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New Approach for Vibration Control of Cantilevered Structures

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1- Abstract

A new approach for suppression and control of mechanical vibration in cantilevered structures undergoing cyclic motion is presented [1]. The proposed model is based on the idea of generating axial uniform distributed forces on the superficial fibers of the vibrating structure. These forces are imposed on structure in such way that their vertical components act in a direction always opposing the rotation of the vibrating elements of the structure. Moreover, the damping level according to this model is dependent on the axial force value.
Equation of motion for the new model are obtained where, the effectiveness of this model for reducing lateral vibration of a base excited cantilevered beam is determined theoretically at different force values. It is shown that the higher the force value, the higher the attenuation percentage. The new model is characterized by its simplicity, which enhances its reliability and reduce its cost, as it provide the desired results with higher reliability and low cost, compared with other approaches of active and intelligent structural designs.

Key Words
Vibration, Control, Passive, Damping and Structure

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2- Introduction

The reduction of vibration responses and the transmission of vibratory energies in structures and mechanical systems have been a subject of investigation for many years. These requirements have motivated different means for vibrations control since the fourties.

In general, this objective can be achieved in a number of ways, such as reducing vibratory energy at the source; designing the system to have specific resonance frequencies to avoid the coincidence of the exciting frequencies and the system natural frequencies, and providing means for dissipating the energy.

One of the simple means for dissipating the vibratory energy in structures is the use of viscoelastic material perfectly glued on the surface of the vibrationg structure. This technique is called the unconstrained layer damping (UCLD). But the UCLD systems are often of low damping ratios, which are unacceptable for many mechanical structures. On 1959, Kerwin, et al [2], first did a fundamental work in what is called passive constrained layer damping (PCLD), and since that time there were many investigators who developed this technique [3-8], but unfortunately, the PCLD systems were not enough to reach the desired attenuation percentage. In addition to that the PCLD systems showed a mode dependency, which reduced its efficiency and increased its production cost.

So, the conventional structural designs are often unacceptable in coping with modern problems of structural resonance caused by the complex nature of the dynamic environment and the requirements of design objectives. These requirements have motivated a new approach to structural design where feedback controls principles and advances in sensors and actuators are applied to the design of high performance structural systems. Active Constrained Layer Damping (ACLD) treatments have been recognized as effective means for damping out the vibration of flexible structures. The effectiveness of the (ACLD) treatment is determined using distributed- parameter methods [9-16], or Finite Element Analysis (FEA) [17,18]. But still these new techniques showed high cost, low reliability and complexity.

In the present paper, a passive control new approach for vibration control in cantilevered structures is presented, showing high damping characteristic, low cost, simplicity and high reliability.

3- Equation of Motion of the New Model

Consider the forces and moments acting on an element of a beam, sandwiched between two elastic damping layers, undergoing cyclic motion, as shown in Figure (1) where $w(x,t)$ is measured from an inertial frame of reference and the beam is in bending during its upwards half cycle. These two elastic damping layers are used to generate the superficial axial forces F_0. Figure (1.b) shows the forces and moments acting on an element dx of the sandwiched beam.

For simplification, it is assumed that

1- The shear strain in the base beam is negligible.
2- The lateral displacements $w(x,t)$ of all points on the same cross-section of the beam are considered to be equal.
3- The effect of the rotary inertia in the beam element is negligible.
4- Axial uniformly distributed force per unit length of the beam is constant along the beam length.

By summing forces in the x-direction, one obtain

$$2\frac{\partial P}{\partial x}+\frac{\partial N}{\partial x}=2F_0 \tag{1}$$

Also, the summation of forces in the y-direction, results in

$$2\left(P\frac{\partial \theta}{\partial x}+\frac{\partial P}{\partial x}\theta\right)+\left(N\frac{\partial \theta}{\partial x}+\frac{\partial N}{\partial x}\theta\right)-\frac{\partial Q}{\partial x}-2F_0\theta=m\frac{\partial^2 w(x,t)}{\partial t^2} \tag{2}$$

Where, $m=2\rho_d A_d+\rho_3 A_3$ is the mass per unit length of the sandwiched beam.
Substitution of equation (1) into equation (2), results in

$$2P\frac{\partial \theta}{\partial x}+N\frac{\partial \theta}{\partial x}-\frac{\partial Q}{\partial x}=m\frac{\partial^2 w(x,t)}{\partial t^2} \tag{3}$$

The summation of the moments about any point on the right face of the beam element, results in

$$Q=\frac{\partial M}{\partial x}+N\theta+2S+2P\theta \tag{4}$$

Substitution of equation (4) into equation (3), results in

$$\frac{\partial^2 M}{\partial x^2}+\frac{\partial N}{\partial x}\theta+2\frac{\partial S}{\partial x}+2\frac{\partial P}{\partial x}\theta=m\frac{\partial^2 w(x,t)}{\partial t^2} \tag{5}$$

From the elementary theory of bending of beams (Euler-Bernoulli thin beam theory), the relationship between moment and deflection can be expressed as

$$M(x,t)=D\frac{\partial^2 w(x,t)}{\partial x^2} \tag{6}$$

Where $D=2E_d I_d+EI$ also $I_d=\frac{1}{12}b_d h_d^3$ and $I=\frac{1}{12}bh^3$
Substitution of equations (1) and (6) into equation (5), results in

$$\frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2 w(x,t)}{\partial x^2}\right)+m\frac{\partial^2 w(x,t)}{\partial t^2}+2\frac{\partial S}{\partial x}+2F_0\theta=0 \tag{7}$$

From the geometry of the sandwiched beam element in bending, the slope angle θ can be expressed as

$$\theta=\frac{\partial w(x,t)}{\partial x} \tag{8}$$

And the shear force S in the damping layer is expressed as

$$S=G_d A_d \gamma_d=G_d A_d\frac{\partial w(x,t)}{\partial x} \tag{9}$$

Substitution of equations (8) and (9) into equation (7), results in the partial differential equation of motion of the new model as

$$D\frac{\partial^4 w(x,t)}{\partial x^4}+2G_d A_d\frac{\partial^2 w(x,t)}{\partial x^2}+2F_0\frac{\partial w(x,t)}{\partial x}+m\frac{\partial^2 w(x,t)}{\partial t^2}=0 \tag{10}$$

If the value of the superficial force vanishes, equation (10) becomes in the following reduced form

$$D\frac{\partial^4 w(x,t)}{\partial x^4}+2G_d A_d\frac{\partial^2 w(x,t)}{\partial x^2}+m\frac{\partial^2 w(x,t)}{\partial t^2}=0 \tag{11}$$

Which is identical to the equation of motion of the Beam/UCLD system in bending vibration.

Considering the coordinate system shown in Figure (2) for base excitation, equation (11) becomes

$$D\frac{\partial^4 w(x,t)}{\partial x^4}+2G_dA_d\frac{\partial^2 w(x,t)}{\partial x^2}+2F_0\frac{\partial w(x,t)}{\partial x}+m\left(\frac{\partial^2 w(x,t)}{\partial t^2}+\frac{\partial^2 y^b(t)}{\partial t^2}\right)=0 \tag{12}$$

4- Solution of Equation of Motion

For harmonic base excitation, the base displacement is given by

$$y^b(t)=y_0e^{j\Omega t} \tag{13}$$

and the corresponding response is assumed to be

$$w(x,t)=\sum_{n=1}^{\infty}W_n(x)q_n(t) \tag{14}$$

For a cantilevered beam, the n[th] mode shape has the form

$$W_n(x)=\cosh\beta_n x-\cos\beta_n x-\sigma_n(\sin\beta_n x-\sinh\beta_n x) \tag{15}$$

Where $\left(\beta_n\right)^4=\frac{m}{D}\left(\omega_n\right)^2,\qquad \sigma_n=\frac{\sinh\beta_nL-\sin\beta_nL}{\cosh\beta_nL+\cos\beta_nL}$

Substitution of equations (13) and (14) into equation (12), it results

$$\begin{aligned}&D\sum_{n=1}^{\infty}\frac{d^4W_n(x)}{dx^4}q_n(t)+2G_dA_d\sum_{n=1}^{\infty}\frac{d^2W_n(x)}{dx^2}q_n(t)+2F_0\sum_{n=1}^{\infty}\frac{dW_n(x)}{dx}q_n(t)\\&+m\sum_{n=1}^{\infty}W_n(x)\frac{d^2q_n(t)}{\partial t^2}=m\Omega^2y_0e^{j\Omega t}\end{aligned} \tag{16}$$

Multiplying both sides of the above equation by $W_n(x)dx$ and integrating over the whole length of the beam, it results in

$$\begin{aligned}&\left[DL\left(\beta_n\right)^4+2G_dA_d\sigma_n\beta_n(2-\sigma_n\beta_nL)+4F_0\right]q_n(t)+mL\frac{d^2q_n(t)}{dt^2}\\&=m\Omega^2y_0e^{j\Omega t}\frac{2\sigma_n}{\beta_n}\end{aligned} \tag{17}$$

Let $q_n(t)=Ae^{j\Omega t}$ (18)

Substitution of equation (18) into equation (17), it results in

$$q_n(t)=\frac{\gamma_n}{\left(\omega_n\right)^2-\Omega^2}e^{j\Omega t} \tag{19}$$

where $\gamma_n=2\frac{\sigma_n}{\beta_n}\frac{y_0}{L}\Omega^2$

and $$\left(\omega_n\right)^2=\frac{D\left(\beta_n\right)^4}{m}+\frac{2G_dA_d\sigma_n\beta_n}{mL}\left(2-\sigma_n\beta_nL\right)+\frac{4F_0}{mL} \tag{20}$$

Therefore, the total response of the harmonically base excited system is

$$w(x,t)=\sum_{n=1}^{\infty}\left(\frac{\gamma_n}{\left(\omega_n\right)^2-\Omega^2}e^{j\Omega t}W_n(x)\right) \tag{21}$$

And the transfer function of the system, $[\alpha(\Omega,x)]$ is

$$\alpha(\Omega,x)=\sum_{n=1}^{\infty}\left[\frac{\left(2\frac{\sigma_n}{\beta_n}\frac{\Omega^2}{L}\right)}{\left(\omega_n\right)^2-\Omega^2}W_n(x)\right] \tag{22}$$

Due to the rheological behavior of the beam material and the damping material, the Young's modulii of the base beam and the damping layers is expressed as follows [18].

$$E=E'(1+\eta j),\ G_d=G_d'(1+\eta_d j)\ \text{and}\ E_d=E_d'(1+\eta_d j) \tag{23}$$

and the superficial forces which are generated in the damping layers, can be expressed as follows

$$F_0=F_0'+jF_0'' \tag{24}$$

where $F_0''=\eta_d F_0'$

Substitution of equation (23) and (24) into equation (22), the natural frequency of the new model becomes

$$\left(\omega_n\right)^2=\left(\omega_n'\right)^2+\left[\left(\beta_n\right)^4\left(\frac{2E_d'I_d\eta_d+E'I\eta}{m}\right)+\frac{2G_d'\eta_d A_d\sigma_n\beta_n}{mL}\left(2-\sigma_n\beta_n L\right)+4F_0''\right]j \tag{25}$$

where $\left(\omega_n'\right)^2=\left(\beta_n\right)^4\left(\frac{2E_d'I_d+E'I}{m}\right)+\frac{2G_d'A_d\sigma_n\beta_n}{mL}\left(2-\sigma_n\beta_n L\right)+4F_0'$

Substitution of equation (25) into equation (22), it results

$$\left|\alpha(\Omega,x)\right|=$$

$$\sum_{n=1}^{\infty}\frac{\left(2\frac{\sigma_n}{\beta_n}\frac{\Omega^2}{L}\right)W_n(x)}{\sqrt{\left[\left(\omega_n'\right)^2-\Omega^2\right]^2+\left[\left(\beta_n\right)^4\left(\frac{2E_d'I_d\eta_d+E'I\eta}{m}\right)+\frac{2G_d'\eta_d A_d\sigma_n\beta_n}{mL}\left(2-\sigma_n\beta_n L\right)+4F_0''\right]^2}} \tag{26}$$

Equation (26) gives the transfer function $\alpha(\Omega,x)$, which represents relative lateral displacement $w(x,t)$ of the Beam/PTLD system element measured from the moving frame (xyz), as shown in Figure (2), divided by the base harmonic displacement $y(t)$ measured from the fixed frame (XYZ).

5- Assessment of the System Performance at Different Values of Superficial Forces

The effect of the generated superficial force on the response of the Beam/PTLD system is assessed. The attenuation percentage is calculated for different superficial forces values, where, the comparison is made with respect to the response of the plain beam as a fixed reference.
The attenuation percentage is calculated from the following formula

$$\Delta_n^t(\omega_n, L) = \frac{\left|\alpha'(\omega_n, L)\right| - \left|\alpha(\omega_n, L)\right|}{\left|\alpha(\omega_n, L)\right|} 100\% \qquad (27)$$

It is necessary before studying the performance of the system, to study the performance of the same system but when the superficial forces vanishes, at this moment, the system response is that response of the Beam/UCLD system, and the damping produced is due to the direct and shear strains in the damping layers, in addition to the structural damping of the base structure. Figure (4) gives the comparison between the Beam/UCLD system and the corresponding plain beam.
Figure (5) gives the comparison between the magnitudes of the frequency response functions of the system at different values of the superficial force within the elastic range of the damping material. These figures indicate the high damping efficiency of the new approach. Table (1) gives the relation between the value of the initial strain, the superficial force and the attenuation percentage of the new model at 1st resonance.

Table (1) the attenuation percentage at different superficial force

Strain	Superficial force [N]	Attenuation ratio[%]
0.002	0.103	63.70%
0.004	0.207	67.54%
0.008	0.415	80.45%
0.016	0.830	89.79%

6- Conclusion

In the present paper, a new passive control model used for suppression of the lateral vibrations of a flexible base excited cantilevered beam, is developed. This new approach, which is called pre-tensioned layer damping (PTLD), is a sort of artificial damping techniques, which is based on the theory of energy dissipation from vibrating systems. The presented model has a closed form solution for the beam/PTLD system. The new approach is simple, reliable and inexpensive technique, which show a major advance in the attenuation of the vibration amplitude. Using this simple model enhance the damping ratio without the complications of control devices used in the case of Active Constrain Layer Damping (ACLD) in addition, it has not the limitation of damping in the case of Passive Constrained Layer Damping PCLD technique. It is noticed that the higher the elastic superficial forces in the pre-tensioned layer damping material, the higher is the attenuation percentage and the damping effectiveness.

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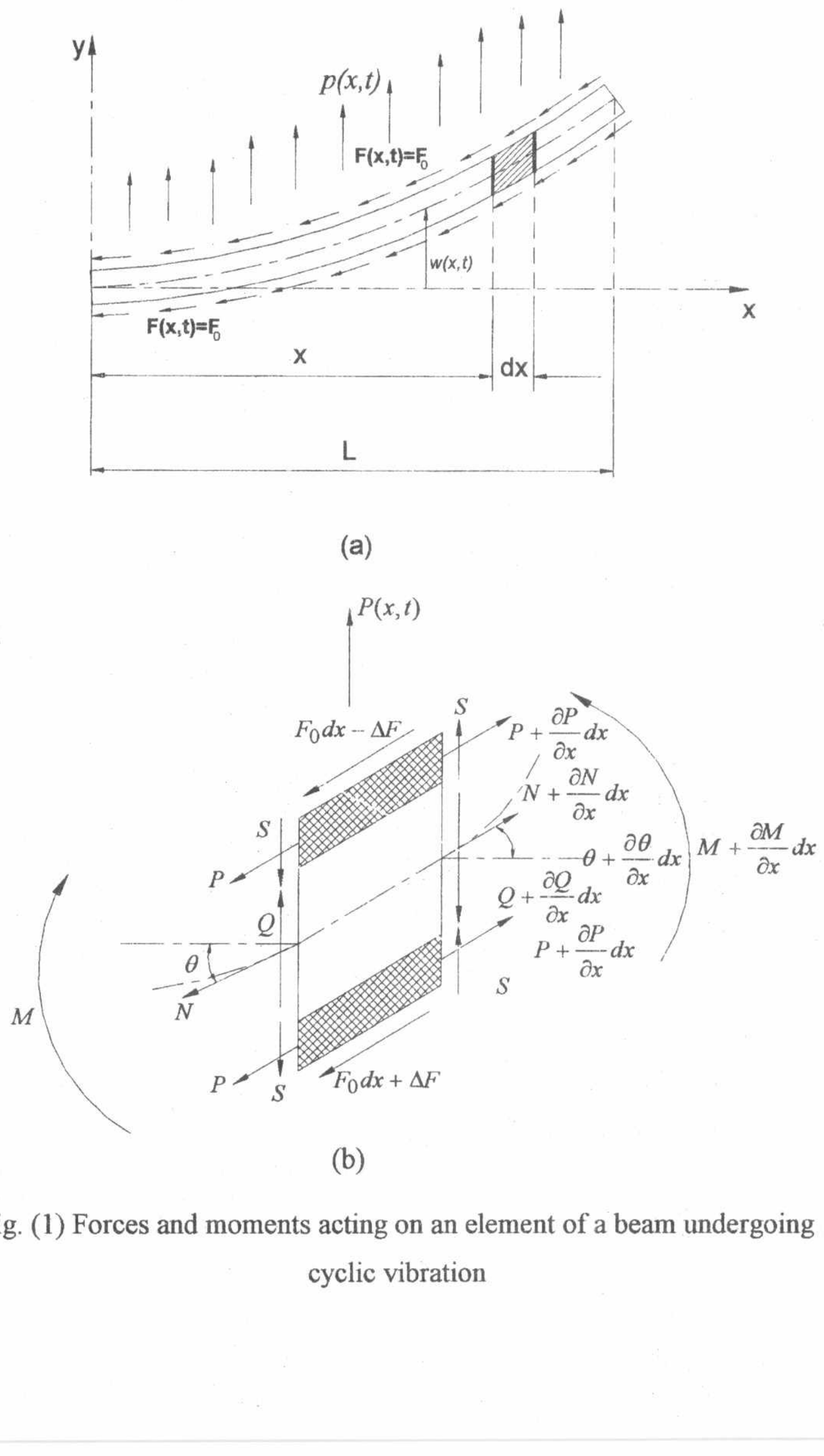


Fig. (1) Forces and moments acting on an element of a beam undergoing cyclic vibration

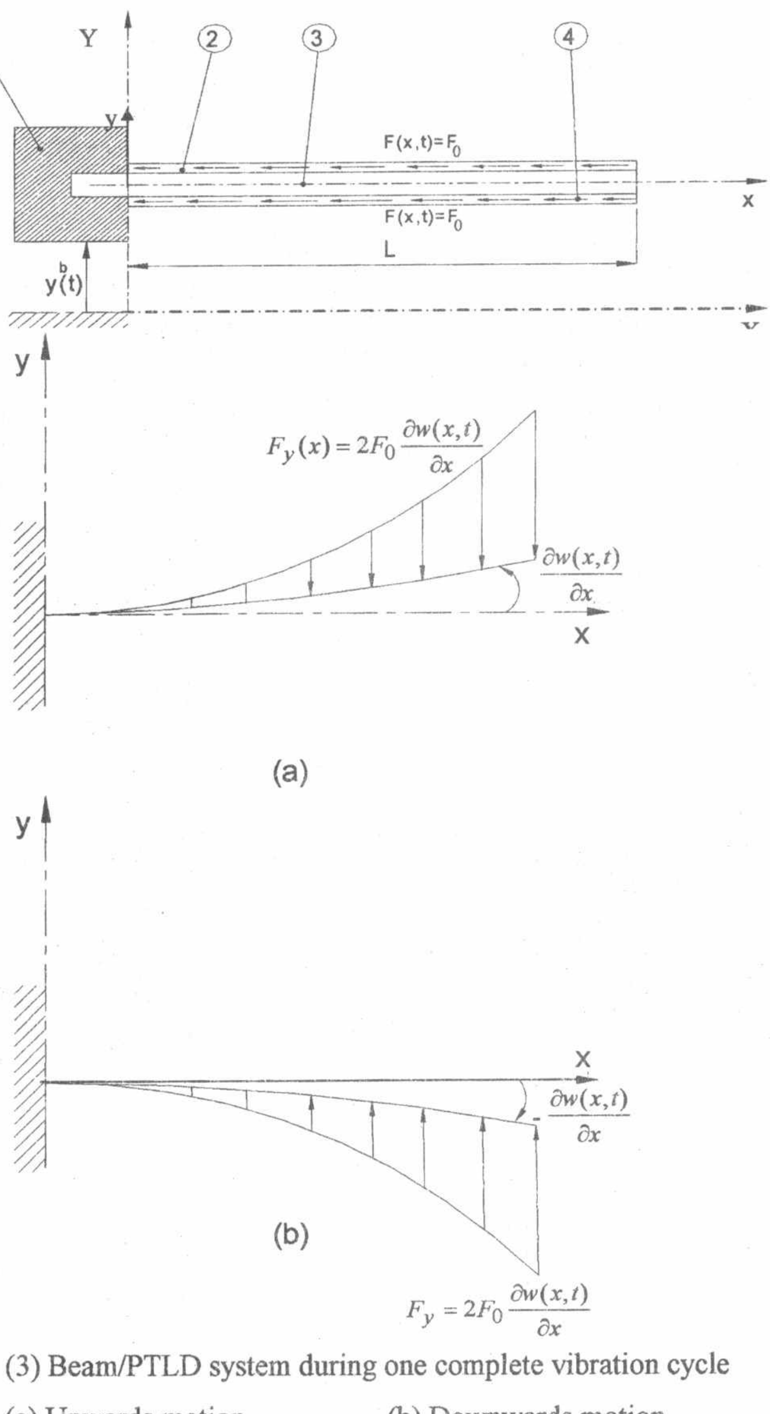


Fig. (3) Beam/PTLD system during one complete vibration cycle

(a) Upwards motion (b) Downwards motion

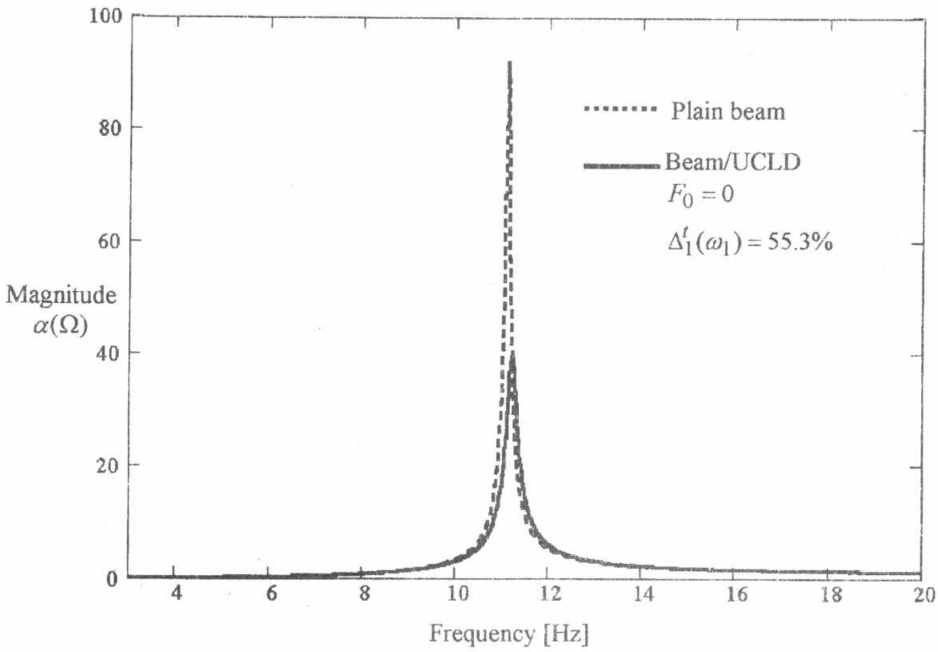
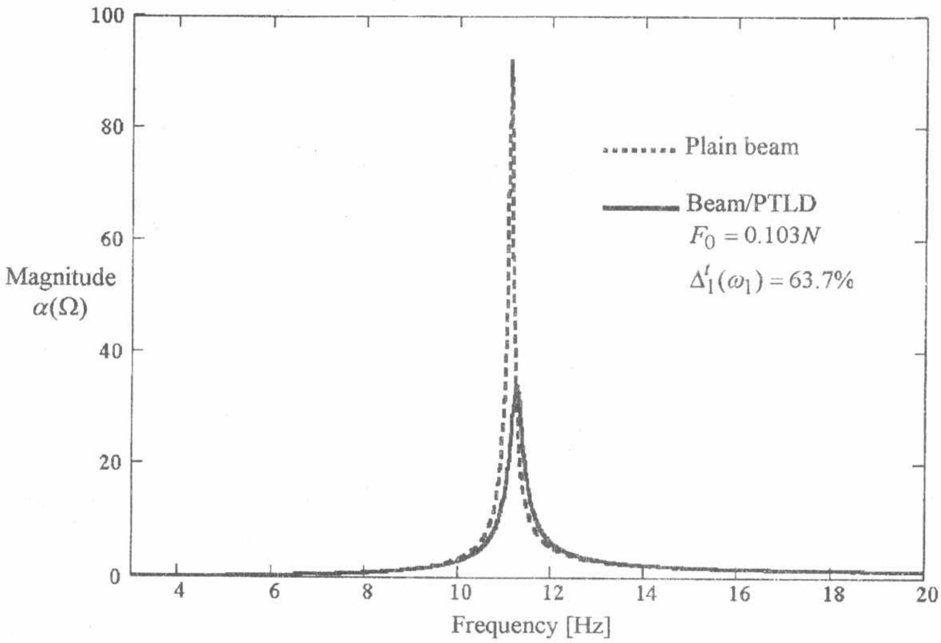


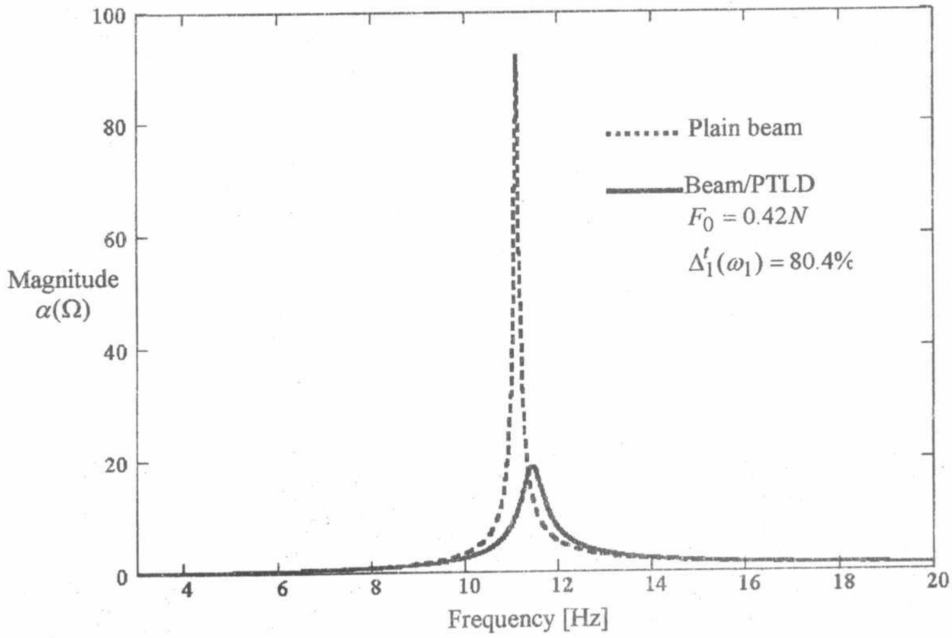
Fig. (4) Attenuation percentage of the Beam/UCLD system at 1st resonance, $\varepsilon_d = 0\mu strain$



(a)

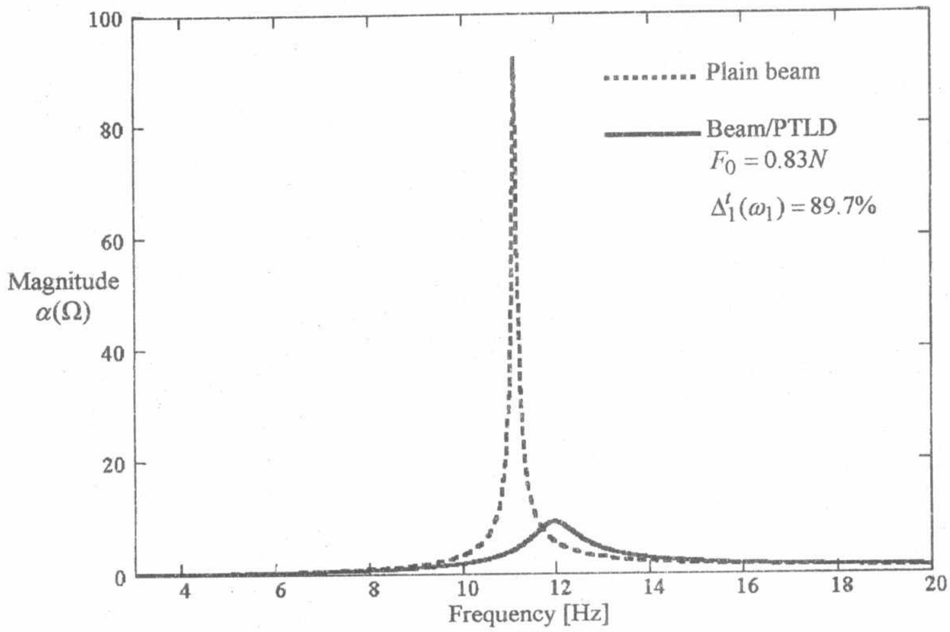
Fig. (5) Attenuation percentage of the Beam/PTLD system at 1st resonance

(a) $\varepsilon_d = 2000\mu\varepsilon$



(b)

Fig. (5) Attenuation percentage of the Beam/PTLD system at 1st resonance



(c)

Fig. (5) Attenuation percentage of the Beam/PTLD system at 1st resonance

(c) $\varepsilon = 16000\mu strain$