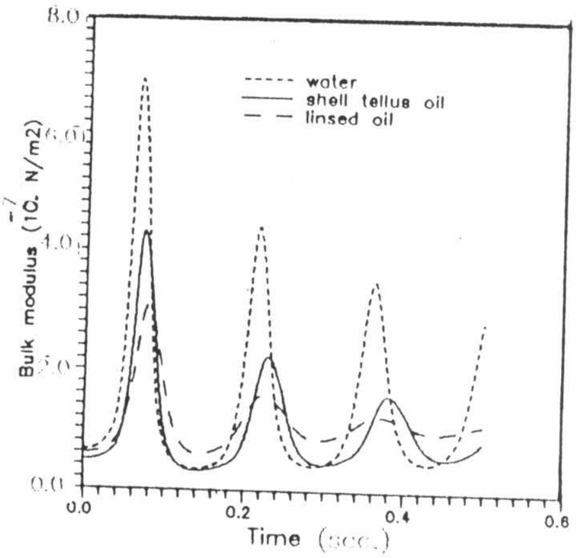

Fig. (13) Bulk modulus variation with time (T/TC). shell tellus oil L1=0.64,L2=3.2,L3=0.16m TC=0.1s α =0.05, HR=25.0m

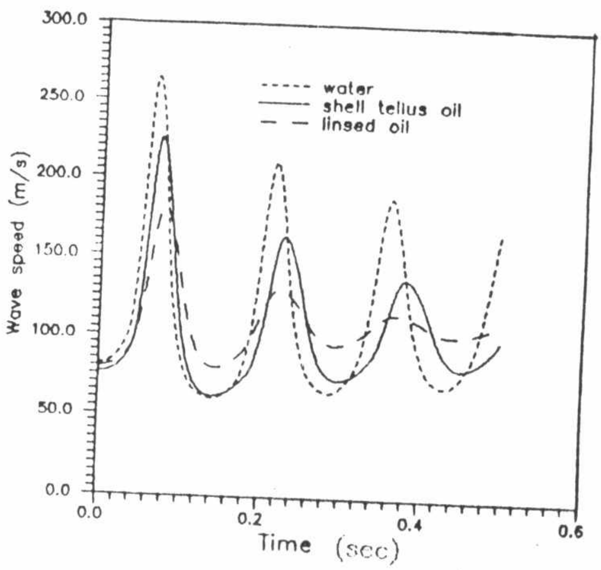

Fig. (14) Variation of wave speed with time for different liquids,TC=0.1s, =0.05 L1=0.64m,L2=3.2,L3=0.16m D=0.0158m

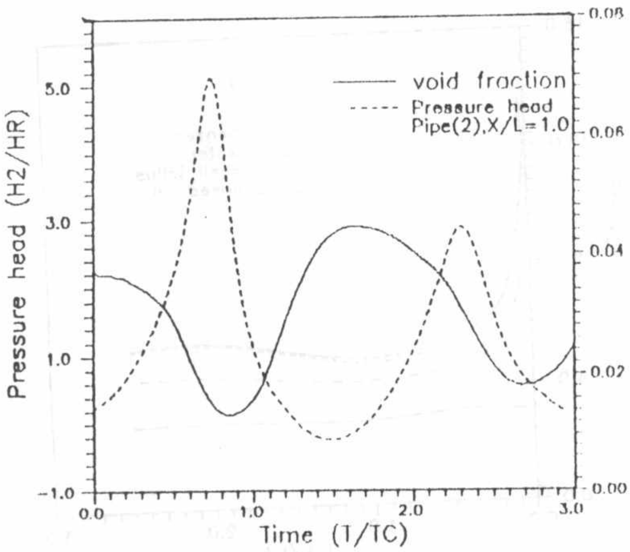

Fig. (15) Variations of pressure head and void fraction with time, shell tellus oil,HR=25.0m,TC=0.1s L1=0.64,L2=3.2,L3=0.16m,D=0.0158m

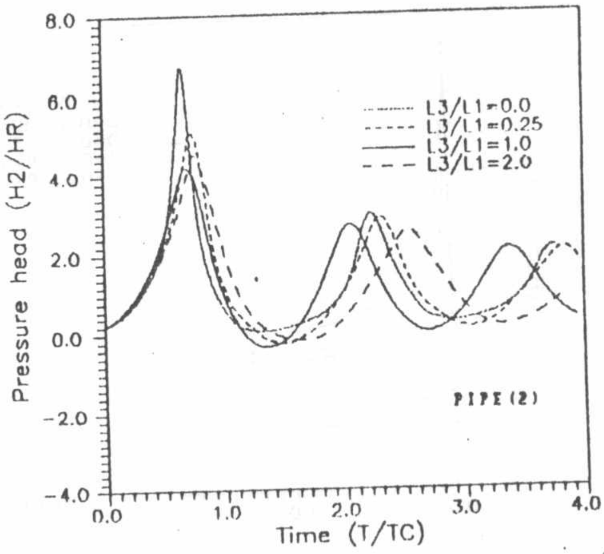

Fig. (16) Pressure head variations (at the valve) with time (T/TC), for different (L3/L1) ratio, shell tellus, D=0.0158m,HR=25.0m TC=0.1s.

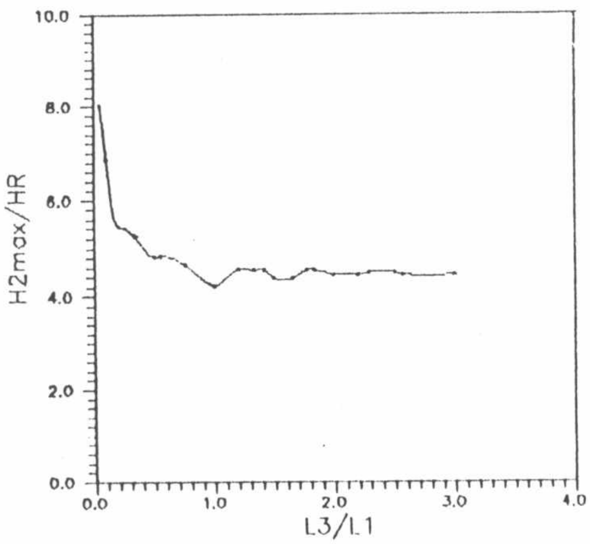

Fig. (17)) Effect of (L3/L1) ratio on max. pressure head, pipe (2), shell tellus oil, HR=25.0m TC=0.1s, L2=3.2, D=0.0158m, α_e = 0.05

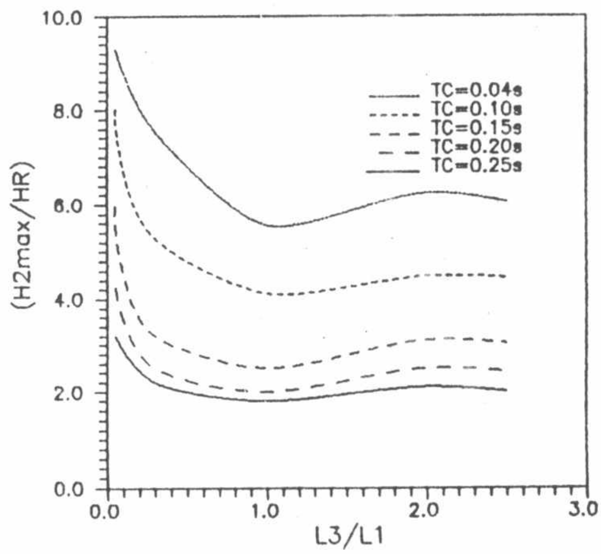

Fig. (18) Effect of (L3/L1) ratio on max. pressure head (H2/HR) for different closure time (TC), pipe(2) shell tellus oil, HR=25.0m L2=3.2m D=0.0158m α_0=0.05

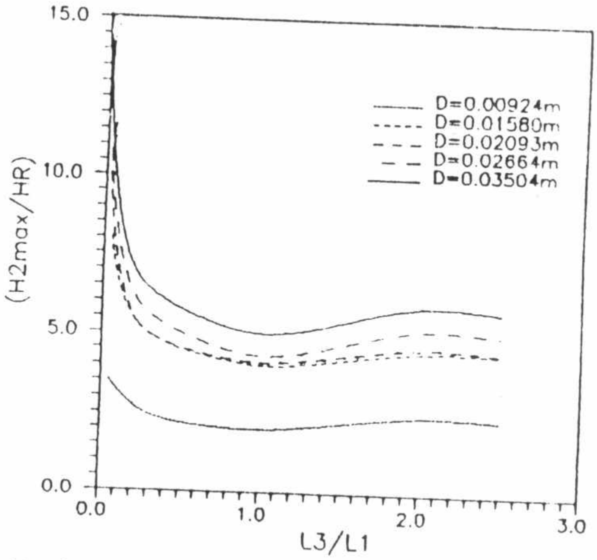

Fig. (19) Effect of (L3/L1) ratio on max. pressure head (H2/HR) for different pipe diameters pipe(2), shell tellus oil, Tc=0.1S, α_g=0.05, HR=25.0m.

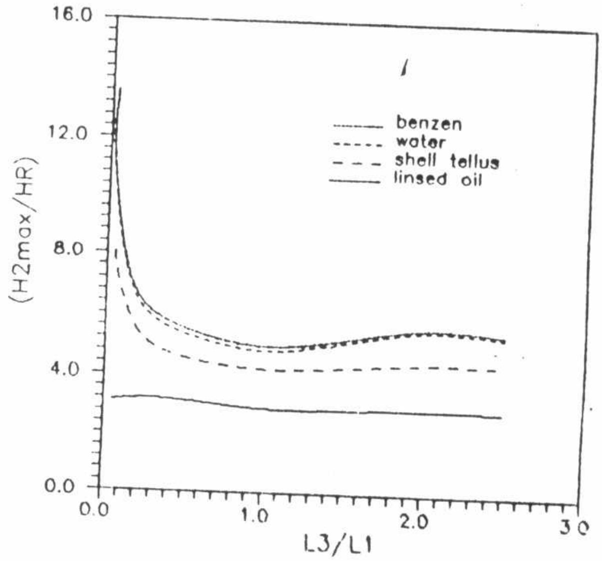

Fig. (20) Effect of (L3/L1) ratio on max. pressure head (H2/HR) for different liquids pipe(2) L1=0.64,L2=3.2,L3=0.16m TC=0.1s α_g=0.05, HR=25.0m.

6- CONCLUSIONS

The problem of pressure transients resulting from sudden valve closure in a two-component, homogeneous (bubbly-liquid) two-phase flow, in a branched pipe is theoretically investigated. The analysis is based on one dimensional flow system taking into account the variation of affecting parameters such as void fraction, friction factor and bulk modulus of the mixture, and numerical solutions of the governing equations are established using the method of characteristics.

The following conclusion can be drawn from the many cases considered for computation :

1- The closed dead-end branched pipe in a hydraulic systems can be used as a simple means for controlling transient pressure. The proper design of branched pipe length as to be equal to the pressure source pipe length may reduce the maximum transient pressure head by about 30 % .

2- The effects of different parameters on the form of transient pressure cycles and the values of maximum pressure head rise are summarized as follows :

a- The maximum pressure head rise is obtained just upstream of the control valve and decreases as we leave the control valve towards the constant pressure head source.

b- The increase of void fraction, in the considered range, leads to a great reduction in maximum pressure head rise and a significant delay of the cyclic pressure response.

c- The liquid phase properties and in particular the viscosity has a substantial effect on the values of maximum pressure head rise. The higher the liquid viscosity the more the reduction in the maximum head rise.

d- The maximum pressure head rise in a branched system increases as the valve closure time decreases.

e- Higher transient pressures are available to larger diameter pipes, and vice versa.

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NOMENCLATURE

- a : Pressure wave speed, (m/s).
- A : Valve opening cross-sectional area at time "t", (m^2).
- D : Pipe diameter, (m).
- e : Pipe wall thickness, (m).
- E : Pipe modulus of elasticity, (N/m^2).
- f : Friction factor.
- H : Pressure head, (m).
- g : Gravitational acceleration, (m/s^2).
- K : Bulk modulus of elasticity of liquid, (N/m^2).
- n : Polytropic exponent.
- P : Transient pressure, (N/m^2).
- Q : discharge through pipe, (m^3/s).
- Re : Reynolds number.
- t,T: Time, (s).
- TC : Valve closure time, (s).
- Tcr : Critical valve closure time (=2L/a), (s).
- δt : Time interval, (s).
- X : Distance along pipe, (m).
- α : Void fraction.
- ρ : Density, (Kg/m^3).
- μ : Dynamic viscosity, ($N.s.m^{-2}$).
- ε : Pipe surface roughness, (m)

Subscripts

- A : Conditions at point A (upstream).
- B : Conditions at point B (downstream).
- P : Conditions at point P (intersection of characteristic lines)
- atm : Conditions at atmospheric pressure.
- av : Average value.
- g : Gas phase.
- l : Liquid phase.
- m : Gas-liquid mixture value.
- o : Initial steady state conditions.

Appendix- A1

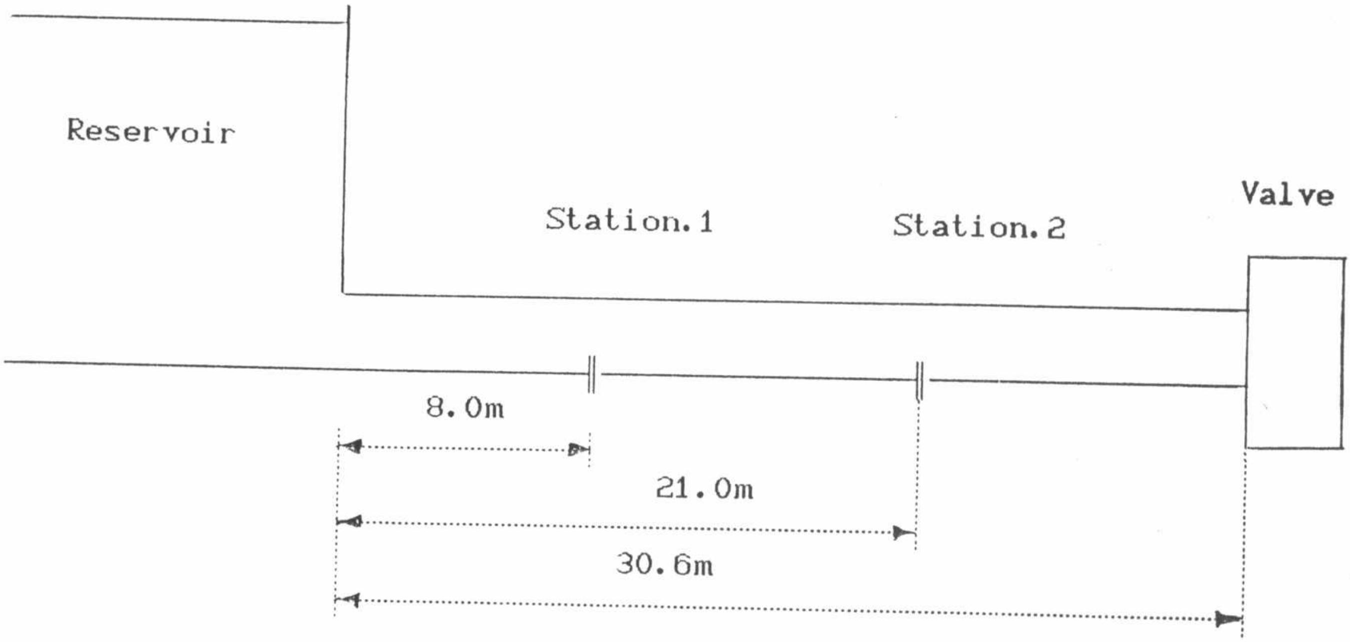
LIQUID PROPERTIES

(at temperature 20 ^{o}C and pressure 1.013 $N.m^{-2}$)

	Benzene	water fresh	shell tells oil	linsed oil
Density ($Kg.m^{-3}$)	900.0	999.8	858.2	955.0
Bulk modulus ($10^{9}.N.m^{-2}$)	1.236	1.962	1.38	1.907
Dynamic viscosity ($10^{-4}.N.s.m^{-2}$)	8.995	17.53	343.28	920.0

Appendix- A2

Experimental Model Description



Appendix-A3

Valve Discharge Characteristics