THIN-FILM MULTILAYER OPTICAL FILTERS; SENSIIVITY ANALYSIS

S.M.S. SAAD*

ABSTRACT

Thin-film multilayer optical filters play an important role in many aeronautic applications. The selection of a specific wavelength signal is very important for aircraft communication, guidance and control. Aerial imaging and reconnaissance require optical filters to reject background radiation. The performance of optical filters is affected by many parameters. The materials' indices of refraction, the layers' thicknesses and the angles of incidence are the most important parameters. During production, these parameters have tolerances within which they can be produced. Here, a sensitivity study is done to show the effect of these parameters on the filter performance. The study covered single-layer, two-layer and three-layer filters. For all the investigated filters neither a variation of up to 5% of the indices of refraction and the thickness of layers nor angles of incidence up to 10° showed considerable effect on both the peak transmission and the shape of transmission curve. The sensitivity analysis helps greatly in studying the mutual relationship between the filter performance and the tolerance of its parameters.

INTRODUCTION

The selection of a band of optical radiation spectrum to be transmitted is an essential need for many aeronautic applications. This is obtained by using optical transmitting filters .The communication between aircrafts and stations using laser signals requires filters to avoid background radiation . Also, guidance and control of aircraft born rockets and bombs using infrared and laser signals makes a great use of such filters [1]. Aircraft imaging and detection gain high resolution by using optical filters[2]. According to the selected wavelength band, the filter may be : short wave-pass, longwave-pass, or band-pass filter. An optical transmitting filter is a stack of optical thin layers with special materials, thicknesses and order chosen according to the selected wavelength band. The thicknesses of the layers of filters studied here are multiples of quarter of the central wavelength of the transmitted band [3,4]. The greater the number of layers, the higher the peak transmission and the narrower the transmission bandwidth.

The high-selection property of optical thin film filters requires them to be produced with high accuracy. The important production parameters of the filters are : the indices of refraction of films and substrate and the layers' thicknesses. The index of refraction of a thin film depends on its thickness, and method of preparation [5,6]. The film thickness depends on the precision of the thickness monitoring device[7]. Also, the

* Assistant Professor, Dept. of optoelectronics, Military Technical College, Cairo, Egypt .

angle of incidence of light affects the filter transmission. This angle depends on the degree of convergence or divergence of light.

In this paper, we investigate the sensitivity of the multiples of quarter wavelength filters to the films' parameters and angles of incidence. The investigation considers the visible band of the spectrum. Single-film, two-film and three-film filters are investigated. In the first part of the paper, we present a theoretical background. This shows that starting with Snell's law of refraction, we can find the system matrix from which the system transmittance is calculated, In the second part, the results of calculation are shown. For each filter, we start by finding the wavelength at which the transmission is maximum and calculating the value of this maximum transmission. Then, the dependence of this wavelength and maximum transmission on the indices of films and substrate, on the films' thicknesses and on the angle of incidence is studied. In the third part, we come to the conclusion of the sensitivity analysis. It showed that optical filters consisting of multiples of quarter wavelength layers show very week dependence on the layers' parameters and the angle of incidence. The peak transmission and the transmission curve shape are almost unchanged if the indices of refraction or films' thicknesses are changed by up to 5% or if the angle of incidence reaches up to 10° .

THEORETICAL BACKGROUND

Consider incident light from a non-absorbing medium of index of refraction N_0 at its boundary with an absorbing medium of complex index of refraction $N_1 = n_1 - i\,k_1$. Starting with Maxwell's equations and applying the boundary conditions, the reflected amplitude and intensity and the transmitted amplitude and intensity , Fig.1 , take the form [8]

$$\rho = (\eta_0 - \eta_1)/(\eta_0 + \eta_1) \tag{1a}$$

$$R = \rho\rho^{*} \tag{1b}$$

$$\tau = 2\,\eta_0/(\eta_0 + \eta_1) \tag{2a}$$

$$T = 4\,\mathrm{Re}(\eta_0)\,.\,\mathrm{Re}(\eta_1)/(\eta_0 + \eta_1)(\eta_0 + \eta_1)^{*} \tag{2 b}$$

If we consider a thin film of index of refraction N_1 and geometrical thickness d_1 between two media of indices N_O and N_2 , Fig. 2, it can be shown that the tangential components of the electric and magnetic fields at the boundaries are related by

$$\begin{bmatrix} E_a \\ H_a \end{bmatrix} = \begin{bmatrix} \cos\delta_1 & i\sin\delta_1/\eta_1 \\ i\eta_1\sin\delta_1 & \cos\delta_1 \end{bmatrix} \begin{bmatrix} E_b \\ H_b \end{bmatrix} \tag{3}$$

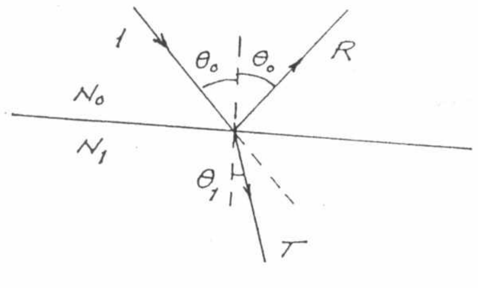

Fig.1. Light reflection and transmission at the boundary between two optical media

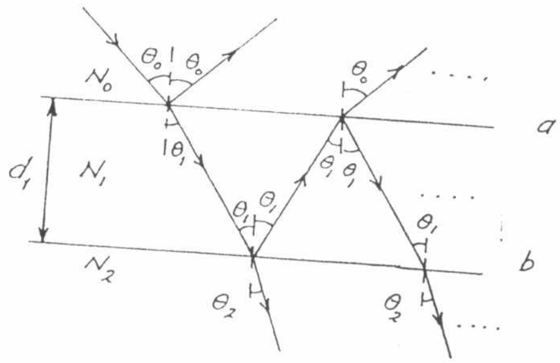

Fig.2. A thin optical film between two optical media

Fig.3. An assembly of two films

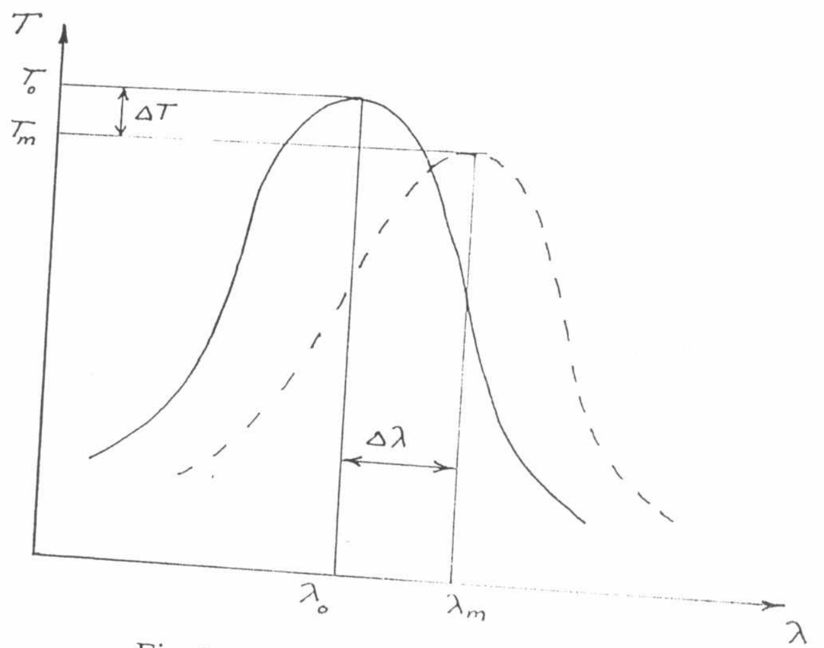

Fig.4. Characteristics of the transmission curve

Where the 2 X 2 matrix on the right - hand side is the characteristic matrix of the thin film, and the reflected amplitude and intensity by the film become those of an interface between two media of admittance η_0 and Y which means

$$\rho_r = (\eta_0 - Y)/(\eta_0 + Y) \quad (4a)$$

$$R_r = \rho_r \cdot \rho_r^{*} \quad (4b)$$

$$E_a \begin{bmatrix} 1 \\ Y \end{bmatrix} = \begin{bmatrix} \cos\delta_1 & (i\sin\delta_1/\eta_1) \\ i\eta_1 \sin\delta_1 & \cos\delta_1 \end{bmatrix} \begin{bmatrix} 1 \\ \eta_2 \end{bmatrix} E_b \quad (5)$$

which gives

$$Y = (\eta_2 \cos\delta_1 + i\eta_1 \sin\delta_1)/(\cos\delta_1 + i(\eta_2/\eta_1 \sin\delta_1)) \quad (6)$$

and the characteristic matrix of the assembly is

$$\begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} \cos\delta_1 & (i\sin\delta_1/\eta_1) \\ i\eta_1 \sin\delta_1 & \cos\delta_1 \end{bmatrix} \begin{bmatrix} 1 \\ \eta_2 \end{bmatrix} \quad (7a)$$

clearly

$$Y = C/B \quad (7b)$$

Further , if we consider two thin films of indices N_1 , N_2 and thicknesses d_1 , d_2 on a substrate of index N_3 and preceded by a medium of index N_o , Fig.3 then at the second boundary we have

$$\begin{bmatrix} E_b \\ H_b \end{bmatrix} = \begin{bmatrix} \cos\delta_2 & i\sin\delta_2/\eta_2 \\ i\eta_2 \sin\delta_2 & \cos\delta_2 \end{bmatrix} \begin{bmatrix} E_c \\ H_c \end{bmatrix} \quad (8a)$$

and using equation (3), the field components at the first boundary become

$$\begin{bmatrix} E_a \\ H_a \end{bmatrix} = \begin{bmatrix} \cos\delta_1 & i\sin\delta_1/\eta_1 \\ i\eta_1 \sin\delta_1 & \cos\delta_1 \end{bmatrix} \begin{bmatrix} \cos\delta_2 & i\sin\delta_2/\eta_2 \\ i\eta_2 \sin\delta_2 & \cos\delta_2 \end{bmatrix} \begin{bmatrix} E_c \\ H_c \end{bmatrix} \quad (8b)$$

and the characteristic matrix of the assembly will be

$$\begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} \cos\delta_1 & i\sin\delta_1/\eta_1 \\ i\eta_1 \sin\delta_1 & \cos\delta_1 \end{bmatrix} \begin{bmatrix} \cos\delta_2 & i\sin\delta_2/\eta_2 \\ i\eta_2 \sin\delta_2 & \cos\delta_2 \end{bmatrix} \begin{bmatrix} 1 \\ \eta_3 \end{bmatrix} \quad (9a)$$

as before

$$Y = C/B \quad (9b)$$

$$\rho = (\eta_0 - Y)/(\eta_0 + Y) \quad (9c)$$

$$R = \rho\rho^{*} \quad (9d)$$

For a general assembly of q-layers, the characteristic matrix is the product of the individual matrices in the correct order, i.e.

$$\begin{bmatrix} B \\ C \end{bmatrix} = \left(\prod_{r=1}^{q} \begin{bmatrix} \cos\delta_r & i\sin\delta_r/\eta_r \\ i\eta_r \sin\delta_r & \cos\delta_r \end{bmatrix} \right) \begin{bmatrix} 1 \\ \eta_m \end{bmatrix} \quad (10)$$

where η_m is the characteristic impedance of the substrate , and the values of θ for m and any r are given from θ_o and Snell's law by

$$N_o \sin\theta_o = N_r \sin\theta_r = N_m \sin\theta_m \quad (11)$$

and for non-absorbing films , the transmitted intensity (transmittance) will be

$$T = 1 - R \quad (12)$$

RESULTS OF THE ANALYSIS

First, we explain the steps of calculations, then, we give examples of the results of calculations using a single-layer, a two-layer and a three-layer filter.

The Steps of Calculations

The steps used for calculation are as follows, Fig.4 :

1- Using the nominal design parameters of the given filter, we determine the wavelength at which the transmission is maximum(λ_o) & calculate the value of this maximum transmission (T_o).

2- Using the parameters of the filter after introducing the indicated percentage error, we determine the wavelength at which the transmission is maximum (λ_m) and the value of this maximum transmission (T_m) .

3- We calculate the percentage change in transmission and wavelength from the following relations.

$$\Delta T = (T_m - T_o) / T_o \quad \% \qquad (13a)$$
$$\Delta l = (\lambda_m - \lambda_o) / \lambda_o \quad \% \qquad (13b)$$

The Results of Calculations

A single- film filter

We study a filter made of magnesium fluoride of index $n_f = 1.38$ on glass substrate of index $n_s = 1.7$, the thickness of the film is $d_1 = 996A^o$, it is found that the peak transmission is $T_o = 0.996784$ at a wavelength of $\lambda_o = 5500A^o$., Fig.5 shows the dependence of ΔT and $\Delta\lambda$ on the filter's parameters and angle of incidence.

A double -film filter

We study a filter with glass substrate of index $n_s = 1.72$, the first film is of magnesium fluoride with index $n_1 = 1.38$ and thickness $d_1 = 996\ A^o$, the second film is of silver chloride with index $n_2 = 2.06$ and thickness $d_2 = 1335\ A^o$. It is found that the peak transmission is $T_o = 1.0$ at the wavelength $\lambda_o = 6410\ A^o$. Fig .6 shows the dependence of ΔT and $\Delta\lambda$ on the filter's parameters and angles of incidence.

A three - film filter

We study a filter with glass substrate of index $n_s = 1.62$, the first film is of magnesium fluoride of index $n_1 = 1.38$ and thickens $d_1 = 996\ A^o$, the second film is of zinc sulfide of index $n_2 = 2.3$ and thickness $d2 = 1197\ A^o$, the third film is of aluminum oxide of index $n_3 = 1.76$ and thickens $d_3 -= 2344\ A^o$. It is found that the peak transmission is $T_o = 0.999996$ at the wavelength $\lambda_o = 4580\ A^o$ Fig. 7 shows the dependence of ΔT and $\Delta\lambda$ on the filters parameters and angle of incidence .

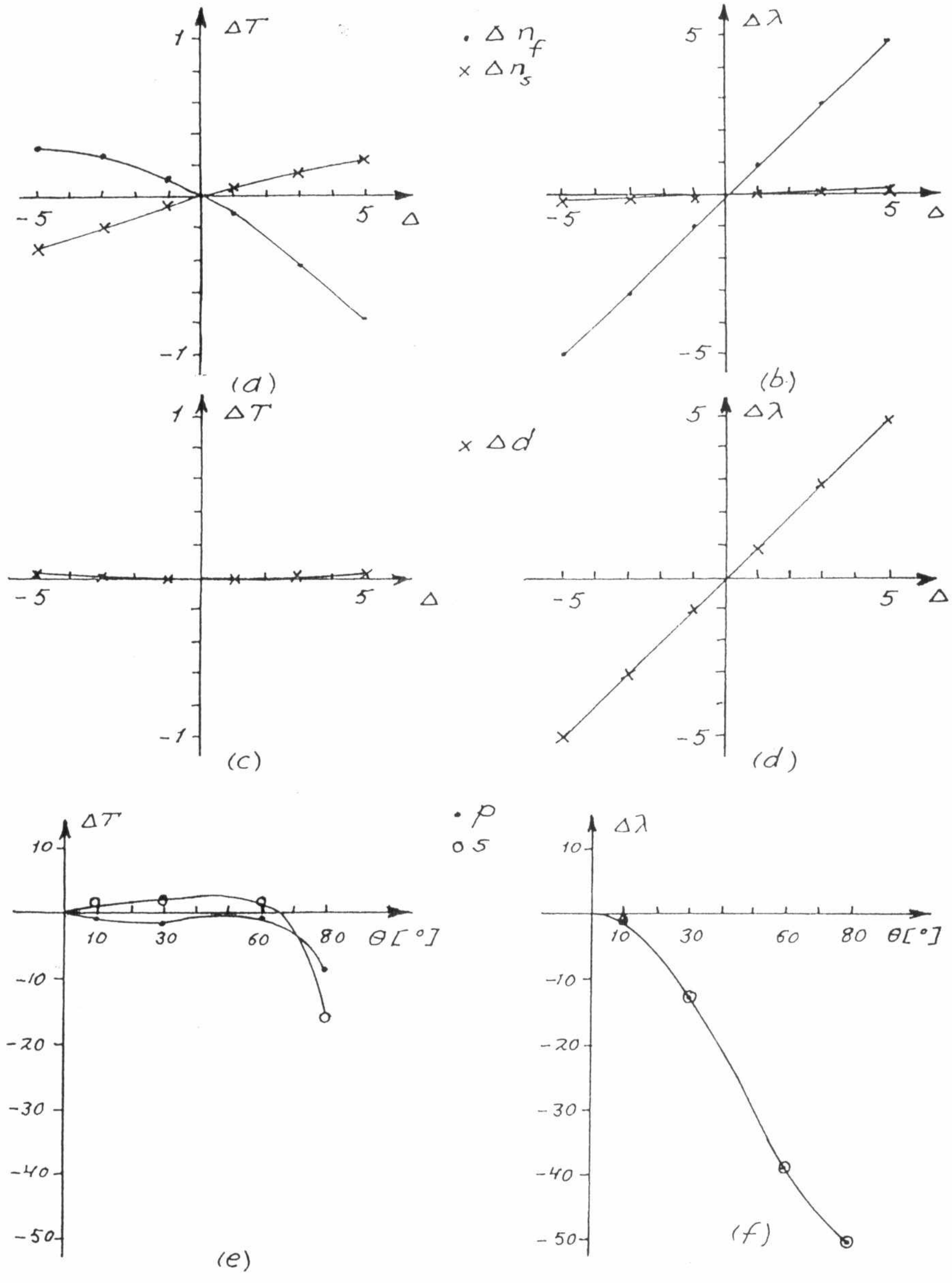

Fig.5. The dependence of ΔT & Δ λ on (a),(b) : the indices of refraction, (c),(d) : the thickness and (e), (f) : the angle of incidence for a single-film filter.

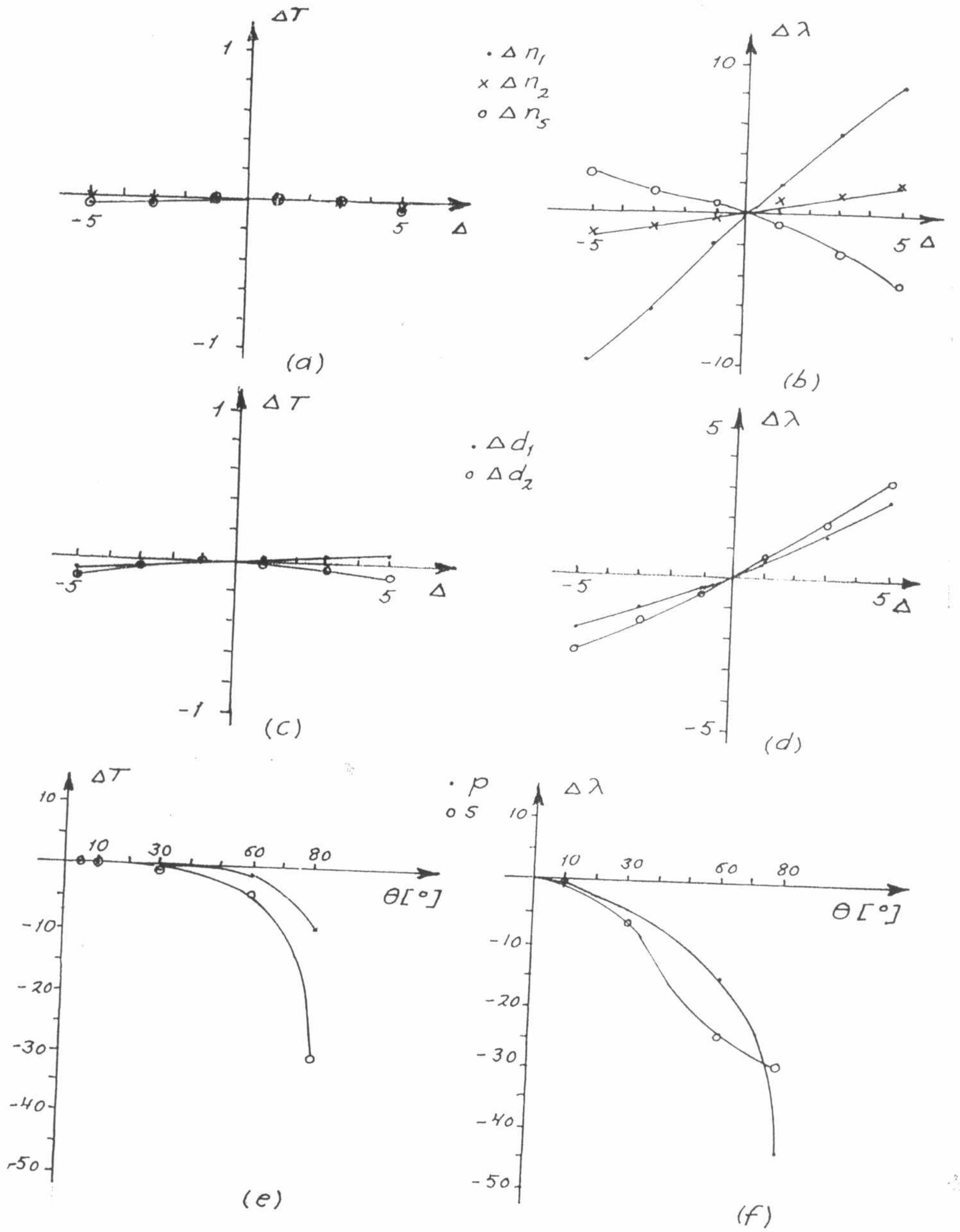

Fig.6. The dependence of ΔT &Δ λ on (a),(b) : the indices of refraction, (c),(d) : the thickness and (e), (f) : the angle of incidence for a double-film filter.

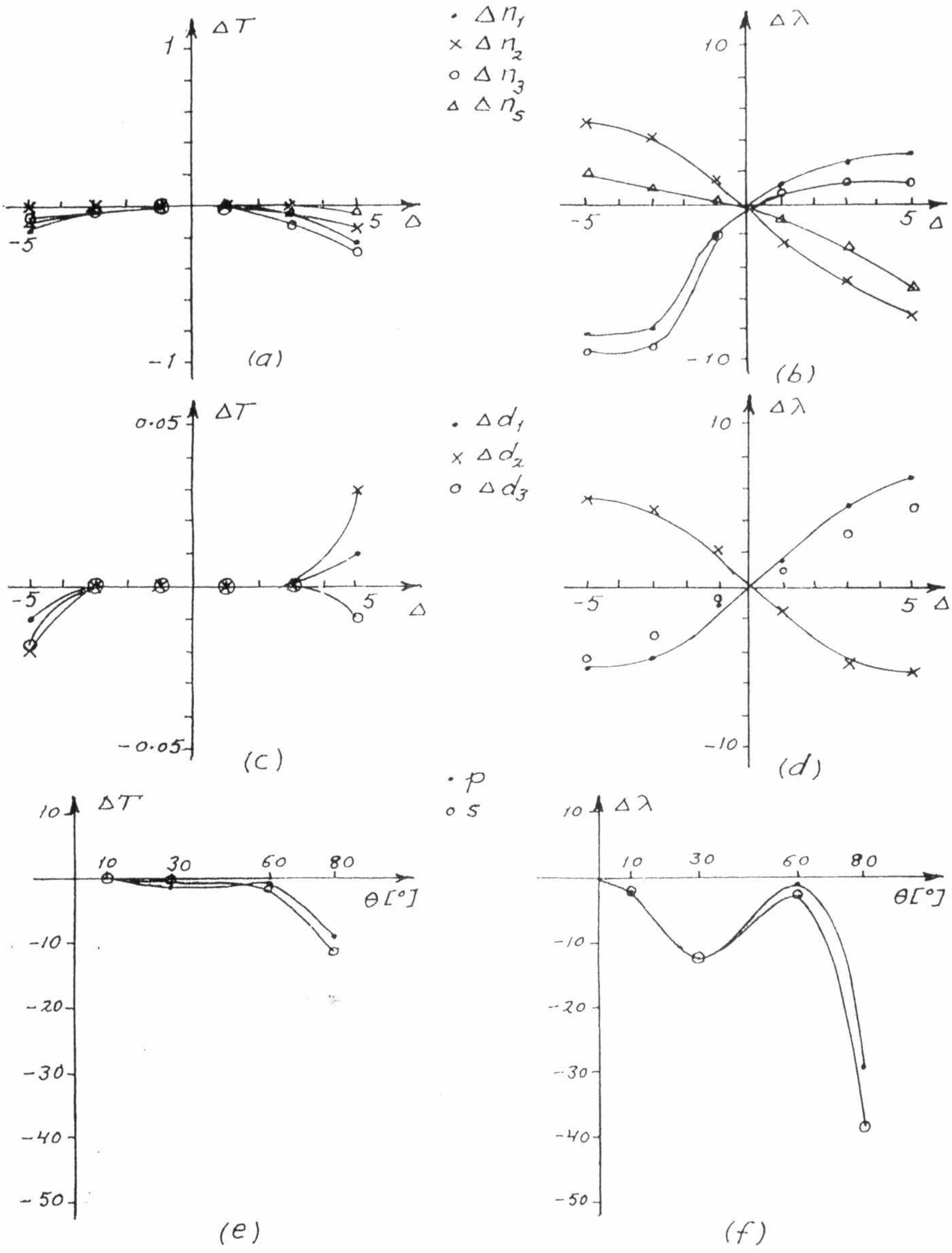

Fig.7. The dependence of ΔT & Δλ on (a),(b) : the indices of refraction, (c),(d) : the thickness and (e),(f) : the angle of incidence for a three -film filter.

CONCLUSION

The analysis of the obtained results show that the multiple quarter wavelength filter is an excellent form of transmission filters. For the investigated filters, the peak transmission is scarcely affected by a change of up to 5% in the indices of refraction of the films, in the index of refraction of the substrate or in the films' thicknesses. The result is in accordance with the result obtained by Heavens [9] for the effect of thickness variation. This property is an advantage over many other formsof optical filters. For example the filters consisting of two four-high-index layers separated by a low-index layer show-as reported by Smiley and Stuart [10] - about 30% drop of the peak transmission if a high-index layer is changed by about 5%. On the other hand, the 5% variation in the films' parameters show a variation of the central wavelength of less than 5% for the single-film filter and less than 10% for the two-film and three-film filters. This change is not to worry about concerning the filter performance as the transmission at the central wavelength remains almost unchanged, hence, the transmission curve shape remains almost unchanged.

Concerning the angle of incidence, the study shows that for all the investigated filters, an angle of up to 10^{0} does not affect the filters performance. However, as the angle increases, the performance deteriorates. For example, an angle of about 80° causes a change in the central wavelength and peak transmission of up to50% Concerning the transmission at such great angles of incidence, it is found that the component of light polarized in the plane perpendicular to the plane of incidence is more affected than the one polarized in the plane of incidence.

Finally, the shown analysis is a good tool for studying the sensitivity of the filter performance to its individual parameters. It can be used for determining the allowed tolerances in the filters' parameters for a given performance level.

REFERENCES

[1] Abdel Dayem , I.A.M, "Performance Improvement of Short-Range-Homing Guided Missiles in Infrared Noisy Conditions" , MSc. Thesis , M.T.C. , Cairo , Egypt (1990).

[2] Seely , J. F . , Holland , G . E . ,and , Giasson , J . V . ,"High-resolution Imaging of Laser-produced plasmas at a wavelength of 130 A° by a Normal Incidence Multilayer-mirror Microscope ", J . of Appl. Opt. ,No. 31 , PP 6294-6302 . (1993)

[3] Heavens, O.S., "Optical Properties of Thin Solid Films", Dover Publications, Inc., New York (1965),

[4] Saad , S.M.S. and El-Mahdy, A.M., " Design and Analysis of Three-layer Transmission Filters in the Near Infrared Band," Proc. of the Fourth ORMA Conf.,PP.425 - 433 , M.T.C.,Cairo,Egypt (1991).

[5] Driscoll, W,G. and Vauhgan,W.,"Handbook of Optics", Mc Graw-Hill Company, New York (1978).

[6] Born, M. and Wolf, E., "Principles of Optics", Pergamon Press, Fourth Edition (1970).
[7] Fengshan Zhang , K . Z . and Yan ,Y . " Problems of Controlling the Optical Thickness of Infrared Coating During Deposition " , J. Appl. Opt. , No. 28 , PP 5579 - 5582 (1993) .
[8] Macleod,H.A., "Thin-film Optical Filters", Adam Hilger Ltd. Bristol, Second Edition (1986).
[9] Heavens, O.S., "All-dielectric High-reflecting Layers", J. Opt.Soc. Am., No. 44, PP 371 - 373 (1954).
[10] Smiley, V.N. and Stuart, F.E., "Analysis of Infrared Inter ference Filters by Means of an Analog Computer", J.Opt.Soc.Am.No.53, PP1078-1083 (1963),

NOMENCLATURE

d	the geometrical thickness of the film
Δd_i	the percentage deviation in d for the i th film
E	the tangential component of the electric field
H	the tangential component of the magnetic field
k	the imaginary part of the complex index of refraction, the extinction coefficient
n	the real part of the complex index of refraction
Δn_i	the percentage deviation for the i th film in n
Δn_s	the percentage deviation in n for the substrate
N	the complex index of refraction
p	the component of polarized light parallel to the plane of incidence
R	the reflected light intensity
Re	real part of
s	the component of polarized light perpendicular to the plane of incidence
T	the transmitted light intensity
y	the optical admittance of the medium, $y = Ny_o$
y_o	the optical admittance of free space, $y_o = \sqrt{\varepsilon_o/\mu_o} = 2.6544\text{x}10^{-3}$ s
Y	the equivalent optical admittance of the thin film or film assembly
δ	the phase change due to crossing the film, $\delta = (2\pi/\lambda)\, Nd\cos\theta$
ε	the permittivity of the medium
ε_o	the permittivity of free space
η	modified optical admittance for oblique incidence, $\eta_p = y/\cos\theta$, $\eta_s = y\cos\theta$
θ	the angle of incidence, reflection or refraction
λ	wavelength of light
μ	permeability of the medium
μ_o	permeability of free space
ρ	the reflected light amplitude
τ	the transmitted light amplitude.