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LINEAR MODEL EVALUATION OF COMMAND GUIDANCE SYSTEM

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ABSTRACT:

The primary design of a missile control system is commonly derived from a linearized model of that system. Gain selection is accomplished by examining both the performance and the relative stability of the linear representation of the system at various operating conditions given by incidence angles, mach number and flight altitude. Most often, classical phase and gain margins are used as measures of relative stability. The linearized model assumes the linearity of hardware comprising the guidance loop and the missile dynamics. The reliability of the linearization assumptions determines the robustness and performance of the primary design. This paper is devoted for the study of the linearization assumptions evaluation. A typical command guidance system is considered as a case study. In spite of the nonlinear behavior of the various subsystems comprising the guidance system, the guidance-commands limiters are assumed the only nonlinear elements in the guidance loop due to their obvious direct effect on the guidance process. Computation analysis shows that the effect of these limiters appears only during the early guidance period which does not exceed 5 percent of the entire flight time. The missile dynamics nonlinearities are also studied. A comparison between the results of the six-degree-of-freedom nonlinear model and the linearized model is carried out. Comparison shows that the linear dynamical model can be relied-on in cases where sudden target maneuver or sudden variation of flight environmental conditions are not considered. However, during steady guidance periods, the linear model results can be fairly adopted.

INTRODUCTION

Guided missile systems are generally nonlinear systems. The nonlinear behavior is caused by the nonlinearity of parts of the hardware and the nonlinearity of the equations of the missile motion [1]- [3]. For nonlinear systems, the concept of the transfer function is not valid. Thus, most of the linear control system design and analysis techniques are not applicable. Therefore; the theoretical design and analysis of missile control systems require certain assumptions. A traditional assumption is one of the hardware linearity i.e.; electronic systems, missile servos, measuring devices, and equations of motion. Indeed, linearity is a necessary constraint for one to use Laplace operator method to analyze the system response [1].

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The general method of designing a missile control system is to consider a typical speed or mach number and flight height. A set of aerodynamic derivatives for zero incidence can be considered in the system design with the assumption that the missile is exercised through small perturbations about zero incidence. Obviously, the control system is designed to meet a specific steady state and transient behavior. A small incidence is then assumed and all the calculations are repeated. The calculations are repeated many times for different combinations of incidence angle, mach number, and height. If the design is satisfactory at all these points, then it is satisfactory at all the intermediate points. The effect of varying C.G. position, mass, and inertia is finally considered [1], [6], [8] and [9].

After the theoretical design of the control system is established, accurate theoretical analysis of that system is initiated. For large nonlinear systems, the analytical analysis is an elaborate task. computer numerical simulation is thus considered the practical tool for the designed system evaluation, analysis, and development[1]. For an accurate estimation of the complete system behavior , the six-degree-of -freedom motion for the missile has to be considered. As well, accurate representation of the individual components that comprises the guidance loop should be involved. In situations where the practical experimentation is an expensive process, extreme accuracy in the system simulation is required. In this case, parts of the system that are hard to model mathematically, are physically inserted in the simulation model. The simulation model-in that case- is known as hardware-in-the-loop model [7].

On the other hand, the design of lateral autopilot is based on lateral channels decoupling (yaw and pitch). Then, the roll channel is designed separately [1]. This decoupling facilitates dynamical equations linearization procedure. However, in a very recent work , a linearization procedure of the time invariant model for fully coupled, high angle of attack six-degree-of-freedom symmetrical missile in trim is presented [9]. The definition of trim is that the moments acting on the missile are zero and the time-rate of change of the incidence angles is zero.

In this paper, the linarization assumptions cited before are analyzed. The analysis is carried out on a command guidance system. The analysis of the present guidance system covers the system hardware and the dynamical equations of the missile flight. The system hardware involves many level-limitation nonlinearities. The effect of these nonlinearities is studied. As well, linearizion of the equations of motion is considered. A comparison between the linear and nonlinear models is carried out. Except for the initial period, analysis reveals that, ignoring the nonlinear limitation of the guidance error signals is of minor importance since the error signals rarely exceed their limit level throughout the flight. This result is particular for the considered system and may not be valid for other systems. In the mean time, the linearized dynamical model of the missile motion can effectively be adopted in the prediction of the missile flight behavior as

long as the guidance error signals are relatively small. During the transient periods of guidance (initial guidance period and sudden target maneuver), the linear model fails to be an effective means for system evaluation. These findings indicate that the control system design based on the linearized model has to be carefully tuned to accommodate the system response during the non-steady guidance periods and high target maneuvers.

GUIDANCE SYSTEM ANALYSIS

The guidance system of concern is a command guidance system. Fig. 1 shows a simplified block diagram of the guidance loop. The angular channel receiver represents the guidance radar which tracks the engaged target and generates signals correspond to the missile and target positions in space. Based on these data and on the predefined guidance method, the guidance computer generates the guidance commands that steer the missile to follow the desired trajectory for target interception [3]. The generated commands are coded and put in a form suitable for transmission through wireless data link in the guidance transmission system. The guided missile-during its flight- receives and decodes the coded guidance commands. These described subsystems are comprised of nonlinear and linear elements. In this work, the input-output relationship of these subsystems is considered linear. However, more practical results would be obtained if their accurate behavior is-instead- considered.

The missile is aerodynamically controlled via two rear pairs of control fins. The missile control system is of acceleration control type. Thus, the guidance commands represent the missile demanded lateral acceleration. The guidance commands drives the control fins which generate an unbalanced moment. This moment turns the missile board which results in additional normal force that steers the missile in the proper direction as demanded.

In command guidance systems, many guidance methods can be employed [10]; however, in the present work, the three-point-guidance method is only adopted. In this method, the missile should always be on the line joining the target and the guidance station (LOS).

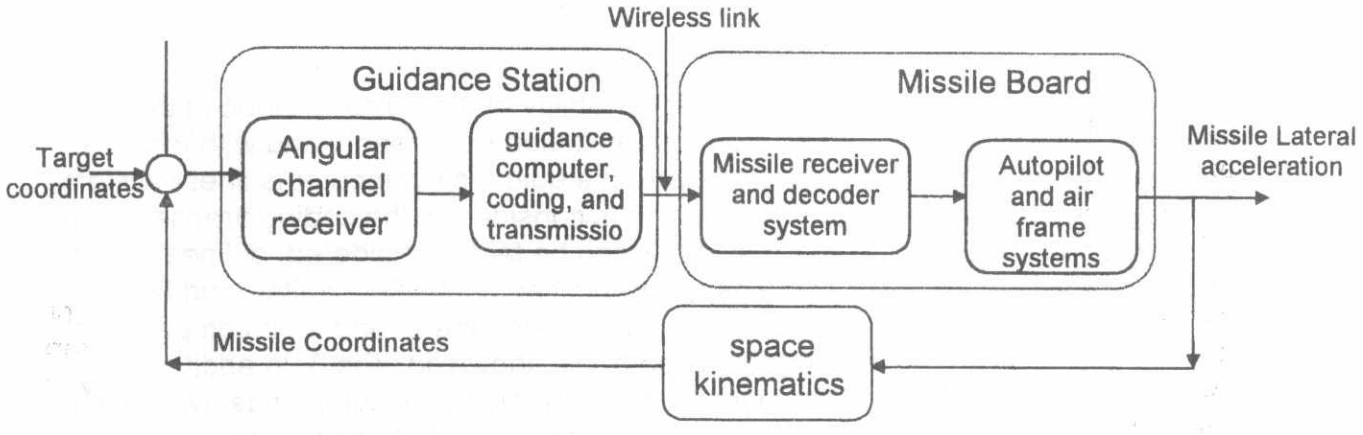

Fig. 1. Simplified block diagram of the command guidance system.

The guidance loop sin Fig. 1 involves linear and nonlinear subsystems. The angular channel receiver output is proportional to the angular difference between the target and missile line-of-sight (LOS) direction. Fig. 2 shows the vertical plane projection of the missile and target instantaneous locations with respect to the guidance point O.

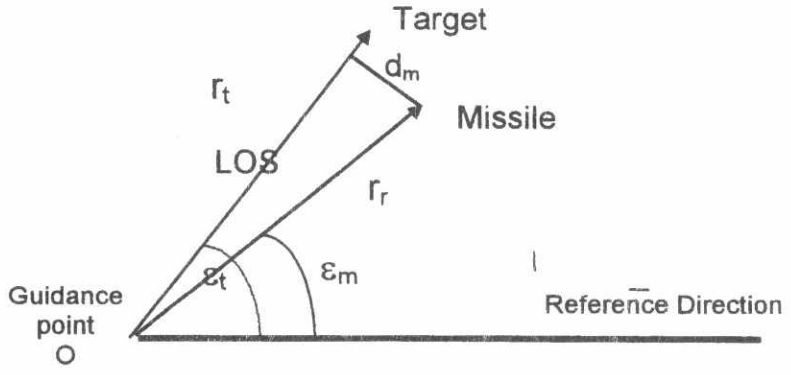

Fig. 2. Missile and target positions in the vertical plane

Thus, the angular channel receiver output V_{or} for vertical plane missile deviation from the LOS can be written as:

$$V_{or} \propto \Delta\varepsilon = \varepsilon_t - \varepsilon_m . \qquad (1)$$

The guidance system generates the guidance commands that are proportional to the missile linear deviation from the target LOS (d_m) ; i.e.,

$$\chi \propto V_{or} \cdot r_m , \qquad (2)$$

where r_m is the missile range measured from the guidance point O. The guidance system usually involves some sort of limitation on the guidance commands levels, thus its output χ_{co} can be written as

$$\begin{aligned} \chi_{co} &= \chi && \text{for } |\chi| < |\chi_{limit}| \\ &= +\chi_{limit} && \text{for } \chi > +\chi_{limit} \\ &= -\chi_{limit} && \text{for } \chi < -\chi_{limit} \end{aligned} \qquad (3)$$

This limitation is necessary for the stability of the guidance loop in the early stage of the guidance. Apart from this limitation nonlinearity and with the three-point guidance method being employed, the guidance computer is linear system. As long as the input-output relation ship is considered, the coding, transmission, missile receiver, and decoding systems can be fairly considered of linear nature too. The missile's autopilot contains electronic servoamplifier, limiter, and fin servo actuator in the forward path. Rate gyro and accelerometer and their compensation circuits are located in the feedback path as shown in Fig.3. In addition to the limitation considered during the generation of the guidance commands, two limiters are included-on the missile board - in the forward path to ensure the stability of

the airframe during the execution of commands. The first is the level limitation of the acceleration error signals. This limitation is necessary for the airframe stability during the initial guidance phase where the missile is highly deviated from its nominal trajectory and the demanded acceleration is high as well. The second limitation is the mechanical limitation of the control fins to insure that the instantaneous high turn rates of the airframe will not affect its stability.

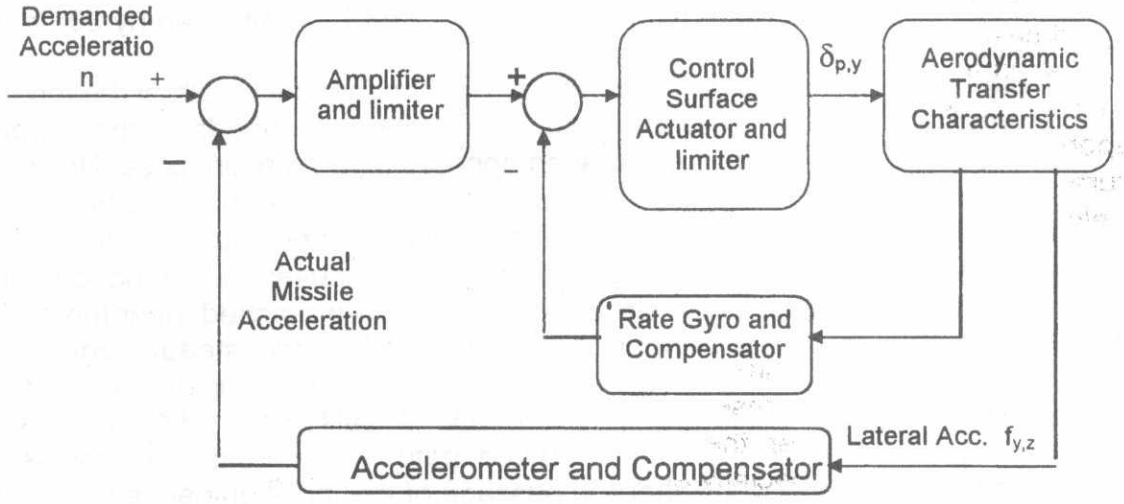

Figure 3. Block Diagram of the lateral Autopilot and airframe.

The operation of the guidance loop shown in Fig. 1 is simulated via a computer code written with Borland C [10]. Modular concept is considered during the code development. The differential equations pertinent to the missile CG motion in the space and the rotation of airframe around CG are solved numerically via Rung-Kutta 4 method. The linear subsystems are simulated by their transfer functions. The guidance system is represented by the equations relating the guidance commands to the missile and target coordinates in space as given by Eqs.1 to 3. The electronic systems that constitute the angular channel receiver, coding, transmission and decoding blocks are considered by their gains; since the delays involved in these systems can be neglected with respect to the missile inertia.

For the purpose of code verifications, zero commands are issued to the missile throughout its flight time. This results in a trajectory identical to the conventional ballistic trajectory. Furthermore; constant acceleration command is issued in the horizontal plane. In response, the missile flies a circular trajectory with radius identical to that obtained from the point mass model.

In order to analyze the effect of nonlinearities caused by the presence of various limiters in the forward path of the guidance loop, severe engagement scenarios are considered; where high speed and highly maneuvering target is encountered. Fig. 4 shows one sample of the obtained results, the vertical plane trajectory of a missile launched against an approaching maneuvering target which executes 10 m/sec^2 maneuver. The target speed is 350 m/sec. The terminal miss

distance is found to be 219 meters. The generated guidance commands and the subsequent rudder deflection are displayed versus time in Fig. 5. The y-axis of the figure represents the guidance commands in volts and the rudder deflection in degrees; simultaneously. It is clear from the figure that the guidance commands attains their limit value immediately after the operation of the guidance loop. The limitation period is relatively short compared with the entire flight time (does not exceed 5 percent). Although the guidance commands attains their limit values, the control rudders deflection do not exceed their allowed limit given by 20° in the present case.

This is attributed to the presence of rate gyro in the feedback path at the input of control surfaces actuator. The presence of this rate gyro speeds up the airframe response to the extent where no saturation appears in the present case. However, saturation of the fin actuator could happen when a maximum commanded acceleration is required in the presence of small dynamic pressure condition. This situation means that instantaneous high missile maneuver is demanded at high flight altitudes which is not usual since high altitudes are reached near the end of engagement with the guidance loop being operated in the steady region. In addition to the case presented in Figs 4 and 5, several hard engagement scenarios are then studied (not presented). The study reveals that disregarding the limitations spreaded over the entire forward path in the guidance loop is an adequate assumption; specially after the passage of the initial guidance phase and the compensatoin of the large initial errors in the guidance loop.

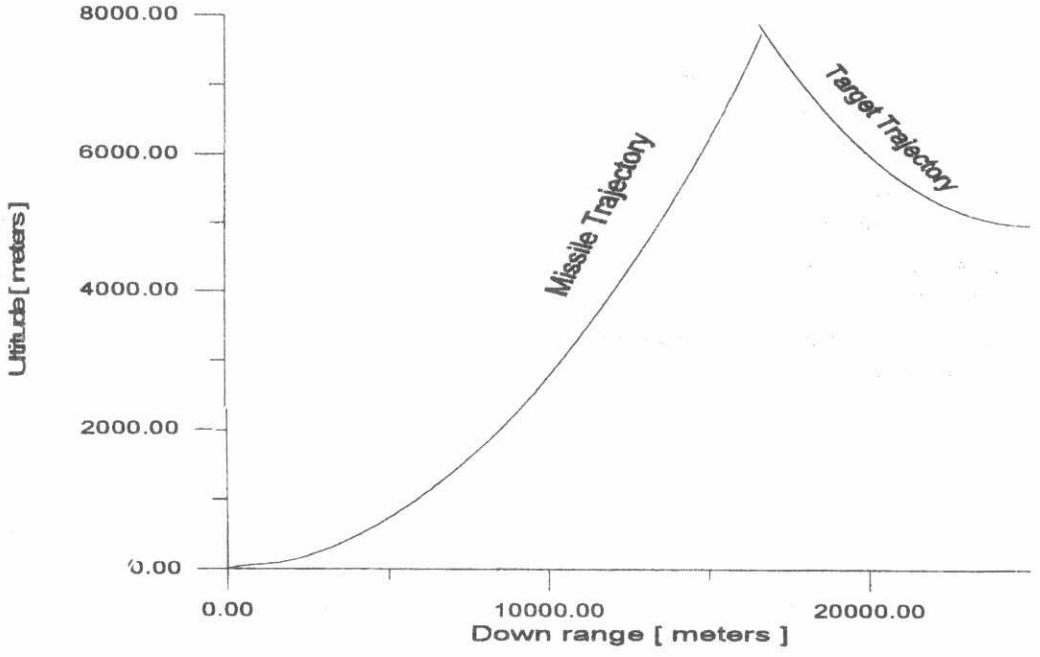

Fig.4. Missile and target trajectories for the approaching maneuvering target with speed 350 m/sec and 10 m/sec^2 normal acceleration.

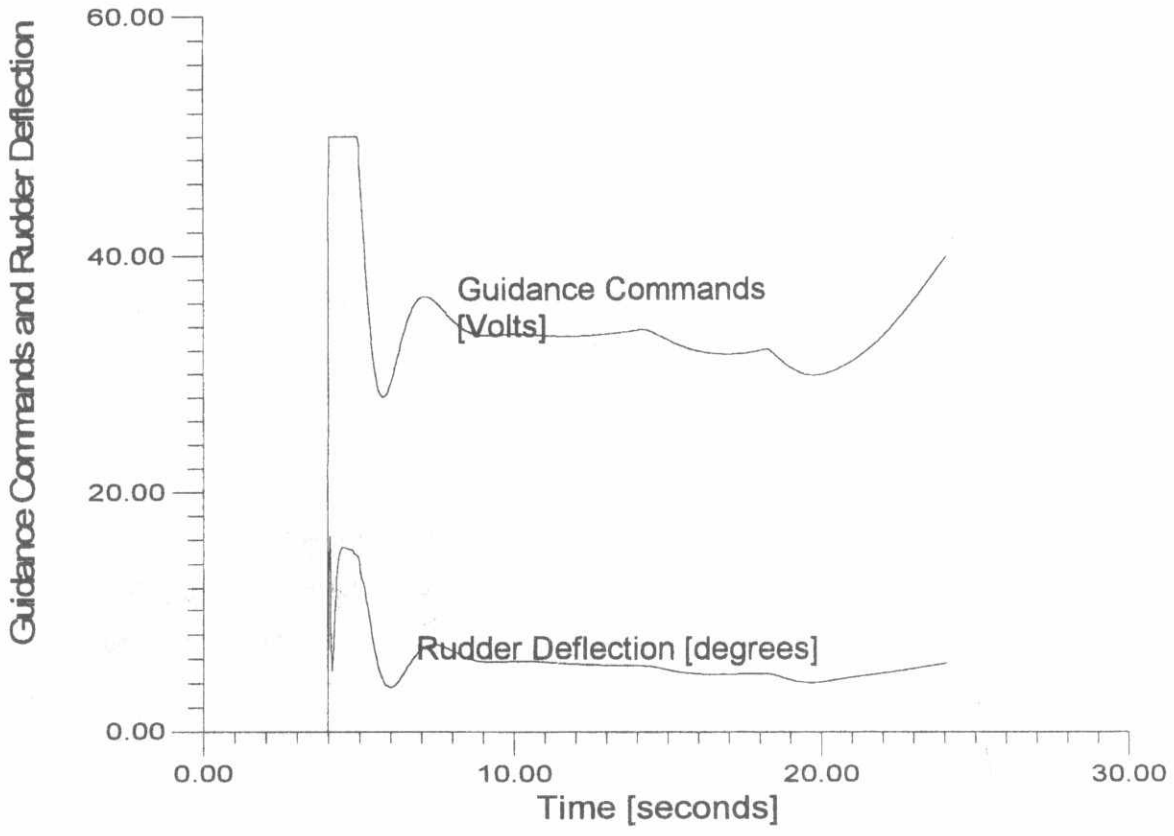

Fig. 5. Guidance commands and the rudder deflection versus time for the missile trajectory in Fig. 4.

SIX-DEGREES-OF -FREEDOM DYNAMICAL ANALYSIS

In this section the lineariztion of the dynamical equations pertinent to the missile flight will be considered. The evaluation of the linearization approximation is achieved by comparison with reference results. These reference results are obtained via the nonlinear model of the guidance loop. The simulation program employed in the missile trajectory calculation is used to calculate the step response of the air frame in the time domain. The guidance loop is opened and the control fins are instructed to turn suddenly with a constant angle. In response to this sudden fin deflection, the missile board vibrates and eventually turns in space. During this process, the missile speed is kept constant. Different fin deflection values given by 5°, 10°, and 20° are considered. As well, the missile response at Different mach numbers is recorded. Figure 6 shows the transient normal acceleration response, obtained via the nonlinear model. It is noted that as the airframe speed increases, higher aerodynamic gains are attained and the airframe response speed increases. This answers the question, why do the transients last longer time for lower M.

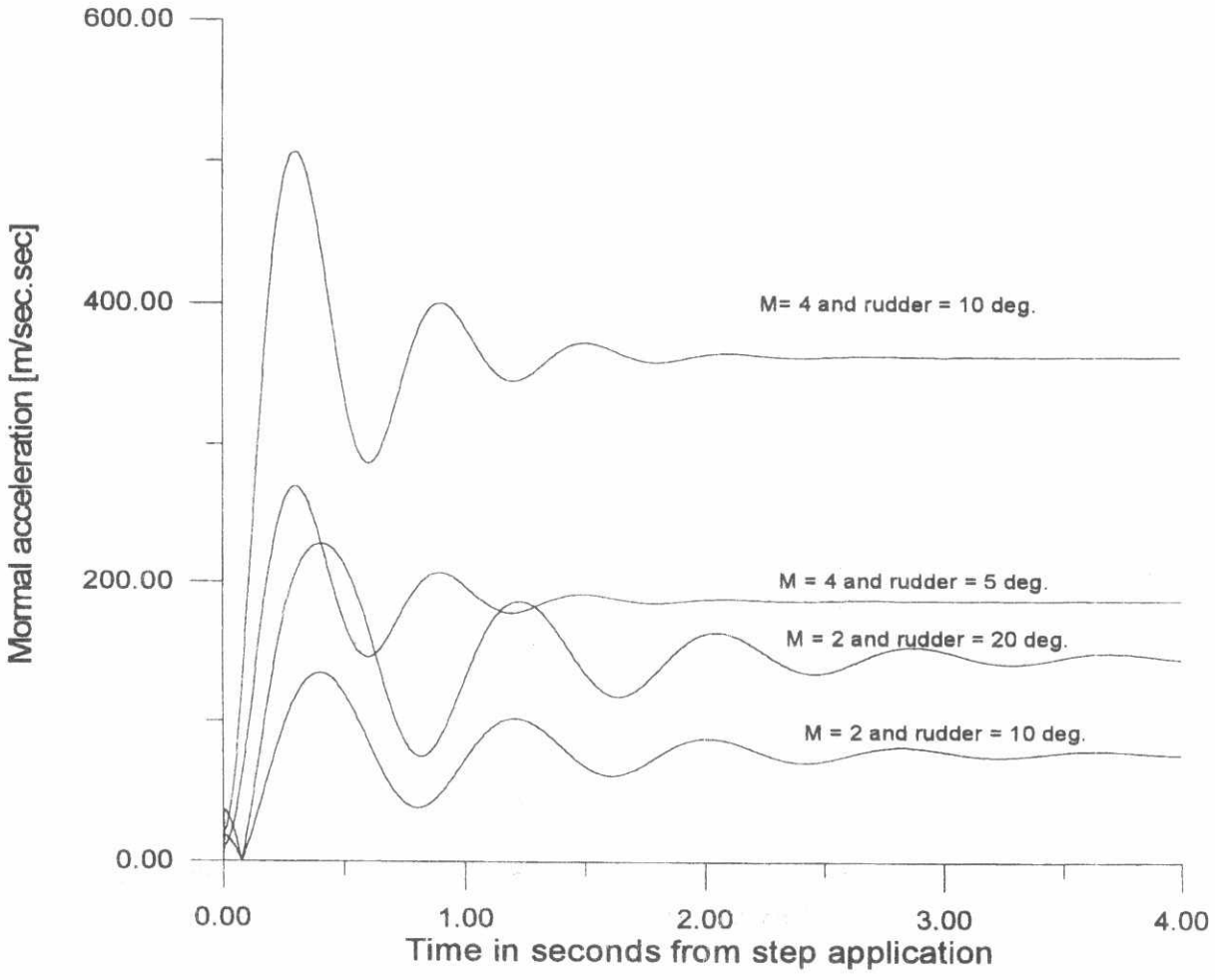

Fig.6. Normal acceleration response due to step rudder deflection for mach number M=2 and 4 (nonlinear model).

Generally, the aerodynamic transfer characteristics of the airframe are of nonlinear nature. This can be seen from the examination of the simplified dynamical equations of the missile airframe in the lateral plane [1];

$$
\begin{aligned}
F_y &= m\,(\,v + U\,.\,r - p\,.\,w) \\
F_z &= m\,(\,w - U\,.\,q + p\,.\,v) \\
My &= B\,.\,q - (C - A)\,p.q \\
M_z &= C\,.\,r - (A - B)\,p.q
\end{aligned}
\qquad (4)
$$

Where F_y , F_z , My, and M_z are the total normal forces and moments acting along the missile lateral axes y and z; respectively. U,v, and w are the missile velocities. p, q, and r are the turn rates. A, B, and C are the missile moments of inertia, as shown in Fig. 7. m is the missile mass.

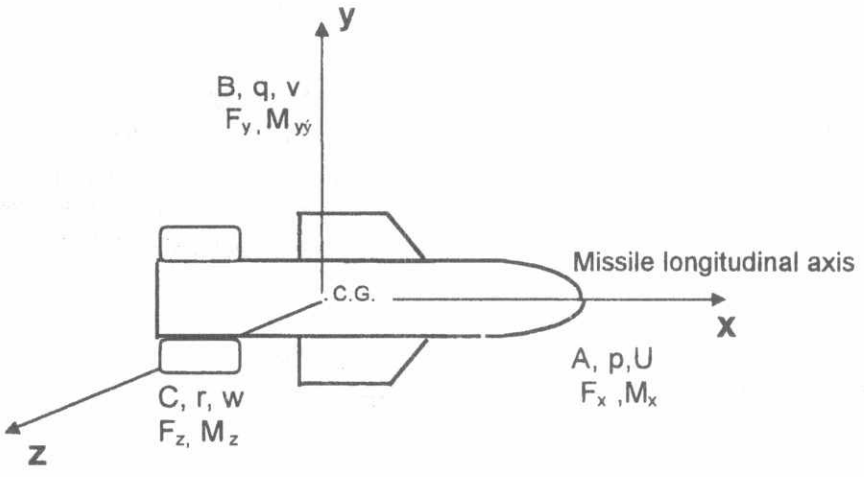

Fig. 7. Coordinate system convention.

One source of nonlinearity is the multiplication process of the flight variables as shown in Eq.4. A second source of nonlinearity is inherent in the dependence of the aerodynamic force and moment coefficients on the incidence angles. In addition to that, the transformations of the flight variables between the reference, board and velocity coordinate systems are non-linear. They involve various sinusoidal functions of the flight angles.

Thus, the linearity of the equations of motion imply linearity of the aerodynamics, linearity of the necessary transformations and ignoring the nonlinear product terms in the dynamical equations. These are acceptable assumptions when the concept of the small perturbation is adhered to. The small perturbation assumption is valid when the missile incidence throughout the flight does not exceed 20° and the attendant body rates are not large (small missile incidence indicates small lateral velocities) . In order to derive the linearized transfer function of the air frame, the gravity and thrust forces are omitted. The exclusion of the gravity force obviates the necessity of having transformation from the reference and board coordinate systems. In the mean time, the effect of the gravity force in the lateral plane is negligible compared with the thrust and aerodynamic forces [10]. The forces and moments of pure aerodynamic origin are only considered. The force equation along the missile longitudinal axis will be omitted as this neither affects the lateral motion. The missile body turn rates and incidence angles are considered small enough such that their product is negligible. Thus equation 4 can be rewritten as [1]:

$$
\begin{aligned}
F_y &= m f_y = m(v + U.r) = Y = Y_v\, v + Y_r r + Y_\zeta \zeta, \\
F_z &= m f_z = m(w - U.q) = Z = Z_w\, w + Z_q\, q + Z_\eta \eta, \\
My &= B.q = M_{yw}\, w + M_{yq}\, q + M_{y\eta}\, \eta,
\end{aligned}
\qquad (5)
$$

and

$$
M_z = C.r = M_{zv}\, v + M_{zr}\, r + M_{z\zeta}\, \zeta .
$$

Where Y_v , Y_r , Y_ζ , Z_w , Z_q , Z_η , M_{yw} , M_{yq} , $M_{y\eta}$, M_{zv} , M_{zr} , $M_{z\zeta}$ are the aerodynamic forces and moments derivatives with respect to their subscript variables. F_y and F_z are the normal accelerations of the missile in the lateral plane. The transfer function that relates the missile normal acceleration along the yaw -axis to the rudder deflection input can be directly derived from equation 5 and is given by [1]:

$$G_{f\zeta} = \frac{f_y}{\zeta} = \frac{y_\zeta s^2 - y_\zeta m_{zr} s - U(m_{z\zeta} y_v - m_{zv} y_\zeta)}{s^2 - (y_v + m_{zr}) s + y_v m_{zr} + U m_{zv}} \qquad (6)$$

The lowercase variables m and y are the normalized uppercase variables M and Y as given in [1]. In order to justify the validity of the assumptions cited before, a comparison is made between the nonlinear model results and the linearized dynamical model given by Eq. 6. At the beginning, the values of steady state gains from both models are compared for various fin deflection values given by 1°, 5°, 10°, and 20° at different speeds given by 2, 3, and 4 M. Table 1 shows this comparison.

Table 1. Aerodynamic normal acceleration gain for the missile air frame: Comparison between linear and nonlinear dynamical models.

	M=2		M=3		M=4	
rudder deflection η°	linear model	nonlinear model	linear model	nonlinear model	linear model	nonlinear model
1°	7.88	7.92	17.08	17.07	37.79	37.68
5°	39.38	39.35	85.40	84.86	189.78	186.85
10°	78.76	77.58	170.80	166.66	371.23	364.14
20°	157.52	146.27	341.60	309.01	755.80	652.60

It is clear that as the rudder deflection increases, the coincidence between both models deteriorates. This is attributed to the presence of large incidence angles and body rates associated with the large demanded rudder deflection. Fig. 8 shows the incidence angle step response. It is clear that the steady state incidence exceeds 10 degrees for rudder deflection 10 and 20 degrees. Thus, the accuracy of the approximations $\sin\alpha = \alpha$ and $\cos\alpha = 1$ necessary for the validation of the linear model is no longer acceptable; and hence, the discrepancy between both models becomes more recognizable. Figure 9 shows the turn rate of the airframe as a result of sudden fin deflection input. It is clear that the turn rate increases in the transient and steady state periods as the fin deflection increases. Thus, with a small induced rolling motion, the products rp

and qp in Eq. 4 significantly contribute to the lateral moments and ignoring them largely reduces the accuracy of the results.

In order to compare the transient behavior of the linear and nonlinear models, the step responses obtained from both models are plotted simultaneously as shown in Fig. 10 .

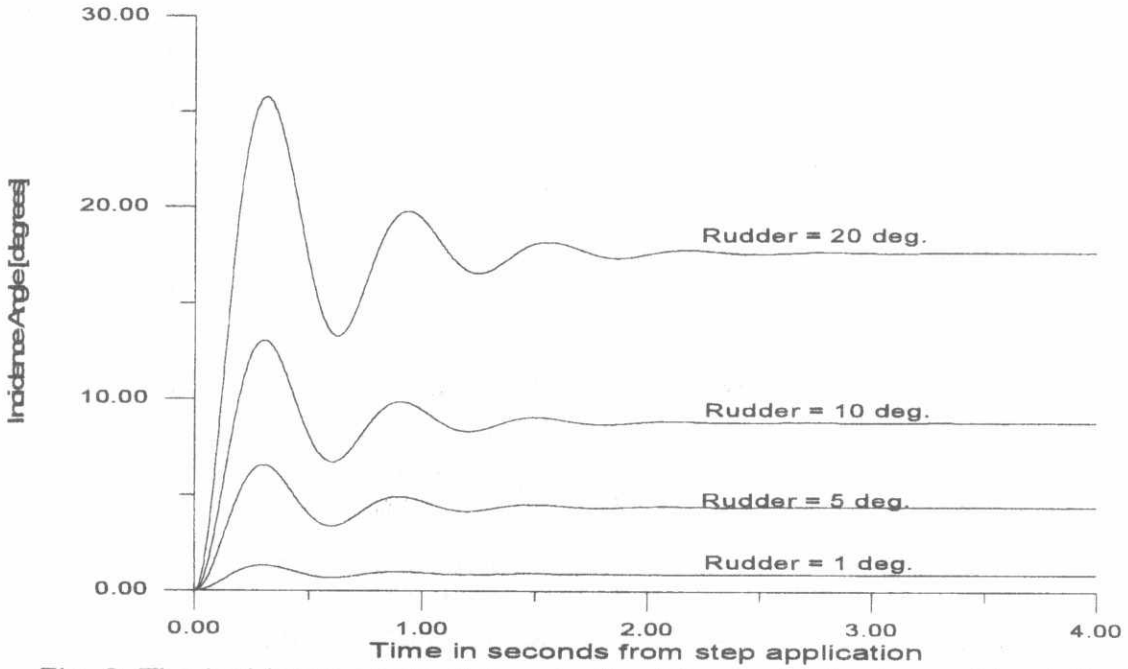

Fig. 8. The incidence angle of the missile airframe due to step rudder deflection input with values as given in the figure and Mach number = 4.

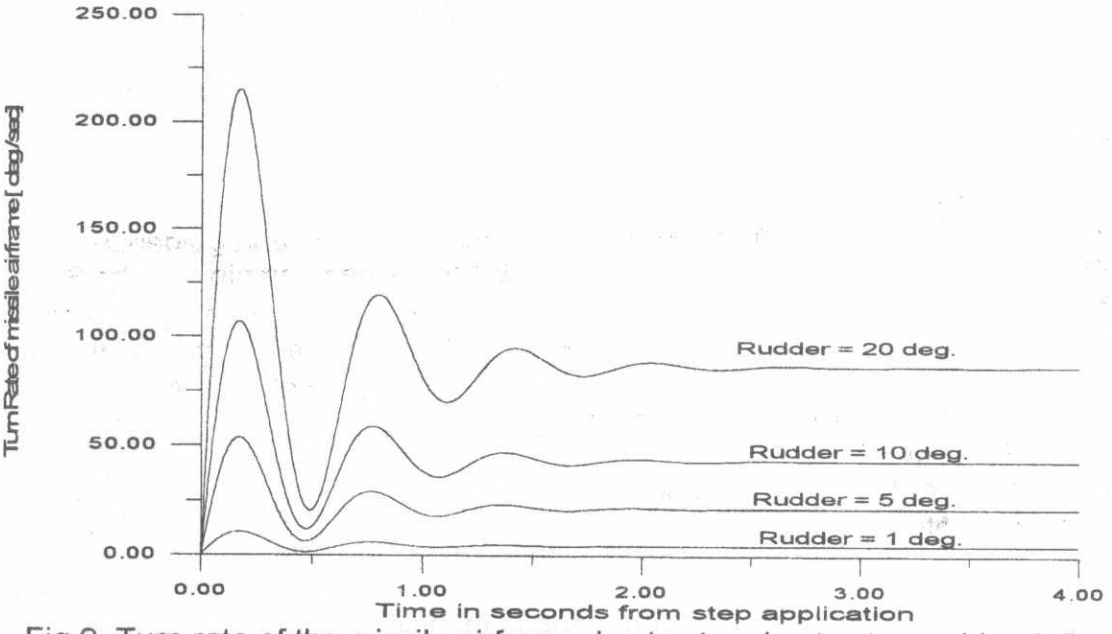

Fig.9. Turn rate of the missile airframe due to step due to step rudder deflection input with values as given in the figure and Mach number = 4.

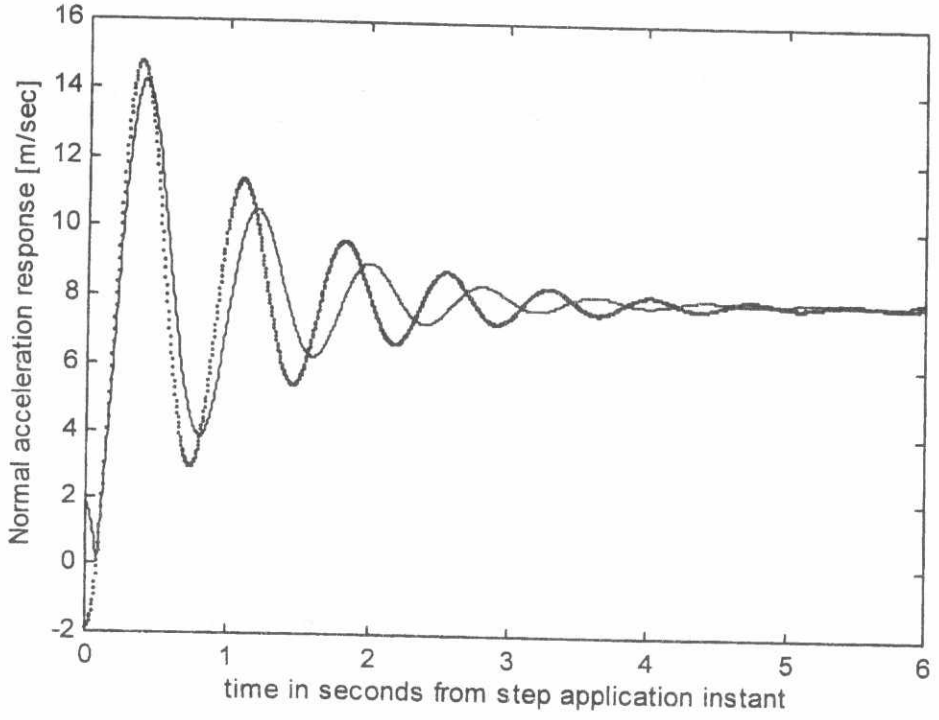

Fig. 10. Normal acceleration response due to 1° step rudder deflection input at M=2.
____ nonlinear model and ..... linear model.

As shown in Fig. 10, the linear and nonlinear models results are in agreement in the steady state period; however, they differ in the transient period. The difference is expected to increase as the excitation rudder deflection increases. This difference is attributed to the behavior of the incidence angles and body rates in the transient period as given by Figs. 8 and 9.

CONCLUSIONS AND FUTURE WORK

A six-degree-of-freedom model is utilized for the investigation of the reliability of guidance loop linearization assumptions. A command guidance system is considered in the investigation. The guidance system employs the 3-point guidance method. A computer code written in Borland C is used for the numerical solution of the nonlinear model The guidance system hardware nonlinearitites and the missile flight dynamical equations nonlinearities are involved. Level-limitation type is considered the only nonlinearity type that exists in the hardware. Investigations show that the effect of this limitation is only obvious in the early time period, just after the start of guidance. As well, the linearization assumptions are no longer accurate during the transient periods of missile sudden maneuvers.

However, during the periods of steady maneuvers, the accuracy of the model depends on the maneuver level. High maneuver levels are associated with lower accuracy. The inclusion of other types of hardware nonlinearities that practically exist in the guidance system represents an attractive future research topic.

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