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Smart Composite Plate Shape Control Using Piezoelectric Materials II. Optimization

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ABSTRACT

For the aircraft industry, the ability to change and control the shape of the structures has been a challenging problem. In the present work the shape control of fiber-reinforced composite plate with embedded piezoelectric actuators and sensors is investigated. For shape control and shape optimization, an optimization algorithm based on finite element technique, is presented which formulate the error between the actual and the desired shape. The error (objective) function is the root mean square of the error between the point in the actual surface and the corresponding point in the desired surface for each element of the finite element grid. The algorithm is able to compute the optimal amount of voltage applied to each actuator to achieve minimum error function, and determine the value of the error function for different actuator(s) position for a specified amount of the applied voltage.

INTRODUCTION

This part of the analysis is directed toward shape control and optimization of the actuator(s) locations. Several works have been completed in vibration control. For example Baz et. al [1], Aditi et. al [2], Tzou [3], Anderson et. al [4], and Hagood et. al [5]. Few studies have been done in the shape control and optimization, Ghosh et. al [6] showed a model for plate shape control by using PZT. Agrawal et. al [7], developed a mathematical model for deflection using finite difference technique and estimated the optimal actuation voltages. Koconis et. al [8], developed analytical method to determine the voltages needed to achieve a specified desired shape. Based on Koconis method, a new model is developed using the finite element method to determine the voltage applied to the piezoelectric actuator to get minimum error between the desired shape and the actual shape with a prescribed actuator(s)

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placement. The mathematical model is developed based on a kinematical relation between points in the actual shape and the desired shape for each element of the finite element grid. The system of equations of the error function is formulated based on a finite element technique. The optimization algorithm is applied for the error function through a Matlab program called 'OPTSHP'. The code performs the analysis using the finite element technique and determine the plate deformation due to the applied voltage to the actuators. It is able to call Matlab optimization functions to compute the minimum voltage applied to each actuator with minimum error function and to determine the value of the error function for different actuator(s) position for a specified amount of the applied voltage.

MATHEMATICAL MODEL

Consider the problem of a plate with known original shape, actuator configuration, and the desired shape is specified. The objective is to find an actuator(s) voltage to achieve the prescribed shape which minimize the error between the desired shape and the actual shape, for a prescribed actuator(s) positions. The model is based on the small deformation theory, therefore the desired shape must be within the region of 'small deformation' from the original shape. The shape of the plate reference surface (could be top or bottom surface) is defined by a single z_0 -coordinate of point of each element of the finite element grid. The original and the desired shapes can be specified by a polynomial function in x and y such as:

$$z_o = l_o + l_1x + l_2y + l_3xy + l_4x^2 + l_5y^2 \quad (\text{length units}) \quad (1)$$

$$z_{des} = m_o + m_1x + m_2y + m_3xy + m_4x^2 + m_5y^2 \quad (\text{length units}) \quad (2)$$

where l_i and m_i are constants.

When voltage is applied to the actuator(s), the plate shape will change and a new configuration will arise which represents the actual shape. An error function is introduced which includes the difference between the actual shape and the desired one and can be written as:

$$\Lambda = \int_{\Omega_e} \bar{\Delta}^2 dx dy \quad (\text{length}^4) \quad (3)$$

where Λ is proportional to the distance between the reference surface of the desired shape and the reference surface of the actual shape, Ω_e is the element domain, and $\bar{\Delta}$ represents the difference between the z -coordinate of a point on the reference surface of the desired shape and the z coordinate of a corresponding point on the actual reference surface. The error function can also be the sum of the error at each element

$$\Lambda = \sum_{i=1}^n \Delta^2 \quad (4)$$

where Δ is account for each element of the finite element grid. From Koconis et. al [8], the error function is given by:

$$A = \int_{\Omega} [(z_0 - z_{des}) + \eta_h w + \eta_2 u + \eta_3 v]^2 dx dy \quad (5)$$

where;

$$\eta_h = 1 + \left(\frac{\partial z_{des}}{\partial x} \right) \left(\frac{\partial z_0}{\partial x} \right) + \left(\frac{\partial z_{des}}{\partial y} \right) \left(\frac{\partial z_0}{\partial y} \right) \quad (6)$$

$$\eta_2 = \left(\frac{\partial z_0}{\partial x} \right) - \left(\frac{\partial z_{des}}{\partial x} \right)$$

$$\eta_3 = \left(\frac{\partial z_0}{\partial y} \right) - \left(\frac{\partial z_{des}}{\partial y} \right)$$

are parameters which depend on the geometry of the original and desired surfaces, and $u, v,$ and w are the displacements at a point in three directions $x, y,$ and z , respectively.

FINITE ELEMENT FORMULATION

A bilinear, isoparametric, rectangular element is used to describe the shape of the element. The actual element coordinates x and y are interpolated using a linear shape function N_i which has the form;

$$N_i = 1/4(1 + \xi\xi_i)(1 + \eta\eta_i) \quad (7)$$

where ξ, η are the master element coordinates. The coordinates x and y can be transformed to the master element as:

$$x = x_c + \xi a / 2; \quad y = y_c + \eta b / 2 \quad (8)$$

where (x_c, y_c) are the plate centroid coordinates, and (a, b) are its dimensions. By substituting equation (8) in equations (1) and (2) the actual and the desired shape equations take the form:

$$z_o = l_o + l_1(x_c + \xi a/2) + l_2(y_c + \eta b/2) + l_3(x_c + \xi a/2)(y_c + \eta b/2) + l_4(x_c + \xi a/2)^2 + l_5(y_c + \eta b/2)^2 \quad (9)$$

and

$$z_{des} = m_o + m_1(x_c + \xi a/2) + m_2(y_c + \eta b/2) + m_3(x_c + \xi a/2)(y_c + \eta b/2) + m_4(x_c + \xi a/2)^2 + m_5(y_c + \eta b/2)^2 \quad (10)$$

(z_o could be taken as zero, for a flat plate, or first order polynomial). By using the chain rule we get:

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial z}{\partial \eta} \frac{\partial \eta}{\partial x} \quad (11)$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial z}{\partial \eta} \frac{\partial \eta}{\partial y}$$

where z can be z_0 or z_{des} . Equations (11) can be used to evaluate the three geometric parameters, η_1 , η_2 , and η_3 which are expressed in equation (6). Thus the objective function equation (5) can be written as:

$$\Lambda = \int_{\Omega} [(z_0(\xi, \eta) - z_{des}(\xi, \eta)) + \eta_1(\xi, \eta)w + \eta_2(\xi, \eta)u + \eta_3(\xi, \eta)v]^2 dx dy \quad (12)$$

The electric excitation may be factored out of displacements u, v , and w in equation (12) by introducing the vectors \tilde{u}, \tilde{v} and \tilde{w} which can be defined as follows.

The degrees of freedom of the system can be given in short form as:

$$\{q\} = [K^*]^{-1} \{F^*\} \quad (13)$$

where $\{q\}$ is the array of the degrees of freedom $\{u, v, \phi_x, \phi_y, w, w_x, \text{ and } w_y\}$, $[K^*]^{-1}$ is the structure stiffness matrix inverse, and $\{F^*\}$ is the applied electric and mechanical loads vector (equations (44) and (47) in part I). The displacement u, v , and w are defined via the interpolation shape function as:

$$u = \sum_{i=1}^4 N_i u_i; \quad v = \sum_{i=1}^4 N_i v_i; \quad \text{and} \quad w = \sum_{i=1}^4 f_i w_i \quad (14)$$

where u_i, v_i , and w_i are the displacements at the nodal points in the x, y , and z direction, respectively, which can be picked out from the deformation array $\{q\}$ given in equation (13) as follows;

$$u_i = \tilde{K}_u \{F^*\}; \quad v_i = \tilde{K}_v \{F^*\}; \quad \text{and} \quad w_i = \tilde{K}_w \{F^*\} \quad (15)$$

where \tilde{K}_u, \tilde{K}_v , and \tilde{K}_w are the flexibility influence coefficients rows corresponding to the displacements u_i, v_i , and w_i in the global matrix inverse $[K^*]^{-1}$. By substituting u_i, v_i and w_i in equation (14) one can obtain;

$$\tilde{u} = \sum_{i=1}^4 N_i u_i = \sum_{i=1}^4 N_i \tilde{K}_u \{F^*\} \quad (16)$$

$$\begin{aligned}\tilde{v} &= \sum_{i=1}^4 N_i u_i = \sum_{i=1}^4 N_i \tilde{K}_v \{F^o\} \\ \tilde{w} &= \sum_{i=1}^4 f_i w_i = \sum_{i=1}^4 f_i \tilde{K}_w \{F^o\}\end{aligned}$$

where; f_i is the shape function used for the deflection w at node i , N_i is the shape function used for the displacement u and v at node i , and \tilde{u}, \tilde{v} , and \tilde{w} are the displacements in x, y , and z directions which are computed from the finite element analysis of the plate. These displacements are functions of electric excitation term applied to each actuator. By using \tilde{u}, \tilde{v} and \tilde{w} values instead of u, v , and w in equation (12), the objective function takes the form;

$$\Lambda = \int_{\Omega} \left[(z_0(\xi, \eta) - z_{des}(\xi, \eta)) + \eta_1(\xi, \eta)\tilde{w}(V) + \eta_2(\xi, \eta)\tilde{u}(V) + \eta_3(\xi, \eta)\tilde{v}(V) \right]^2 dx dy \quad (17)$$

which is function of the amplitude of the electric potential distribution (V in volts).

NUMERICAL INTEGRATION

Equation (17) can be integrated numerically using Gauss-Legendre quadrature. A full integration technique of 3×3 Gauss points is used for each element. Thus the error function for an element can be written as;

$$\Lambda = \int_{-1}^{+1} \int_{-1}^{+1} \left[(z_0(\xi, \eta) - z_{des}(\xi, \eta)) + \eta_1(\xi, \eta)\tilde{w}(V) + \eta_2(\xi, \eta)\tilde{u}(V) + \eta_3(\xi, \eta)\tilde{v}(V) \right]^2 |J| d\xi d\eta \quad (18)$$

where $|J|$ is the determinant of the Jacobian matrix. The structure objective function is defined as the summation of the error functions for each element such as[9]:

$$\bar{\Lambda} = \sum_{i=1}^m \Lambda \quad (19)$$

subjected to:

$$\text{Lower limit} \leq V \leq \text{Upper limit}$$

where m is the number of elements of the finite element grid.

OPTIMIZATION ALGORITHM

A Matlab function f_{min} and f_{mins} are used to find the minimum of a scalar function. The f_{min} function is used to minimize a function of one variable on a fixed interval. The function is used to optimize the objective function in case of constant voltage applied to the each actuator (i.e. V is scalar). The function f_{mins} is used when a different values of the voltages are applied to the piezoelectric actuators (V is a vector).

SOLUTION PROCEDURE

To solve an optimization problem, the flow chart of solution procedure was developed as shown in Figure 1. The procedure is as follows:

* Input the structural data such as plate and patches dimensions, materials properties (graphite epoxy and piezoelectric), boundary conditions and the applied voltages to the actuator(s).

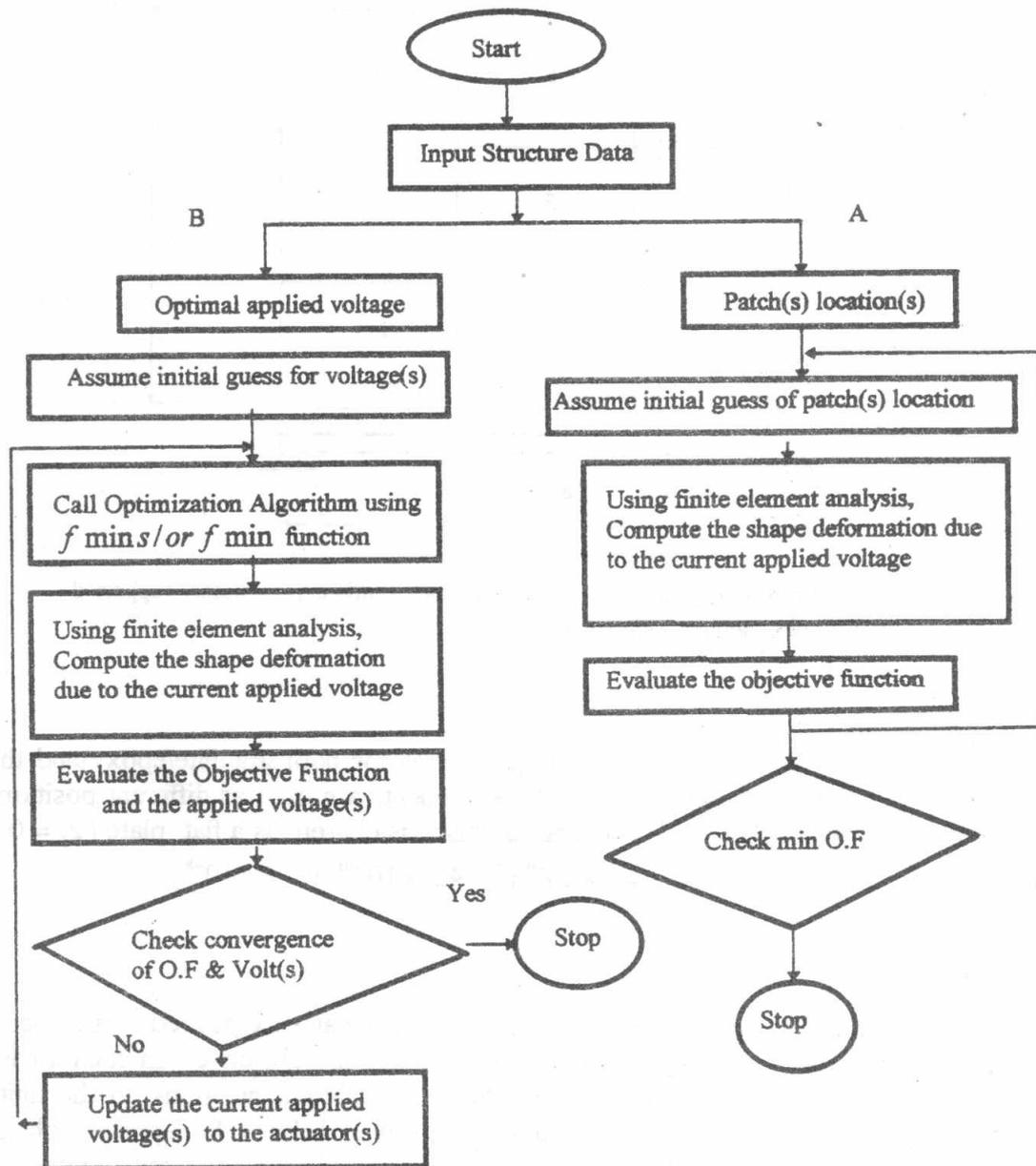
Path A: Location of the patch(s);

1. Assume initial guess of the patch(s) location on the finite element grid.
2. Compute the plate shape deformation using the finite element analysis, due to the current applied voltage(s) and patch(s) location.
3. Evaluate the objective function value at the current voltage(s) and patch(s) location.
4. Repeat steps 1-3 for each patch(s) position in the finite element grid and compute the corresponding objective function each time
5. Compare the values of the objective functions for each patch(s) position, the minimum value of the objective function means, the minimum error between the actual shape and the desired shape at the corresponding patch(s) position for a prescribed applied voltage.

Path B: Optimal voltage(s) applied to the actuator(s) to minimize the objective (error) function.

1. Assume initial guess of voltage applied to the actuator(s) at a prescribed position.
2. Call optimization algorithm using a Matlab function f_{mins} or f_{min} .
3. Compute the plate deformation due to the current voltage using finite element analysis.
4. Evaluate the objective function and applied voltage(s).
5. Check the convergence of the objective function and voltage(s), if converge the program will terminate.
6. Otherwise, update the applied voltage(s) to the actuator(s) and the go to step 2 in path B and repeat the procedure.

Figure 1: Flow chart of solution procedure



VALIDATION

A simply supported square fiber-reinforced composite plate with three layers (0⁰/90⁰/0⁰) and two piezoelectric layers patches on the top and bottom surfaces was used. The length to

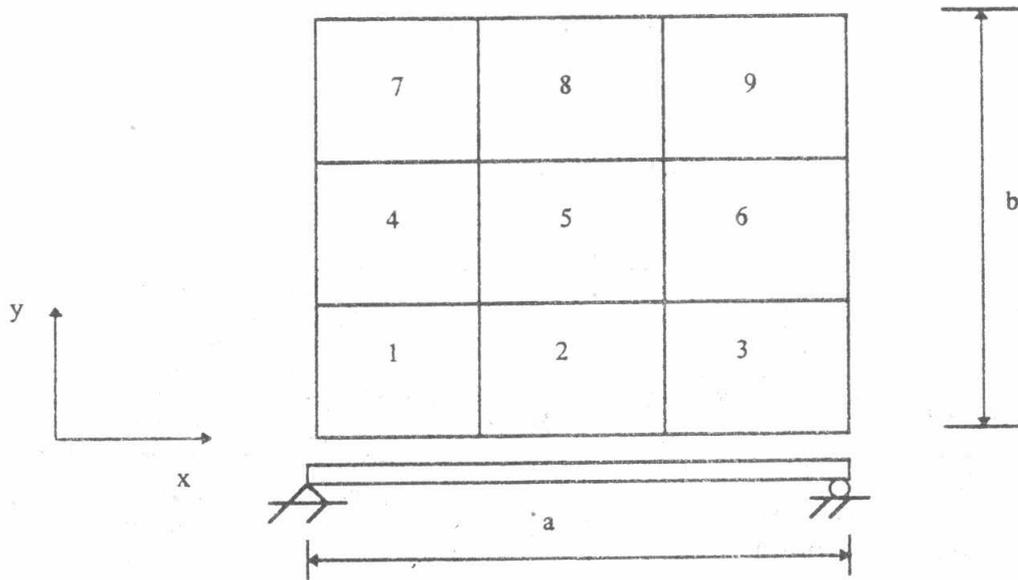


Figure 2. Elements numbering for different actuators positions for a simply supported composite plate ($a = b$).

thickness ratio is equal to fifty and the materials properties for both graphite/epoxy and the PVDF material are the same as used in part I. Nine elements are used as different positions for the actuators as shown in Figure 2. The original shape is chosen as a flat plate ($z_0 = 0$), and the desired shape is selected as: $z_{des} = 1 \times 10^{-9} x^2 - 4.2 \times 10^{-10} xy + 6 \times 10^{-8} y^2$.

Example 1.

In path A, when the minimum error between the actual shape and the desired shape due to specified amount of the applied voltage to the actuator is the only objective. In such a case the error values are computed by placing the actuator(s) at different positions on the finite element grid (Figure 2). The error function values are given in Table 1 for applied voltage equal 100 volts to each actuator position. The applied voltage and the actual and the desired transverse deflections (in-plane deformation are not include) are computed for two cases. Figures 3 shows the result when one actuator is used at element number 5. When two actuators are placed at the elements number 2 two and 8, the error is shown in Figure 4. It is shown from the Figures that the error between the actual and the desired transverse deflection is smaller in case of using two actuators than the case of using one actuator.

Example2.

For path B as shown in Figure 1., the minimum error function with optimal applied voltage(s) to the actuator(s) is computed, the Matlab f min function is used with certain upper and lower limits of the applied voltage such as $-100 < V < 100$. The optimal voltage applied to the actuator and the corresponding minimum value of the error function for different actuator positions on the grid are given in Table 2. Table 3 shows the results for two actuators used at a time. For the case of four actuators used at a time the voltages and error function values are given in Table 4. The results in both cases are obtained by using the Matlab function f mins with unconstrained value on the applied voltages. The results show that the more actuators used the minimum error obtained with optimal applied voltage(s) with a dependence on the selected actuator(s) positions. The reported values are global minimum. This was accomplished by applying different values of voltages (i.e. -500 to 500 Volt.) to the actuator and determining the numerical values of the objective function at these voltages. The minimum value of the error function found was identical to the value computed by the optimization algorithm.

CONCLUSION

An optimization algorithm based on a finite element technique is developed. Kinematical relations were used to formulate the objective (error) function which is the summation of the mean square error between a point on the actual surface and a corresponding point on the desired surface for each element of the finite element grid. The optimal voltage applied at each actuator to achieve a specified shape with minimum error function was computed. A Matlab code 'OPTSHP' is developed in conjunction with Matlab functions which gave a very satisfactory results. The code is able to determine the value of the error function for different actuator(s) positions for a prescribed amount of the applied voltage(s). The model is applied for two cases either constant voltage applied to the actuator(s) or different voltages applied to each actuator. It was demonstrated from the examples that the desired shape is made closely to the actual shape which satisfy the small deformation theory. The solution of the problem was global optimal and unique. The procedure proposed in this work was implemented for a composite plate with any number of piezoelectric actuators. The procedure has been tested for more general layout of actuators. It can be conclude that the model can be used for subsequent algorithm for actuator(s) positions using a neural networks on line with optimization algorithm, and for a closed loop shape control. Also the model can be extended to a structure having a rigid body motion.

Table 1: Error function values at different actuator positions for applied voltage equal 100 volt.

Actuator Position	Error Function Value
Element # 1	$f = 5.87619804 \text{ e-18}$
Element # 2	$f = 5.05453907 \text{ e-18}$
Element # 3	$f = 4.44646994 \text{ e-18}$
Element # 4	$f = 7.68074043 \text{ e-18}$
Element # 5	$f = 9.30776749 \text{ e-18}$
Element # 6	$f = 4.28947367 \text{ e-18}$
Element # 7	$f = 5.86952426 \text{ e-18}$
Element # 8	$f = 5.04220635 \text{ e-18}$
Element # 9	$f = 4.44063580 \text{ e-18}$

- The applied voltage at each actuator is 100 volt.

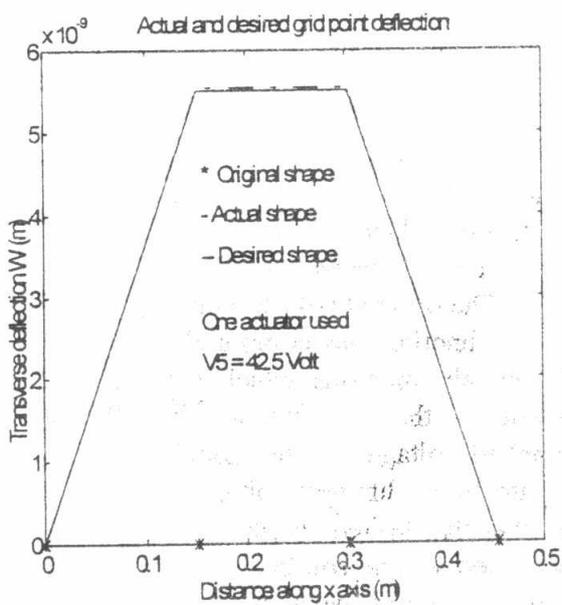


Figure 3: Actual and desired transverse deflection at the grid point for actuator at fifth element.

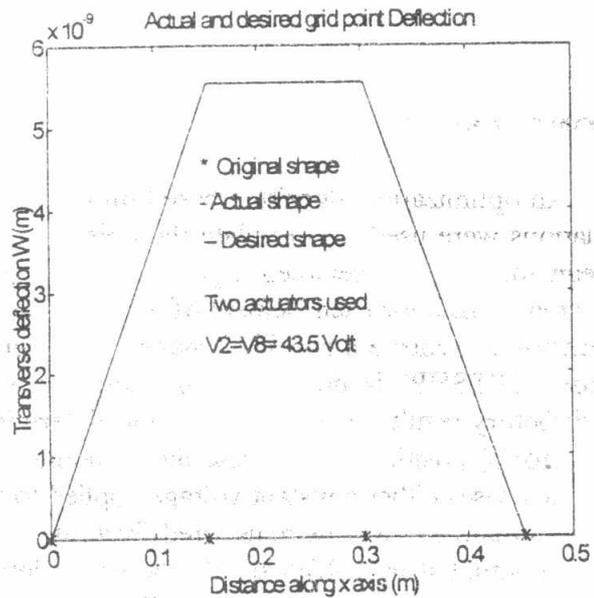


Figure 4: Actual and desired transverse deflection at the grid point for actuators at elements two and eight

Table 2: Optimal applied voltage and error function at different actuator positions

<i>Actuator Position</i>	<i>Minimum Applied Voltage (Volt.)</i>	<i>Minimum Error Function</i>
Element # 1	V = 78.226	f = 5.83495108e-18
Element # 2	V = 70.944	f = 4.78995246e-18
Element # 3	V = 99.999	f = 4.44647064e-18
Element # 4	V = 34.161	f = 5.88349351e-18
Element # 5	V = 37.346	f = 4.74690856e-18
Element # 6	V = 75.062	f = 4.03165923e-18
Element # 7	V = 78.611	f = 5.82972138e-18
Element # 8	V = 71.141	f = 4.78117803e-18
Element # 9	V = 99.999	f = 4.44063651e-18

Table 3: Optimal applied voltages and error functions for the case of pair of actuators used at a time

<i>Actuators Positions</i>	<i>Minimum Applied Voltages (Volt.)</i>	<i>Minimum Error Function</i>
Elements # 1 & 7	V ₁ = 41.612 V ₇ = 47.265	f = 5.78007896e-18
Elements # 2 & 8	V ₂ = 35.943 V ₈ = 39.934	f = 4.74678213e-18
Elements # 3 & 9	V ₃ = 85.710 V ₉ = 90.547	f = 4.0583368e-18
Elements # 1 & 3	V ₁ = -26.368 V ₃ = 175.055	f = 4.10114654e-18
Elements # 4 & 6	V ₄ = -8.231 V ₆ = 79.660	f = 4.01544062e-18
Elements # 7 & 9	V ₇ = 26.0895 V ₉ = 175.193	f = 4.09288468e-18
Elements # 1 & 9	V ₁ = -17.007 V ₉ = 170.053	f = 4.10970645e-18
Elements # 3 & 7	V ₃ = 169.712 V ₇ = -16.138	f = 4.12087738e-18

Table 4: Optimal applied voltages and error functions for the case of four actuators used at a time

<i>Actuators Positions</i>	<i>Minimum Applied Voltages (Volt.)</i>	<i>Minimum Error Function</i>
Elements # ^s 1, 3, 7 & 9	$V_1 = -21.060$ $V_3 = 98.104$ $V_7 = -16.473$ $V_9 = 101.482$	$f = 4.0531138e-18$
Elements # ^s 2, 4, 8 & 6	$V_2 = 18.404$ $V_4 = -29.429$ $V_6 = 65.262$ $V_8 = 22.349$	$f = 4.01531283e-18$

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