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NEW APPROACH FOR REGULARIZED IMAGE INTERPOLATION

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ABSTRACT

This paper is concerned with solving the image interpolation problem as an inverse problem using a regularized interpolation algorithm. The objective of the paper is how to solve the image interpolation problem as an inverse problem in an efficient manner. The paper suggests a new implementation for the regularized image interpolation algorithm. In this suggested implementation, a single matrix inversion process of small dimensions is required in the interpolation process avoiding the large computational complexity due to the matrices of large dimentions involved in the interpolation process. The performance of this suggested image interpolation algorithm is compared to the standard polynomial interpolation algorithm such as the bilinear, the bicubic and the cubic spline algorithms. Results iluustrate that the suggested implemetation of the regularized image interpolation algorithm is superior to the traditional implementation from the mean square error point of view and the computational complexity point of view as well.

Key Words:

Splie interpolation, Bicubic interpolation , Bilinear Interpolation, Regularized interpolation.

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Introduction

Image interpolation is the process by which a high-resolution image can be obtained from a low resolution one. Image interpolation has a wide range of applications in numerous fields such as medical image processing, military applications, space imagery, image decompression and digital HDTV.

The image interpolation problem has been intensively treated in the literature [1-5]. Conventional interpolation algorithms such as the bilinear, bicubic and cubic spline algorithms have been widely used in image interpolation [4-8]. These conventional algorithms are space invariant algorithms based on a basis function interpolation. They don't take into consideration the spatial activity of the image to be interpolated. They also don't take into consideration the mathematical model by which the imaging sensors capture the image.

Spatially adaptive variants of the above mentioned algorithms have been developed also [8]. Although these adaptive algorithms improve the quality of the interpolated image especially near edges, they still don't take into consideration the mathematical model by which the image capturing devices operate.

In fact, most image capturing devices are composed of charge-coupled devices. (CCD). In CCD imaging, there is an interaction between the adjacent points in the object to be imaged to form a pixel in the obtained image [9, 10]. If this model of interaction is taken into consideration in image interpolation, the interpolation process will be similar to a process of imaging with a high resolution-imaging device to a great extent and better results are expected to occur.

Some image interpolation algorithms have been introduced taking into consideration this interaction process [9, 10]. The linear minimum mean square error (LMMSE) image interpolation algorithm is one of these algorithms. Another interpolation algorithm is the regularized image interpolation algorithm. This regularized interpolation algorithm has been previously solved in a successive approximation manner to avoid the matrix inversion process [10].

In this paper, we suggest a new implementation of the regularized image interpolation algorithm. In this suggested implementation, we solve the problem using a non-iterative inverse solution. This implementation requires a single matrix inversion of small dimensions if a global regularization parameter is used.

Polynomial Image Inerpolation

The process of image interpolation aims at estimating intermediate pixels between the known pixel values as shown in Fig.(1). To estimate the intermediate pixel at position x , the neighboring pixels and the distance s are incorporated into the estimation process.

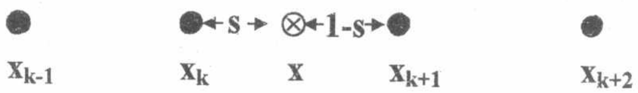

Fig.(1) 1-D signal interpolation. The Pixel at position x is estimated using its neighborhood pixels and the distance s.

For equally spaced 1-D sampled data, $f(x_k)$, many interpolation functions can be used. The value to be interpolated, $\hat{f}(x)$, can, in general, be written in the form [6]:

$$\hat{f}(x) = \sum_{k=-\infty}^{\infty} c_k \beta(x - x_k) \qquad (1)$$

where $\hat{f}(x)$ is the corresponding interpolated function, $\beta(x)$ is the interpolation kernel , and x and x_k represent continuous and discrete spatial distance, respectively.

From the classical Sampling theory, if $f(x)$ is band limited to $(-\pi, \pi)$, then [6] :

$$\hat{f}(x) = \sum_k f(x_k)\,\mathrm{sinc}(x - x_k) \qquad (2)$$

This is known ideal interpolation. From numerical computations view, the ideal interpolation formula is not practical due to the slow rate of decay of the interpolation kernel sinc(x). So, approximations such as the bilinear, bicubic and cubic spline interpolation techniques are used as alternatives [6].
As shown in Fig.1, we define the distance between x, x_k and x_{k+1} as [8]:

$$s = x - x_k \quad , \quad 1 - s = x_{k+1} - x.$$

For the bilinear, Bicubic and Cubic spline image interpolation algorithms we have [8]:

i- **Bilinear**

$$\hat{f}(x) = (1-s)f(x_k) + sf(x_{k+1}) \qquad (3)$$

ii- **Bicubic**

$$\begin{aligned}\hat{f}(x) &= f(x_{k-1})(-s^3 + 2s^2 - s)/2 + f(x_k)(3s^3 - 5s^2 + 2)/2 \\ &+ f(x_{k+1})(-s^3 + 4s^2 + s)/2 + f(x_{k+2})(s^3 - s^2)/2\end{aligned} \qquad (4)$$

iii- **Cubic Spline**

$$\hat{f}(x) = f(x_{k-1})[(3+s)^3 - 4(2+s)^3 + 6(1+s)^3 - 4s^3]/6$$
$$+ f(x_k)[(2+s)^3 - 4(1+s)^3 + 6s^3]/6 + f(x_{k+1})[(1+s)^3 - 4s^3]/6 \quad (5)$$
$$+ f(x_{k+2})s^3/6$$

For 2-D image interpolation, these techniques are applied along rows and then along columns [3].

Low Resolution Image Degradation Model

In the imaging process, when a scene is imaged by a high resolution imaging device, the captured high resolution image can be named $x(n_1,n_2)$ where n_1, n_2=0,1,2,....N-1. If the same scene is image by a low resolution imaging device, the resulting image can be named $y(m_1,m_2)$ where m_1, m_2=0,1,2,....M-1. Here M=N/R, where R is the ratio between the sizes of $x(n_1,n_2)$ and $y(m_1,m_2)$. The relationship between the low-resolution image and the high resolution image can be represented by the following mathematical model [9, 10]:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \quad (6)$$

where $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{v}$ are lexicographically ordered vectors of the unknown high resolution image, the measured low resolution image and additive noise values respectively. The matrix $\mathbf{H}$ represents the filtering and down sampling process, which is assumed to transform the high-resolution image to the low-resolution image.

Under separability assumption, the model of filtering and down sampling processes which transforms the NXN high resolution image to the MXM low resolution image is shown in Fig.(2) . Here M=N/2.

The vector $\mathbf{x}$ is of size N^2X1 and the vectors $\mathbf{y}$ and $\mathbf{v}$ are of size $(N/2)^2$X1. The matrix $\mathbf{H}$ is of size $(N/2)^2$XN^2 which can be written as [10]:

$$\mathbf{H} = \mathbf{H}_1 \otimes \mathbf{H}_1 \quad (7)$$

where $\otimes$ represents the Kronecker product, and the N/2 X N matrix $\mathbf{H}_1$ represents the one dimensional (1-D) low pass filtering and down sampling by a factor of 2 and is given by:

$$\mathbf{H}_1 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 \end{bmatrix} \quad (8)$$

From the above model, it is clear that the process of obtaining a high-resolution image from a low-resolution image is an inverse problem, which requires inverting the operator $\mathbf{H}$.

It is clear that, the matrix $\mathbf{H}$ is not a square matrix, so its direct inversion is not possible.

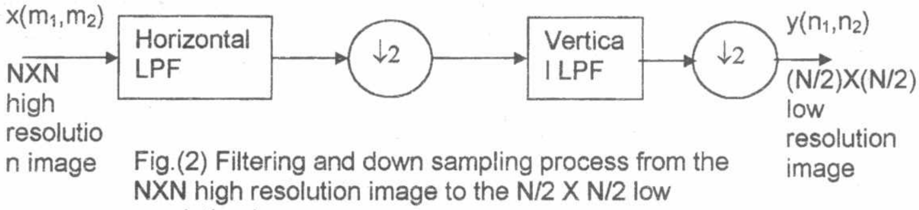

Fig.(2) Filtering and down sampling process from the NXN high resolution image to the N/2 X N/2 low resolution image.

Regularized Image Interpolation

Regularization theory was first introduced by Tikhonov and Miller. It provides a formal basis for the development of regularized solutions of ill posed problems [12, 13]. The stabilizing function technique is one of the basic methodologies for the development of regularized solutions. According to this approach, an ill-posed problem can be regarded as the constrained minimization of a certain function, called stabilizing function. The specific constraints imposed by the stabilizing function technique on the solution depend on the form and the properties of the stabilizing function used.

According to the regularization approach, the solution of eq.(6) is obtained by the minimization of the following cost function [11,12,13]:

$$\Psi(\mathbf{x}) = \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 + \lambda\|\mathbf{C}\mathbf{x}\|^2 \quad (9)$$

where $\mathbf{C}$ is the regularization operator and λ is the regularization parameter .
This minimization is accomplished by taking the derivative of the cost function yielding:

$$\frac{\partial\Psi(\mathbf{x})}{\partial\mathbf{x}} = \mathbf{0} = 2\mathbf{H}^t(\mathbf{g} - \mathbf{H}\hat{\mathbf{x}}) - 2\lambda\mathbf{C}^t\mathbf{C}\hat{\mathbf{x}} \quad (10)$$

Solving for that $\hat{\mathbf{x}}$ that provides the minimum of the cost function yields [11,12,13]:

$$\hat{\mathbf{x}} = (\mathbf{H}^t\mathbf{H} + \lambda\mathbf{C}^t\mathbf{C})^{-1}\mathbf{H}^t\mathbf{y} \quad (11)$$

The solution of the regularized image interpolation problem can't be solved as a direct inversion process for the whole image due to the large computational complexity of the inversion process. One of the possible previously suggested solutions to this problem is to use a successive approximation for the solution.

In this paper, we suggest another solution to the regularized image interpolation problem, This solution is implemented by the segmentation of the low resolution image into overlapping segments and interpolating each segment alone using equation (11) as an inverse problem. It is clear that, if a global regularization parameter is used a single matrix inversion process for a matrix of small dimensions is required because the term $(\mathbf{H}^t\mathbf{H}+\lambda\mathbf{C}^t\mathbf{C})^{-1}$ is independent on the image to be interpolated. Thus the algorithm is efficient from the point of view of computational complexity.

Experimental Results

In this section, the suggested implementation for the regularized image interpolation algorithm is tested on a low-resolution image. The result is compared to the interpolation results obtained using the traditional polynomial image interpolation algorithms such as the bilinear, the bicubic and the cubic spline algorithms.

The low-resolution image is obtained through a filtering and down sampling. In this paper, we perform a process of down sampling by 2 and the suggested technique can be implemented for any down sampling factor. The down sampled image is illustrated in Fig. (2). Figure (3) gives the original aero plane image of size 256 X 256. Figure (4) gives the low resolution degraded image down sampled by a factor of 2 and contaminated by additive Gaussian noise of variance 10^{-4}. This image is of size 128 X 128. The image in Fig. (4) is interpolated using the bilinear, the bicubic and the cubic spline algorithms in Figs.(5) , (6) and (7) respectively.

The regularized image interpolation algorithm is tested on the same low-resolution image with a global regularization parameter λ=0.001. In our experiment, the low-resolution image is segmented into overlapping blocks of size 12X12 pixels each. Each block is interpolated separately to the size of 24 X 24 pixels and 8 pixels are removed from the four sides of each block to yield a small block of size 8 X 8 in order to avoid the edge effects. The regularized interpolation result is illustrated in Fig.(8). By the process of segmentation and the usage of a global regularization parameter, this technique requires a single matrix inversion of size 256X256, which is a moderate size. The same result can be obtained for interpolating an image of any size. It is clear that the result obtained using this technique is the best result from the MSE point of view.

It is also clear that the suggested implementation for image interpolation is better than the traditional image interpolation algorithms from the computational complexity point of view since it interpolates the image on a block by block basis and requires a single matrix inversion for the whole image interpolation process.

Conclusion

This paper suggests an efficient implementation of the regularized image interpolation problem as an inverse problem. The suggested implementation reduces the computational complexity of the image interpolation problem to a single matrix inversion problem of moderate dimensions. The results using the regularized image interpolation algorithm is compared to the results using the traditional polynomial image interpolation algorithms. The regularized image interpolation algorithm with our simplified suggested implementation has proved to be superior from the MSE point of view and from the visual quality point of view.

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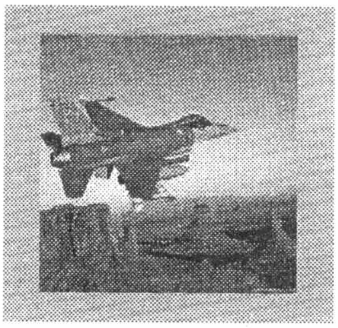

Fig.(3) Original aero plane image.

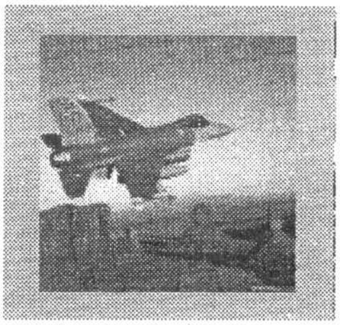

Fig.(4) Down sampled noisy image.

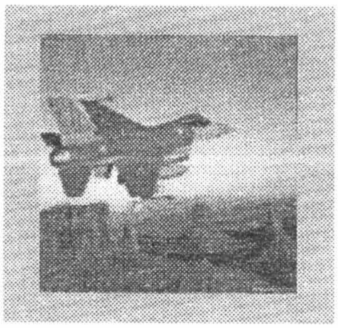

Fig.(5) Bilinear Interpolated image. MSE= 153.77

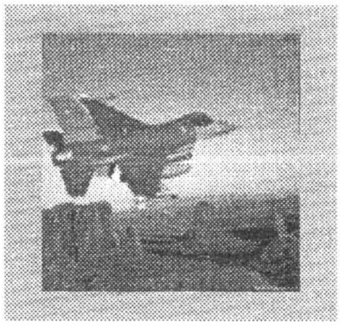

Fig.(6) Bicubic Interpolated image. MSE= 148.89

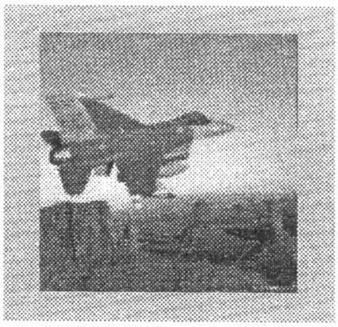

Fig.(7) Cubic Spline Interpolated image. MSE= 157.03

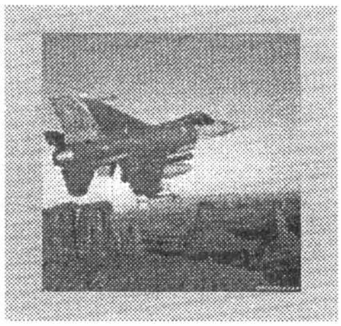

Fig.(8) Regularized Interpolated image. MSE= 80.83