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THE EFFECT OF THE EARTH'S OBLATENESS AND ATMOSPHERIC DRAG ON AN ARTIFICIAL SATELLITE IN TERMS OF THE EULER PARAMETERS

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KEYWORD

Artificial satellite,Earth's oblateness,atmospheric drag,Euler parameters.

ABSTRACT

This paper is concerned with an orbit prediction using the Euler variables. Perturbations due to the Earth's gravitational field up to the fourth zonal harmonic and the atmospheric drag with rotating & non-rotating atmosphere are considered. The method developed is illustrated by application to a typical satellite orbit. A final state of accuracy 10^{-3} m is obtained.

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1. INTRODUCTION

The computations and predication of artificial satellites orbits is now of the most important problems, this is due to their wide applications in scientific researches, remote sensing, military purposes, etc. As far as the computation techniques are concerned, the applications of special perturbation methods to the equations of motion in terms of some set of regular variables, provide one of the most powerful and accurate techniques predications for satellite ephemeris with respect to any type of perturbing forces (cf. Sharaf et Al. [6]; Awad [1]).

It is well known that the solutions of the classical Newtonian Equations of motion are unstable and that these equations are not suitable for long-term integrations. Many transformations have recently emerged in the literature aiming at stabilizing the equations of motion either to reduce the accumulation of local numerical errors or to allow using larger integration step sizes (in the transformed space) or both. The most popular one is known as the Euler transformation (or the Euler parameters). The connections between orbit dynamics and rigid dynamics were developed throughout the Euler parameters as reported in Sharaf et Al. [7, 8, 9] and Awad [2, 3, 4].

The equations of motion of an artificial satellite are generally written in the form

$$\ddot{\vec{x}} + \frac{\mu}{r^3}\vec{x} = -\frac{\partial V}{\partial \vec{x}} + \vec{P}^* \qquad (1.1)$$

where $\vec{x}$ is the position vector in a rectangular frame, r is the distance from the origin, μ is the gravitational parameter, V is the perturbed time independent potential, $\vec{P}^*$ is the resultant of all non-conservative perturbing forces and forces derivable from a time dependent potential. Then, what concerns us in the present paper are the equations of satellite motion in terms of the Euler parameters (Sharaf et Al. [9])

$$u'_1 = \tfrac{1}{2}(W_1 u_4 + W_3 u_2), \qquad (1.2.1)$$

$$u'_2 = \tfrac{1}{2}(W_1 u_3 - W_3 u_1), \qquad (1.2.2)$$

$$u'_3 = \tfrac{1}{2}(-W_1 u_2 + W_3 u_4), \qquad (1.2.3)$$

$$u'_4 = \tfrac{1}{2}(-W_1 u_1 - W_3 u_3), \qquad (1.2.4)$$

$$u'_5 = u_6, \qquad (1.2.5)$$

$$u'_6 = -\frac{P_\xi}{\mu u_5^2 u_7} - u_5 + \frac{2V}{\mu u_5 u_7} + \frac{1}{u_7} - \frac{1}{2}\frac{g_2 u_6}{u_7}, \qquad (1.2.6)$$

$$u'_7 = g_2 = \frac{2 W_3 P_\eta^*}{\mu u_5^3} + \frac{2}{u_5^4\sqrt{\mu^3 u_7}}\frac{\partial V}{\partial t} - \frac{2u_6}{\mu u_5^3}\left[< \frac{\partial V}{\partial \vec{x}}, \vec{x} > + 2V\right], \qquad (1.2.7)$$

$$u'_8 = \frac{1}{u_5^2\sqrt{\mu u_7}}, \qquad (1.2.8)$$

$$(') \equiv \frac{d}{d\tilde{\phi}},$$

where
$$W_1 = \frac{P_\zeta}{\mu\, W_3\, u_7\, u_5^3}, \tag{1.3.1}$$

$$W_3 = \sqrt{\left[1 - \frac{2V}{\mu\, u_5^2\, u_7}\right]}, \tag{1.3.2}$$

$$P_\xi = P_1\, C_{11} + P_2\, C_{12} + P_3\, C_{13}\ , \tag{1.3.3}$$

$$P_\eta^* = P_1^*\, C_{21} + P_1^*\, C_{22} + P_3^*\, C_{23}\ , \tag{1.3.4}$$

$$P_\zeta = P_1\, C_{31} + P_2\, C_{32} + P_3\, C_{33}\ , \tag{1.3.5}$$

$$\vec{P}_j = \vec{P}_j^* - \frac{\partial V}{\partial \vec{x}_j}; \qquad j = 1, 2, 3 \tag{1.3.6}$$

and

$$C_{11} = u_1^2 - u_2^2 - u_3^2 + u_4^2, \tag{1.4.1}$$

$$C_{12} = 2\,(u_1\, u_2 + u_3\, u_4), \tag{1.4.2}$$

$$C_{13} = 2\,(u_1\, u_3 - u_2\, u_4), \tag{1.4.3}$$

$$C_{21} = 2\,(u_1\, u_2 - u_3\, u_4), \tag{1.4.4}$$

$$C_{22} = -\,u_1^2 + u_2^2 - u_3^2 + u_4^2, \tag{1.4.5}$$

$$C_{23} = 2\,(u_2\, u_3 + u_1\, u_4), \tag{1.4.6}$$

$$C_{31} = 2\,(u_1\, u_3 + u_1\, u_4), \tag{1.4.7}$$

$$C_{32} = 2\,(u_2\, u_3 - u_1\, u_4), \tag{1.4.8}$$

$$C_{33} = -\,u_1^2 - u_2^2 + u_3^2 + u_4^2, \tag{1.4.9}$$

also

$$C_{11} = x_1/u_5, \tag{1.5.1}$$

$$C_{12} = x_2/u_5, \tag{1.5.2}$$

$$C_{13} = x_3/u_5, \tag{1.5.3}$$

$$r = \sqrt{x_1^2 + x_2^2 + x_3^2} = 1/u_5. \tag{1.6}$$

2. THE FORCES ACTING ON THE SATELLITE MOTION

In the following two subsections the geopotential and the atmospheric drag force are outlined.

2.1 THE PERTURBED GRAVITY OF THE EARTH'S OBLATENESS

The geopotential (V) can be described in the form

$$V = -\frac{\mu}{R}\sum_{i=2}^{\infty} J_i \left(\frac{R}{r}\right)^{i+1} P_i(\sin\phi) \tag{2.1}$$

where

- R is the equatorial radius of the Earth;

- r, ϕ and λ are respectively the geocentric distance, latitude and east longitude of the subvehicle point;

- J_i, i = 2(1)N (N $\rightarrow \infty$), are dimensionless numerical coefficients and their values are known (Awad [1]) up to N = 36;

- $P_n(x)$ is the Legendre polynomial in x of order n defined A,x ε[-1,1]

as

$$P_n(x) = \sum_{m=0}^{[n/2]} \frac{(-1)^m (2n-2m)!}{2^n\, m!\,(n-m)!\,(n-2m)!} x^{n-2m}$$

hence, [q] denotes the largest integer ≤q.

We will be concerned with the potential including the zonal harmonics (J_2, J_3 and J_4) only, i.e., the potential V can be described as

$$\begin{aligned} V = {} & \mu R^2 J_2 r^{-3} P_2(\sin\phi) + \\ & \mu R^3 J_3 r^{-4} P_3(\sin\phi) + \\ & \mu R^4 J_4 r^{-5} P_4(\sin\phi), \end{aligned}$$

since $\sin\phi = x_3/r,$

$$P_2(x_3/r) = \tfrac{1}{2}(3 x_3^2 r^{-2} - 1),$$

$$P_3(x_3/r) = \tfrac{1}{2}(5 x_3^3 r^{-3} - 3 x_3 r^{-1}),$$

and $P_4(x_3/r) = \tfrac{1}{8}(35 x_3^4 r^{-4} - 30 x_3^2 r^{-2} + 3);$

then

$$\begin{aligned} V = {} & \tfrac{3}{2} Q_2 x_3 r^{-5} - \tfrac{1}{2} Q_2 r^{-3} + \tfrac{5}{2} Q_3 x_3^3 r^{-7} - \\ & \tfrac{3}{2} Q_3 x_3 r^{-5} + \tfrac{35}{8} Q_4 x_3^4 r^{-9} - \\ & \tfrac{15}{4} Q_4 x_3^2 r^{-7} + \tfrac{3}{8} Q_4 r^{-5}, \end{aligned} \qquad (2.2)$$

where

$$Q_2 = \mu R^2 J_2, \qquad (2.3.1)$$

$$Q_3 = \mu R^3 J_3, \tag{2.3.2}$$

$$Q_4 = \mu R^4 J_4 . \tag{2.3.3}$$

2.2 THE ATMOSPHERIC MODEL

The chief aerodynamic force acting on an artificial Earth satellite is the drag force. It acts opposite to the direction of motion and is proportional to the square of the modulus of the spacecraft velocity. The aerodynamic lift-force acting perpendicular to the velocity vector usually produces only small short-periodic effects and is negligible for most satellites. The perturbing acceleration due to air drag is expressed as

$$\vec{D} = -\frac{1}{2} C_D \frac{A}{M} \rho \left\|\vec{V}\right\|^{\alpha} \vec{V} \tag{2.4}$$

where

- C_D is the non-dimensional drag coefficient depending on the satellite geometry, the mode of reflection of the particles off its surface and the temperature of the incident and re-emitted molecules. For a spherical satellite in free-molecule flow, $C_D \approx 2.2$.

- A is a reference surface of the satellite which is commonly chosen as the mean cross-section perpendicular to the direction of motion.

- M is the spacecraft mass. The ratio A/M is assumed to be constant. This is the main assumption in the theory which might not hold in some cases, e.g., for a sun-pointing satellite in a circular orbit. But it is applicable to more than half of the satellites currently in orbit and it is certainly valid for uncon-trolled satellites or debris on their final decay trajectories.

- α is an arbitrary constant, in this paper we will take its value equal to one which corresponds to the chief aerodynamic force acting on an artificial Earth satellite, that is the drag force.

- V is the velocity of the spacecraft relative to the ambient gas, and is computed through

$$\vec{V} = \vec{\upsilon} - \vec{W}_a \times \vec{r} \qquad (2.5)$$

where $\vec{\upsilon}$ is the spacecraft velocity with respect to the Earth center, $\vec{W}_a$ is the west-to-east angular velocity of the atmosphere and $\vec{r}$ is the position vector of the satellite.

- ρ is the density function depends primarily on the altitude and to a lesser extent on the solar and geomagnetic activity, and also the angular separation from the Sun. This function is represented either by an exponential variation with height or a power law, e.g.,

$$\rho = \rho_0 \left(\frac{r_0 - s}{r - s} \right)^{\tau} \tag{2.6}$$

where ρ_0 is the value of ρ at the reference level r_0, and s and τ are two adjustable parameters. They can be adapted to the estimated or observed variations of the solar activity and is periodically updated so that the dynamics of the atmosphere is taken into account. The value of s is approximately equal to the mean Earth equatorial radius and τ equals the inverse of the gradient of the density scale height and can take values in the range from 3 to 9 (Delhaise [5]).

3. THE EQUATIONS OF MOTION

Including the Earth's oblateness and air drag, the differential equations of motion (1.2) of an artificial satellite in terms of the Euler parameters are

$$u'_1 = \frac{1}{2}(W_1 u_4 + W_3 u_2), \tag{3.2.1}$$

$$u'_2 = \frac{1}{2}(W_1 u_3 - W_3 u_1), \tag{3.2.2}$$

$$u'_3 = \frac{1}{2}(-W_1 u_2 + W_3 u_4), \tag{3.2.3}$$

$$u'_4 = \frac{1}{2}(-W_1 u_1 - W_3 u_3), \tag{3.2.4}$$

$$u'_5 = u_6, \qquad (3.2.5)$$

$$u'_6 = -\frac{P_\xi}{\mu u_5^2 u_7} - u_5 + \frac{2V}{\mu u_5 u_7} + \frac{1}{u_7} - \frac{g_2 u_6}{2u_7}, \qquad (3.2.6)$$

$$u'_7 = g_2 = \frac{2 W_3 P_\eta^*}{\mu u_5^3} - \frac{2u_6}{\mu u_5^3}\left[< \frac{\partial V}{\partial \vec{x}}, \vec{x} > + 2V \right], \qquad (3.2.7)$$

$$u'_8 = \frac{1}{u_5^2 \sqrt{\mu u_7}}, \qquad (3.2.8)$$

where

$$W_1 = \frac{P_\zeta}{\mu W_3 u_7 u_5^3}, \qquad (3.2.1)$$

$$W_3 = \sqrt{\left[1 - \frac{2V}{\mu u_5^2 u_7}\right]}, \qquad (3.2.2)$$

$$P_\xi = P_1 C_{11} + P_2 C_{12} + P_3 C_{13}, \qquad (3.2.3)$$

$$P_\eta^* = P_1^* C_{21} + P_1^* C_{22} + P_3^* C_{23}, \qquad (3.2.4)$$

$$P_\zeta = P_1 C_{31} + P_2 C_{32} + P_3 C_{33}, \qquad (3.2.5)$$

$$\frac{\partial V}{\partial x_1} = -\frac{3}{2} Q_2 x_1 r^{-5} + \frac{15}{8} Q_4 x_1 r^{-7} - \frac{15}{2} Q_3 x_1 x_3 r^{-7} + \frac{15}{2} Q_2 x_1 x_3^2 r^{-7} - \frac{105}{4} Q_4 x_1 x_3^2 r^{-9} + \frac{35}{2} Q_3 x_1 x_3^3 r^{-9} + \frac{315}{8} Q_4 x_1 x_3^4 r^{-11}, \qquad (3.3.1)$$

$$\frac{\partial V}{\partial x_2} = -\frac{3}{2} Q_2 x_2 r^{-5} + \frac{15}{8} Q_4 x_2 r^{-7} - \frac{15}{2} Q_3 x_2 x_3 r^{-7} + \frac{15}{2} Q_2 x_2 x_3^2 r^{-7} - \frac{105}{4} Q_4 x_2 x_3^2 r^{-9} + \frac{35}{2} Q_3 x_2 x_3^3 r^{-9} + \frac{315}{8} Q_4 x_2 x_3^4 r^{-11}, \tag{3.3.2}$$

$$\frac{\partial V}{\partial x_3} = -\frac{9}{2} Q_2 x_3 r^{-5} + \frac{3}{2} Q_3 r^{-5} + \frac{75}{8} Q_4 x_3 r^{-7} - 15 Q_3 x_3^2 r^{-7} + \frac{15}{2} Q_2 x_3^3 r^{-7} - \frac{175}{4} Q_4 x_3^3 r^{-9} + \frac{35}{2} Q_3 x_3^4 r^{-9} + \frac{315}{8} Q_4 x_3^5 r^{-11}, \tag{3.3.3}$$

$$P_1^* = -\gamma \, (r - R)^{-\tau} \, V V_1, \tag{3.4.1}$$

$$P_2^* = -\gamma \, (r - R)^{-\tau} \, V V_2, \tag{3.4.2}$$

$$P_3^* = -\gamma \, (r - R)^{-\tau} \, V V_3, \tag{3.4.3}$$

$$\gamma = \frac{1}{2} C_D \frac{A}{M} \rho_0 \, (r_0 - R)^{\tau}, \tag{3.5}$$

$$V_1 = \dot{x}_1 + x_2 W_{a3}, \tag{3.6.1}$$

$$V_2 = \dot{x}_2 - x_1 W_{a3}, \tag{3.6.2}$$

$$V_3 = \dot{x}_3, \tag{3.6.3}$$

in case of rotating ambient gas. Since the components of the angular velocity of the atmosphere (W_{a1} & W_{a2}) are negligible, then

$$V_1 = \dot{x}_1, \tag{3.7.1}$$

$$V_2 = \dot{x}_2, \tag{3.7.2}$$

$$V_3 = \dot{x}_3, \tag{3.7.3}$$

in case of non-rotating ambient gas.

Since $$V^2 = V_1^2 + V_2^2 + V_3^2. \tag{3.8}$$

4. COMPUTATIONAL DEVELOPMENTS

A computer program was applied for the solution of the new system (3.1) using a fixed step size, fourth order Runge-Kutta method. To get the solution we need to compute the following subsection.

4.1 COMPUTATION OF THE INITIAL VALUES AT T=0

Knowing the position & velocity vectors ($\vec{x}_0$ & $\dot{\vec{x}}_0$) at the instant $t=0$ we get the initial values of u's, we compute:

1) $r_0 = \sqrt{x_{01}^2 + x_{02}^2 + x_{03}^2}$

2) $u_5 = 1/r_0$

3) $\dot{r}_0 = u_5 \, (x_{01}\,\dot{x}_{01} + x_{02}\,\dot{x}_{02} + x_{03}\,\dot{x}_{03})$

4) $$V_0 = \tfrac{3}{2}\mu R^2 J_2 u_5^5 x_{03} - \tfrac{1}{2}\mu R^2 J_2 u_5^3 + \tfrac{5}{2}\mu R^3 J_3 u_5^7 x_{03}^3 - \tfrac{3}{2}\mu R^3 J_3 u_5^5 x_{03} + \tfrac{35}{8}\mu R^4 J_4 u_5^9 x_{03}^4 - \tfrac{15}{4}\mu R^4 J_4 u_5^7 x_{03}^2 + \tfrac{3}{8}\mu R^4 J_4 u_5^5$$

5) $u_7 = \dfrac{\dot{x}_{01} + \dot{x}_{02} + \dot{x}_{03} - \dot{r}_0^2 + 2\,V_0}{\mu\, u_5^2}$

6) $u_6 = -\dfrac{\dot{r}_0}{\sqrt{\mu\, u_7}}$

7) $C_{1i} = x_{0i}\; u_5 \qquad i = 1, 2, 3$

8) $p = \dfrac{\dot{x}_{01} + \dot{x}_{02} + \dot{x}_{03} - \dot{r}_0^2}{\mu\, u_5^2}$.

9) $C_{2i} = \dfrac{\dot{x}_{0i}/u_5 - \dot{r}_0\, x_{0i}}{\sqrt{\mu\, p}}, \qquad i = 1, 2, 3$

10) $C_{31} = C_{12}\,C_{23} - C_{13}\,C_{22}$

11) $C_{32} = C_{13}\,C_{21} - C_{11}\,C_{23}$

12) $C_{33} = C_{11}\,C_{22} - C_{12}\,C_{21}$

13) $u_4 = \frac{1}{2}\sqrt{1 + C_{11} + C_{22} + C_{33}}$

14) $u_1 = \dfrac{C_{23} - C_{32}}{4\,u_4}$

15) $u_2 = \dfrac{C_{13} - C_{13}}{4\,u_4}$

16) $u_3 = \dfrac{C_{12} - C_{21}}{4\,u_4}$.

4.2 COMPUTATION OF $\vec{X} \& \dot{\vec{X}}$ AT ANY TIME

Knowing the initial values of the u's at any time, to get the position and velocity vectors ($\vec{x} \& \dot{\vec{x}}$), we compute:

1) $C_{11} = u_1^2 - u_2^2 - u_3^2 + u_4^2$

2) $C_{12} = 2(u_1 u_2 + u_3 u_4)$

3) $C_{13} = 2(u_1 u_3 - u_2 u_4)$

4) $x_1 = C_{11}/u_5$

5) $x_2 = C_{12}/u_5$

6) $x_3 = C_{13}/u_5$

7) $$V = \tfrac{3}{2}\mu R^2 J_2 u_5^5 x_{03} - \tfrac{1}{2}\mu R^2 J_2 u_5^3 + \tfrac{5}{2}\mu R^3 J_3 u_5^7 x_{03}^3 -$$
$$\tfrac{3}{2}\mu R^3 J_3 u_5^5 x_{03} + \tfrac{35}{8}\mu R^4 J_4 u_5^9 x_{03}^4 -$$
$$\tfrac{15}{4}\mu R^4 J_4 u_5^7 x_{03}^2 + \tfrac{3}{8}\mu R^4 J_4 u_5^5$$

8) $$p = u_7 - \frac{2V}{\mu u_5^2}$$

9) $C_{21} = 2(u_1 u_2 - u_3 u_4)$

10) $C_{22} = - u_1^2 + u_2^2 - u_3^2 + u_4^2$

11) $C_{23} = 2(u_2 u_3 + u_1 u_4)$

12) $\dot{x}_1 = C_{21}\, u_5 \sqrt{\mu p} - x_1\, u_5\, u_6 \sqrt{\mu u_7}$

13) $\dot{x}_2 = C_{22}\, u_5 \sqrt{\mu p} - x_2\, u_5\, u_6 \sqrt{\mu u_7}$

14) $\dot{x}_3 = C_{23}\, u_5 \sqrt{\mu p} - x_3\, u_5\, u_6 \sqrt{\mu u_7}$.

4.3 COMPUTATION OF THE STEP SIZE

As we know the problem of step size Δt of the time t has poor conven-tional numerical solution. So, a step size $\Delta\tilde{\phi}$ is adopted as

$$\Delta\tilde{\phi} = u_5^2 \sqrt{\mu\, u_7}\, \Delta t.$$

4.4 COMPUTATION OF ACCURACY CHECKS

The accuracy of the computed values of the u's variables at any time $t \neq t_0$ could be checked by the relation

$$u_1^2 + u_2^2 + u_3^2 + u_4^2 = 1.$$

4.5 NUMERICAL EXAMPLE

We'll take as the numerical example the Indian Satellite RS-1 at 300 Km height (Sharma and Mani [10]) which has

- mass (m) = 35.443 Kg,
- cross sectional area (A) = 0.319019E-06 Km^2,

and its initial position and velocity components are

- x_{01} = 1626.742 Km,
- x_{02} = 6268.094 Km,
- x_{03} = - 1776.018 Km,
- $\dot{x}_{01}$ = - 5.920522 Km/sec,
- $\dot{x}_{02}$ = 0.239214 Km/sec,
- $\dot{x}_{03}$ = - 5.158830 Km/sec,

also one orbital revolution is described in 1.588352085 hours; since the adopted physical constants are

- R = 6778.135 Km,
- μ = 398600.8 Km^3/sec ,
- J_2 = 1.08263E-03,
- J_3 = - 2.53648E-06,
- J_4 = - 1.62330E-06,
- W_{e3} = 7.292115833E-05 rad/sec (Awad [1]);

where

- ρ_0 (at 300 Km) = 0.0337 Kg/Km^3,
- drag coefficient = 2.2,
- the parameter s = 1 Earth radius,
- the parameter τ = 4 (Delhaise [5]),

we use the above values to compute the step size $\Delta\tilde{\phi}$, the position & velocity vectors and the accuracy check of the u's variables at any time $t \neq t_0$. The results of the computations are illustrated in Figures (1 & 2) and are supplemented in Tables (1, 2 & 3). The Figures show the variations of the two orbital elements (a = semi-major axis and e = eccentricity) with the time, while the supplemented Tables show the accuracy checks in each revolutions with the time.

Table 1. One hundred revolutions.

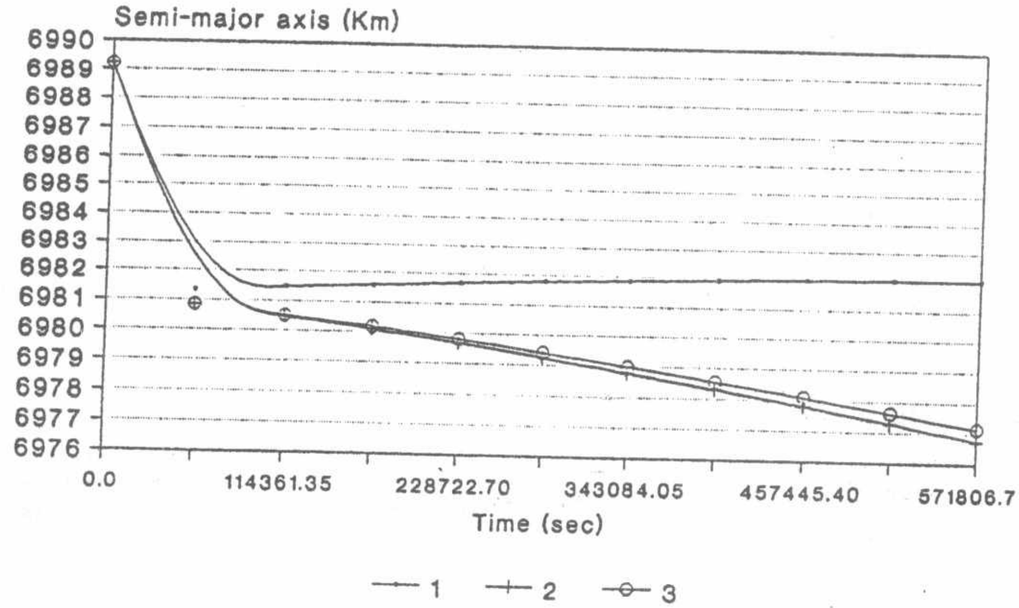
Time (sec.)	The Check Relation		
	With Non-Conservative Force	With Atmospheric Drag and Without Rotation	With Atmospheric Drag and With Rotation
0.0	1.0000000	1.0000000	1.0000000
114361.3501	0.9999998	1.0000000	0.9999976
228722.7002	1.0000000	1.0000020	0.9999990
343084.0503	0.9999998	1.0000010	0.9999984
457445.4004	1.0000010	1.0000010	0.9999971
571806.7505	1.0000020	0.9999985	0.9999977

Table 2. Five hundred revolutions.

Time (sec.)	The Check Relation		
	With Non-Conservative Force	With Atmospheric Drag and Without Rotation	With Atmospheric Drag and With Rotation
2287227.002	1.0000010	1.0000030	0.9999921
2401588.352	0.9999998	1.0000020	0.9999922
2515949.702	0.9999993	1.0000050	0.9999921
2630311.052	0.9999979	1.0000050	0.9999924
2744672.402	0.9999959	1.0000040	0.9999931
2859033.753	0.9999960	1.0000050	0.9999938

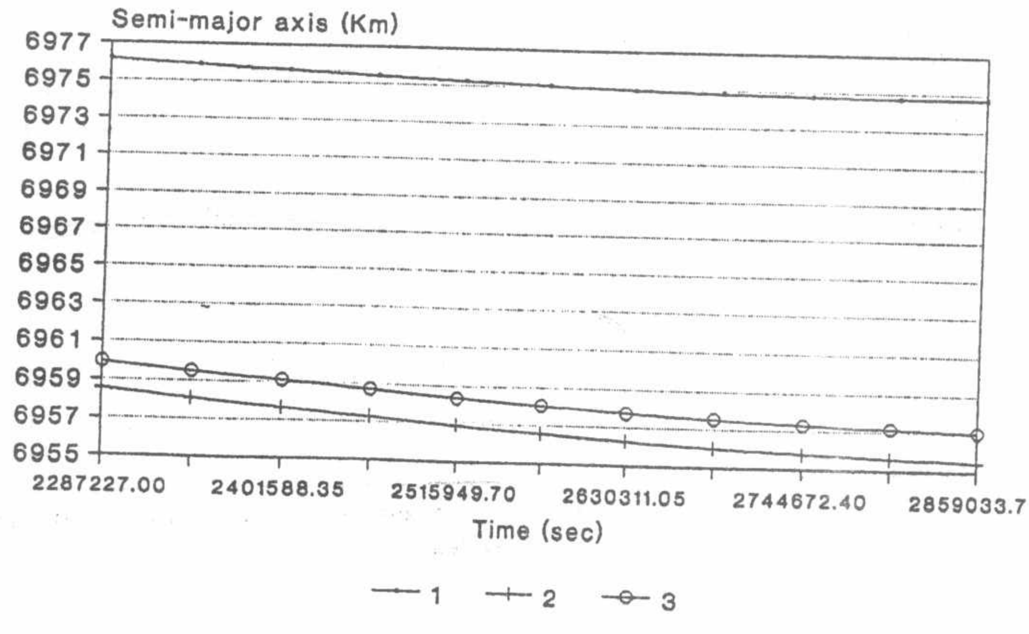
Table 3. One thousand revolutions.

Time (sec.)	The Check Relation		
	With Non- e Force	With Atmospheric Drag and Without Rotation	With Atmospheric Drag and With Rotation
5146260.755	1.0000040	1.0000070	0.9999967
5260622.105	1.0000050	1.0000060	0.9999980
5374983.455	1.0000040	1.0000040	0.9999967
5489344.805	1.0000050	1.0000030	0.9999944
5603706.155	1.0000060	1.0000020	0.9999924
5718067.505	1.0000050	0.9999995	0.9999912



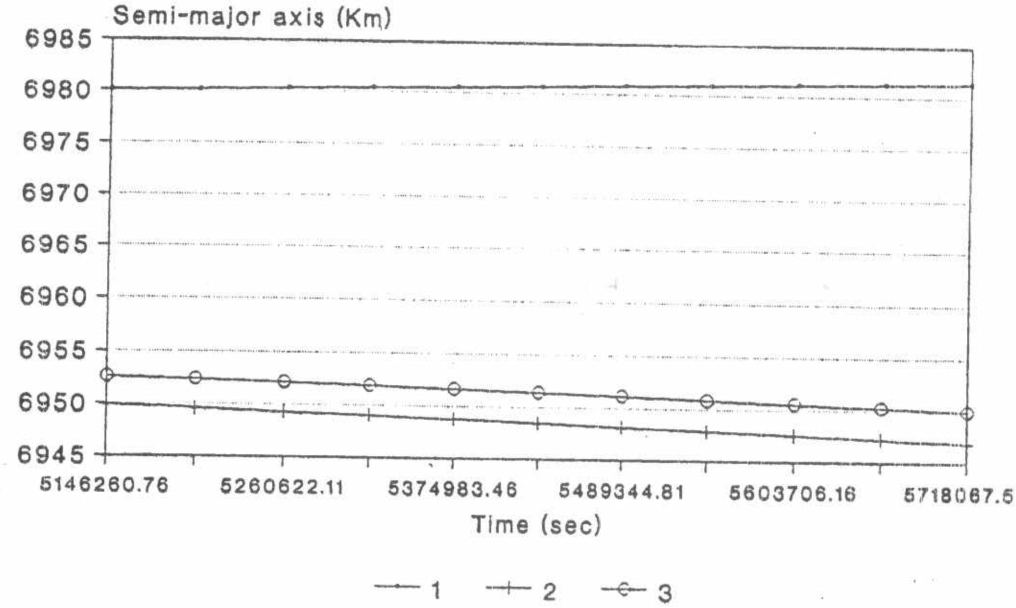
1 Without non-conservative forces
2 With atmosphere & without rotation
3 With atmosphere & with rotation

Fig. 1-a . One hundred revolutions.



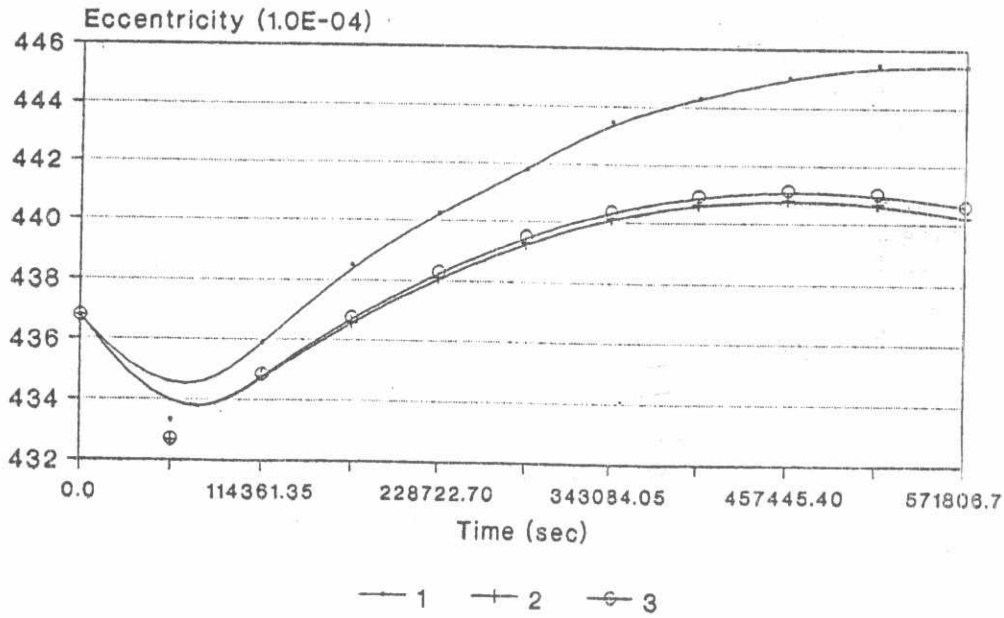
1 Without non-conservative forces
2 With atmosphere & without rotation
3 With atmosphere & with rotation

Fig. 1-b . Five hundred revolutions.



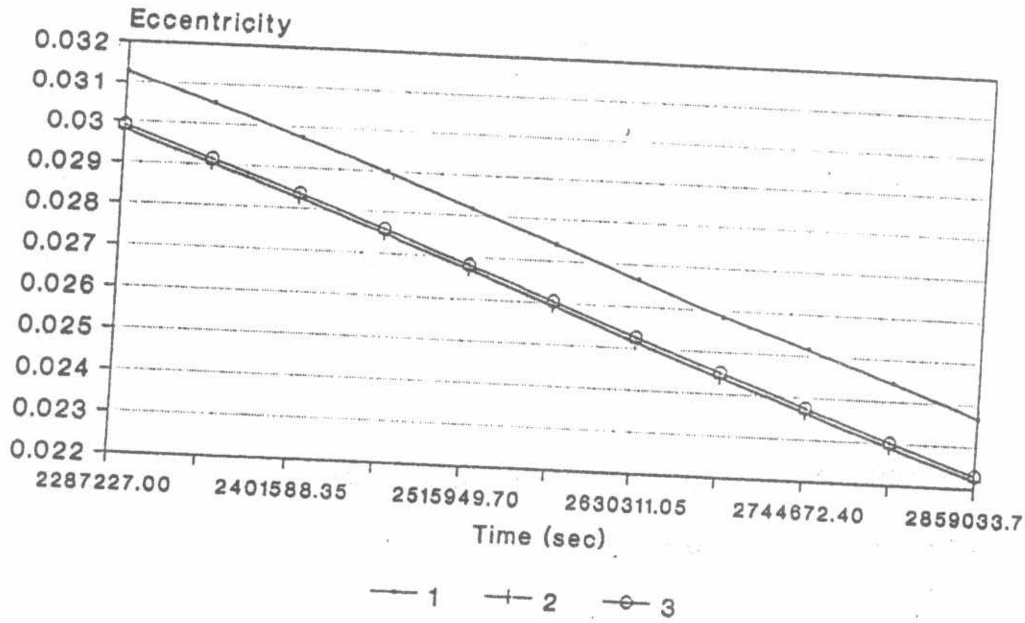
1 Without non-conservative forces
2 With atmosphere & without rotation
3 With atmosphere & with rotation

Fig. 1-c . One thousand revolutions.



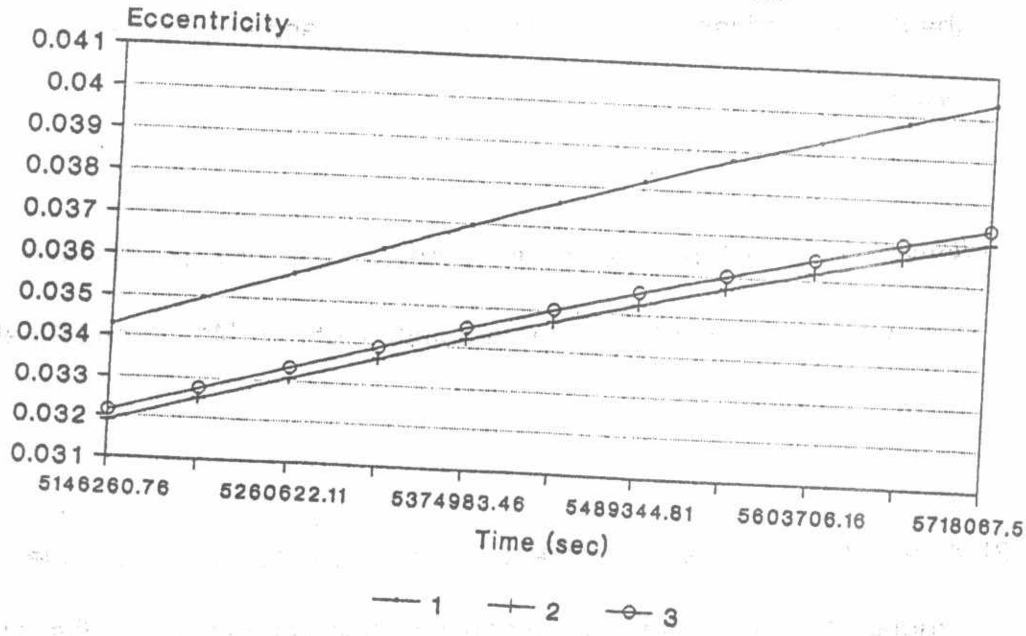
1 Without non-conservative forces
2 With atmosphere & without rotation
3 With atmosphere & with rotation

Fig. 2-a . One hundred revolutions.



1 Without non-conservative forces
2 With atmosphere & without rotation
3 With atmosphere & with rotation

Fig. 2-b . Five hundred revolutions.



1 Without non-conservative forces
2 With atmosphere & without rotation
3 With atmosphere & with rotation

Fig. 2-c . One thousand revolutions.

5. RESULTS

The results of the present work agree with previous works in the size of the atmospheric drag effects and its great enhancement at lower altitudes. The results also reveal that the effect of the rotation of the atmosphere is relatively small and can be neglected for short time predications, though it is to be included for longer time predications and when the area-to-mass ratio is large.

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