MATHEMATICAL MODEL & COMPUTER PROGRAM DEVELOPED FOR CALCULATING THE TOTAL RADIATION ON SLOPED SOLAR SURFACE COLLECTORS BASED ON OPTIMUM TILT ANGLE IN AMMAN, JORDAN

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ABSTRACT

Global radiation received on an inclined surface is composed of beam and diffuse radiation as well as reflected radiation from the surroundings. The main subjects of the analysis on global radiation on a tilted surface is

1. daily optimum tilt angle
2. monthly optimum tilt angle
3. yearly optimum tilt angle

Models available in literature to calculate the global radiation on an inclined surface are similar except for the diffuse radiation component. Among these approaches the simplification of the distribution of diffuse radiation in the atmosphere to be isotropic is the simplest and has a minor effect on the accuracy of prediction of the global radiation. because the diffuse radiation contributes 20% of global radiation in clear skies.

A computer program is constructed on mathematical formulation to find the hourly, daily and monthly average values of optimum tilt angle, optimum orientation and maximum total radiation on the tilted surface of the collector. The daily and monthly average values of optimum tilt angle are calculated, so that the average values of the maximum total radiation will be near to its hourly values.

The following results have been drawn:

1. The optimum slope is found to depend on several parameters, namely $\beta_{opt} = f(\phi, n, \gamma, \rho, C, Z, A_1, W_p)$ the first three of these parameters are the most important; the effect of the remaining parameters is not so strong.

Since diffuse radiation decreases $\beta_{opt}$ and reflected radiation increases $\beta_{opt}$ combined they tend to neutralize each other's effect, resulting in surprisingly good estimates for $\beta_{opt}$.

2. The daily optimum tilt angle, for a flat plate collector facing south and concentrating, changes throughout the year with its minimum value in June and maximum value in December.

3. The yearly average of the daily optimum tilt angles is found to be 32.63° and 54.19° for a flat plate collector facing south and a collector with the optimum orientation, respectively.

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KEY WORD: optimum tilt angle, optimum surface azimuth angle, flat plate collector, mathematical modeling, solar radiation.

NOMENCLATURE

\( a_0, a_1 = \) constants for the standard clear atmosphere with 23 km visibility
\( a_0^*, a_1^* = \) constants for the standard clear atmosphere for altitudes less than 2.5 km
\( A = \) altitude of the observer (km)
\( C = \) Diffuse Sky Factor
\( D = \) Daily Radiation Received on a surface
\( H_f = \) daily total radiation on a tilted surface (MJ m\(^2\))
\( I = \) the direct solar radiation on the at normal incidence
\( I_b = \) hourly beam radiation on a horizontal surface (MJ/m\(^2\))
\( I_{bt} = \) hourly beam radiation on a tilted surface (MJ/m\(^2\))
\( I_{c b} = \) hourly clear sky beam radiation on a horizontal surface (MJ/m\(^2\))
\( I_{cd} = \) hourly clear sky diffuse radiation (MJ/m\(^2\))
\( I_d = \) hourly diffuse radiation on a horizontal surface (MJ/m\(^2\))
\( I_e = \) hourly extra-terrestrial radiation on a horizontal surface (MJ/m\(^2\))
\( I_{es} = \) hourly extra-terrestrial radiation, measured on the plane normal to the radiation (MJ/m\(^2\))
\( I_{sc} = \) solar constant (1353 W/m\(^2\) - 4.8708 MJ/h.m\(^2\))
\( I_T = \) hourly total radiation on a tilted surface (MJ/m\(^2\))
\( D(\beta, \alpha) = \) the diffuse solar radiation,
\( R(\beta, \alpha) = \) the reflected global radiation
\( I(\beta, \alpha) = \) the direct solar radiation on the tilted plane
\( k = \) constant for the standard clear atmosphere with 23 km visibility
\( K^* = \) constant for the standard clear atmosphere for altitudes less than 2.5 km
\( n = \) day of the year
\( r, r_k, r_o = \) correction factors for climate types
\( R_g = \) geometric factor,
\( \frac{I_{bf}}{I_b} = \) the ratio of beam radiation or the tilted surface to that on a horizontal surface

Greek letters

\( \beta = \) collector tilt angle (slope), that is, the angle between the collector and the horizontal (degrees)
\( \beta_{opt} = \) optimum tilt angle (degrees)
\( \gamma = \) surface azimuth angle (degrees)
\( \gamma_{opt} = \) optimum surface azimuth angle (degrees)
\( \eta = \) the angle between the direction of the sun and the normal of the tilted plane.
\( \psi = \) solar azimuth angle
\( \gamma = \) solar elevation angle
\( \alpha = \) azimuth angle of the normal of the plane
\( \delta = \) declination angle (degrees)
\( \theta = \) incidence angle (degrees)
\[ \theta_z = \text{zenith angle (degrees)} \]
\[ \rho = \text{diffuse ground reflectance} \]
\[ \tau_b = \text{atmospheric transmittance for beam radiation} \]
\[ \tau_d = \text{atmospheric transmittance for diffuse radiation} \]
\[ \Phi = \text{latitude angle (degrees)} \]
\[ \omega = \text{hour angle (degrees). the phase type.} \]
\[ W_s = \text{is the sunset hour.} \]

**INTRODUCTION**

The solar collectors concentrate sunlight to heat a heat transfer fluid to a high temperature. The hot heat transfer fluid is then used to generate steam that drives the power conversion subsystem, producing electricity. Thermal energy storage provides heat for operation during periods without adequate sunshine.

Another way to generate electricity from solar energy is to use photovoltaic cells; magic slivers of silicon that convert the solar energy falling on them directly into electricity. Large scale applications of photovoltaic for power generation, either on the rooftops of houses or in large fields connected to the utility grid are promising as well to provide clean, safe and strategically sound alternatives to current methods of electricity generation.

**ECONOMIC AND ENVIRONMENTAL CONSIDERATIONS**

The most important factor driving the solar energy system design process is whether the energy it produces is economical. Although there are factors other than economics that enter into a decision of when to use solar energy; i.e. no pollution, no greenhouse gas generation, security of the energy resource etc., design decisions are almost exclusively dominated by the levelized energy cost. This or some similar economic parameter, gives the expected cost of the energy produced by the solar energy system, averaged over the lifetime of the system. In the following chapters, we will provide tools to aid in evaluating the factors that go into this calculation.

Commercial applications from a few kilowatts to hundreds of megawatts are now feasible, and plants totaling 354 MW have been in operation in California since the 1980s. Plants can function in dispatchable, grid-connected markets or in distributed, stand-alone applications. They are suitable for fossil-hybrid operation or can include cost-effective storage to meet dispatchability requirements. They can operate worldwide in regions having high beam-normal insolation, including large areas of the southwestern United States, and Central and South America, Africa, Australia, China, India, the Mediterranean region, and the Middle East. Commercial solar plants have achieved levelized energy costs of about 12-15¢/kWh, and the potential for cost reduction are expected to ultimately lead to costs as low as 5¢/kWh.

It is our hope that once the simplicity of solar energy system design is understood, engineers and manufacturers will provide new system designs that will expand the solar market worldwide and permit all to benefit from this clean, sustainable and distributed source of energy.
SOLAR POTENTIALS IN JORDAN

Jordan is one of the few countries in the world that hasn’t approval commercial technology readily available of energy (oil and gas). It has also experienced in the last five years a great development in the energy sector whereas the energy demand has increased by approximately 90% annually.

To face this situation it was essential that Jordan should direct some efforts towards developing and utilizing its potential indigenous sources of energy in the most appropriate, efficient and accelerated manner. Among the indigenous sources of energy, solar and wind technology rank high for applications in rural areas.

LITERATURE REVIEW

The energy absorbed by a solar collector depends upon its angle of tilt. The best way to collect maximum daily energy is to use tracking systems, but they are expensive, need energy for their operation, and are not always applicable. Therefore it is often more practicable to orient the absorber plate at an optimum tilt angle, and to correct the tilt from time to time. For this purpose, one should be able to determine the optimum slope of the absorber plate at any latitude, for any surface azimuth angle, and on any day of the year.

A flat plate collector is the simplest means available for solar energy collection. It is widely employed for applications such as water heating. The performance of a flat plate collector is highly influenced by its orientation and its angle of tilt with the horizontal. This is due to the fact that both the orientation and tilt angle change the solar radiation reaching the surface of the absorber and the overall heat losses. Some collectors are moved in prescribed ways to track the beam radiation to varying degrees. Tracking solar collectors are almost always of the concentrating type.

Several interesting articles have been devoted to this problem. Most of these articles treat the problem qualitatively and quantitatively. Practice for a long time has shown that in the northern hemisphere the optimum orientation is south facing and the optimum tilt depends on the latitude. Various papers have also pointed out this fact, and have made different recommendations for different locations.

Kern and Harris have optimized the slope based on only beam radiation and for equator-facing collectors, and obtained the following formula,

\[ \beta_{opt} \]

Chiou and El-Naggar have given an analytical method to determine the optimum tilt angle of an equator facing absorber plate only in the heating seasons i.e. from 21 September to 21 March in the Northern Hemisphere, and from 21 March to 21 September in the southern Hemisphere. They came up with expression of \( \beta_{opt} \).

The disadvantages of analytical methods suggested are:
- The variation of the clearness index is not taken into account
- They can be applied only to equator facing collectors
- To use then one must know \( H, H_b, \) and \( H_d \)

In addition Kern and Harris do not consider the effect of sky and ground diffuse radiation.

Koronakis has carried out a computer simulation to determine the total solar insolation on a south facing inclined surface and has used it to calculate optimum absorber tilt in Athens basin area.
Ladsaongikay and Parikh found that the tilt angle is a function of latitude and declination angles. They also observed that, the effect of tilt is more pronounced for places away from the equator, and that it is more advantageous to tilt the surfaces with the horizontal more during autumn and winter than summer.

Garg and Gupta concluded that, for average winter and summer performance, the optimum tilts are (latitude angle +15°) and (latitude angle — 15°), respectively, and for year round performance, the best tilt is 0.9 times the latitude angle.

Villarrubia et al. calculated the monthly averages of hourly irradiance upon surfaces tilted towards the south in Barcelona, Spain by using global and diffuse average hourly solar irradiance on a horizontal surface. They determined the optimum slopes corresponding to maximum incident energy, calculated from the annual average of daily solar radiation.

Willmott developed a numerical climatic model for computing total solar irradiance on the surface of a flat plate collector, positioned at any tilt and azimuth.

Badescu who performed calculations by using actinometric and thermal data measured at Bucharest, Romania, showed that solar air heater performances were strongly dependent on tilt, and orientation. Better performances of solar air heaters were obtained in the case of S, SE and SW orientations. Also, he concluded that optimum tilt angle was a function of orientation and there were no significant difference between the tilt angles, which optimize the incident solar radiation and the energy supplied by the collector. Of broader interest were the studies on the determination of the optimum slope of solar radiation receiving surfaces.

The aforementioned papers clearly indicate that, usually, the flat plate collector is oriented north-south to avoid the complicated task of tracking the sun and that the inclination of the solar collector to the horizontal influences the irradiance on that tilted surface. Review of literature further shows:

1. there is a wide range of $\beta_{\text{opt}}$ as recommended by different authors, and they are mostly for specific locations
2. none of analytical methods use $\gamma \neq 0$.

Therefore, in the present work, the attention is focused to find the optimum tilt angle for a flat plate collector facing south and the optimum orientation and tilt angle for concentrating or flat plate solar collectors tracking the sun on the basis of maximizing the total solar radiation reaching the collector surface over a specified period (a day, a month, a season, a year, etc.). The collectors are located in Amman (latitude of 32 N, Jordan).

EXPERIMENTAL MEASUREMENT OF OPTIMUM SLOPE

Recently The experimental optimum slopes were obtained in Jordan in 2006-2007. For this purpose, 6 identically manufactured plane solar collectors were used. The insolated surface of each of these collectors was equal $1m^2$. All collector had one glass cover. 6 collectors were placed at tilt angles 15,25,35,45, 55, and 65 to the horizontal surface south ($\gamma=0$).
MATHEMATICAL FORMULATION

Global radiation received on an inclined surface is composed of beam and diffuse radiation as well as reflected radiation from the surroundings. The beam component depends on geometrical considerations whereas the diffuse radiation depends on the distribution of diffuse radiation in the surrounding atmosphere as well as on the shape factor of the inclined surface. Reflected radiation, on the other hand is a function of the characteristics of the surrounding surfaces are diffuse reflectors.

Models available in literature to calculate the global radiation on an inclined surface are similar except for the diffuse radiation component. Among these approaches the simplification of the distribution of diffuse radiation in the atmosphere to be isotropic is the simplest and has a minor effect on the accuracy of prediction of the global radiation. This is because the diffuse radiation contributes only about 20% of global radiation in clear skies.

The main subjects of the analysis is as follows:

Optimum Tilt Angle For The Collection of Beam Radiation

1. Daily Optimum Tilt Angle
2. Monthly Optimum Tilt Angle
3. Yearly Optimum Tilt Angle

Daily Optimum Tilt Angle

The optimum tilt angle for any given day can be obtained by differentiating with respect to s and equating to zero

$$\frac{d}{dS} H_T = \frac{d}{d\beta} \left\{ \frac{12 \times 3600}{\pi} \left( \int_{-L}^{L} (l_{bt} + l_{drt} + l_{rT})d\omega \right) \right\} = \frac{d}{dS} \left\{ \frac{12 \times 3600}{\pi} \int_{-L}^{L} I_T d\omega \right\} = 0 \quad (1)$$

This yields the following equation:

$$S_{opt} = \tan^{-1} \left\{ \frac{[F_1 \cos L + F_2 \sin L]}{[F_1 \sin L + F_2 \cos L + F_3]} \left(1 - \frac{\rho}{2}\right) \right\} \quad (2)$$

where $L$ is sunset angle (rad) for horizontal surface

where

$$F_1 = \sin(\delta) \int I(b).d\omega$$

$$F_2 = \cos(\delta) \int I(b).\cos(w).d\omega$$

$$F_3 = (C/2) \int I(b).d\omega$$

$$C = \int I(b).d\omega$$

$$\rho = \frac{\beta}{\beta_{max}}$$

$$\omega = \frac{180}{\lambda} \times \cos L$$

$$\beta_{max} = \frac{90}{\lambda}$$

$$\lambda = 0.5296$$
where C is the sky diffuse factor. All integrations are performed between hour angles of sunrise to sunset.

**Monthly Optimum Tilt Angle**

The monthly total global radiation received on a tilted surface mounted towards the south can be obtained by integrating the daily total radiation for a given month as:

$$M(T) = \int \int G(T).d\omega .d\delta$$

where \(\delta_1\) and \(\delta_2\) refers to the solar declination angle at the beginning and end of the month, respectively.

The monthly optimum tilt angle can be obtained by differentiating equation with respect to \(s\) and equating to zero, which gives the following equation:

$$S_{opt} = \tan^{-1}\left\{ \frac{[-R_1 . \cos L + R_2 . \sin L]}{\{[R_1 . \sin L + R_2 . \cos L + R_3]\} \{1 - \frac{\rho_1}{2}\}} \right\}$$

where

\(R1 = \int \int I(b).\sin \delta .d\omega .d\delta\)

\(R2 = \int \int I(b).\cos \delta .d\omega .d\delta\)

\(R3 = (\frac{C}{2}) \int \int I(b).d\omega .d\delta\)

the limits of the first integral is between \(\omega(s,s)\) to \(\omega(s,r)\), whereas the limits of the second integral is between \(\delta_1\) and \(\delta_2\), where \(\delta_1\) and \(\delta_2\) refer to the solar declination at the beginning and at the if the month, respectively.

**Seasonal Optimum Tilt Angle**

The total global radiation intercepted by a tilted surface during a season can be obtained by performing the summation of the monthly totals of the global radiation reaching the tilted surface throughout the season i.e.

$$G(s) = \sum M(T[i])$$

where \(I\) refers to the sequence of a month in one year, i.e., \(i=1\) for January and \(i=12\) for December; \(M\) and \(N\) refers to the first and last month in a season, respectively.

$$G(s) = \sum S_i \left\{ R_{1i} \{\sin(1-s) + \left(\frac{\rho}{2}\right)(1-\cos(s)) \} + R_{2i} \{\cos(L - s) + \left(\frac{\rho}{2}\right)(1-\cos(s)).\cos(L)\} \right\}$$

$$G(s) = \sum R_{3i} \left\{ \left(\frac{C}{2}\right)(1+\cos(s)) + \left(\frac{\rho C}{2}\right)(1-\cos(s)) \right\}$$
The seasonal optimum tilt

\[
S_{opt} = \tan^{-1} \left\{ \frac{-\cos L \sum R_{ii} + \sin L \sum R_{ii}}{\left[\sin L \sum R_{ii} + \sum R_{2i} \cos L + R_{3i} \right] + \sum R_{2i} \cos L + R_{3i}} \right\} (9)
\]

where the limits of the summation is for \(i=M \) to \(i=N\)

**Yearly Optimum Tilt Angle**

The yearly optimum tilt angle can be obtained from equation (9) but the limits of \(i\) are for \(i=1 \) to \(i=12\).

Certain solar energy collection systems, such as concentrating collectors, utilize beam radiation only. For non-tracking concentrating systems, the optimum tilt angle is different from that of flat plate collectors which collect beam and diffuse radiation.

**Daily Optimum Tilt Angle**

The daily optimum tilt angle for any day of the year can be obtained by optimizing the collected beam radiation for that day. The daily total beam radiation received on a tilted surface may be obtained by eliminating \(I_d\) from equation and integrating from sunrise to sunset, i.e.

\[
\beta(T) = \int \left\{ (I(b) \cos(\theta(T)) + \rho \cdot I(b) \cos(\theta_z) \left(\frac{1-\cos(s)}{2}\right)) \right\} d\omega
\]

where \(\delta\) is the tilt angle.

The daily optimum tilt angle for beam radiation can be obtained similar to that for the global radiation, this is given by the following equation:

\[
\beta_{opt} = \tan^{-1} \left\{ \frac{-F_1 \cos L + F_2 \sin L}{F_1 \sin L + F_2 \cos L + F_3 (1 - \frac{\rho}{2})} \right\}
\]

**Monthly Optimum Tilt Angle**

The monthly optimum tilt angle for beam radiation may be obtained by maximizing the monthly summation of beam radiation on the tilted surface. The monthly total is given by the following equation:

\[
R_1 = \int \int \left\{ (I(b) \cos(\theta(T)) + \rho \cdot I(b) \cos(\theta_z) \left(\frac{1-\cos(s)}{2}\right)) \right\} d\omega \cdot d\delta
\]

The limits of integration are the same as that of equation . The monthly optimum tilt angle for beam radiation is obtained from equation
Seasonal Optimum Tilt Angle of Beam Radiation

The yearly optimum tilt angle for the collection of beam radiation can be obtained from equation (8). This can be done by eliminating \( R_3 \) which accounts for the diffuse radiation; the limits of the summation is for \( i=1 \) to \( i=12 \).

FORMULATION

Global Radiation On A Tilted Plane

\[
G(\beta, \alpha) = I(\beta, \alpha) + D(b, a) + R(\beta, \alpha) \tag{14}
\]

\[
I(\beta, \alpha) = I \cdot \cos(\eta) \tag{15}
\]

\[
\cos(\eta) = \sin(\gamma) \cdot \cos(\beta) + \cos(\gamma) \cdot \sin(\beta) \cdot \cos(\alpha - \psi) \tag{16}
\]

\[
I = \frac{B}{\sin(\gamma)} = \frac{G - D}{\sin(\gamma)} \tag{17}
\]

\[
D(\beta, \alpha) = D \cdot \cos2\left(\frac{\beta}{2}\right) \tag{18}
\]

\[
R(\beta, \alpha) = R \cdot \left[1 - \cos2\left(\frac{\beta}{2}\right)\right] = R \cdot \sin2\left(\frac{\beta}{2}\right) \tag{19}
\]

\[
R = Q_s \cdot G \tag{20}
\]

\[
D(\beta, \alpha) = D \cdot \cos2\left(\frac{\beta}{2}\right) \cdot \left[\frac{1 + \sin3\left(\frac{\beta}{2}\right)}{2}, (1 + \cos2(\eta) \cdot \cos3(\gamma))\right] \tag{21}
\]

\[
R(b, a) = R \cdot \sin2\left(\frac{\beta}{2}\right) \cdot \left[1 + \sin2\left(\frac{90° - \gamma}{2}\right) \cdot \cos(\alpha - \psi)\right] \tag{22}
\]

\[
F = 1 - \left(\frac{D}{G}\right)^2 \tag{23}
\]

\[
D(\beta, \alpha) = D \cdot \cos2\left(\frac{\beta}{2}\right) \cdot \left[1 + F \cdot \sin3\left(\frac{\beta}{2}\right), (1 + F \cdot \cos2(\eta) \cdot \cos3(\gamma))\right] \tag{24}
\]

\[
R(b, a) = R \cdot \sin2\left(\frac{\beta}{2}\right) \cdot \left[1 + F \cdot \sin2\left(\frac{90° - \gamma}{2}\right) \cdot \cos(\alpha - \psi)\right] \tag{25}
\]
Finally we end up with the optimum tilt angle, which gives the maximum total radiation on a flat plate solar collector,

$$\beta_{opt} = \tan^{-1} \left\{ \frac{\cos\gamma (\cos\delta \sin \phi \cos \omega - \sin \delta \cos \phi)}{\tau_1 (1-\rho) \frac{\cos \theta_z}{2} \tau_b - \rho \cos \theta_z + \sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega} \right\}$$  \hspace{1cm} (26)

Substituting $\beta_{opt}$ the maximum total radiation on a flat plate solar collector with optimum tilt angle is found, and the optimum surface azimuth angle (orientation) is given by

$$\gamma_{opt} = \tan^{-1} \left\{ \frac{\sin \omega}{\sin \phi \cos \omega - \tan \delta \cos \phi} \right\}$$  \hspace{1cm} (27)

A computer program is constructed on the basis of the aforementioned mathematical formulation to find the hourly, daily and monthly average values of optimum tilt angle, optimum orientation and maximum total radiation on the tilted surface of the collector. The daily and monthly average values of optimum tilt angle are calculated, so that the average values of the maximum total radiation will be near to its hourly values.

**RESULTS AND DISCUSSIONS**

Figure 1 shows the hourly variation of the maximum total solar radiation on a flat plate solar collector tilted with the optimum tilt angle and facing south in Amman for the average day of the selective months. It is clear from the figure that minimum and maximum radiations occur in December and June.

Figure 2 shows the hourly variation of the optimum tilt angle for the average day for each month in the year for a flat plate solar collector facing south in Amman. The average day of the month is that day which has the extra-terrestrial radiation closest to the average for the month. The figure shows that optimum tilt angle increases with the day time, reaching a maximum value at solar noon and then decreases during the months of April-September. Meanwhile, the optimum tilt angle decreases with the day time, reaching a minimum value at solar noon and then increases during the months of October-February. During March the optimum tilt angle is constant with the day time. The minimum and maximum variations of the optimum tilt angle during the year occurs in June and December, respectively.

Figure 3 shows the variations of the daily average optimum tilt angle and maximum total radiation throughout the year for a flat plate collector tilted with the daily average optimum tilt angle and facing a flat plate collector with a fixed tilt angle 27. It is observed that the daily optimum tilt angle varies from about 0 to 59, being a minimum in June and a maximum in December.

These results are least squares fitted to the following equation to find the daily optimum tilt angle as
\[ \beta_{oa} = 32.18 + 1.388 \sin(n + 10) + 30.8765 \cos(n + 10) \]  \hspace{1cm} (28)

where \( n \) is the day of the year starting from 1 January.

The yearly average of the daily optimum tilt angles was found to be 32.63°, which is near to the local latitude angle of Amman (32°), as commonly proposed by many researchers. The daily total radiation for a flat plate collector with a fixed tilt angle equal to 27° and facing south is also shown in Fig. 3. It is obvious that the yearly maximum total radiation calculated at the daily optimum tilt angle is greater than the yearly total radiation calculated at fixed tilt angle. The fixed tilt angle for each month was determined as the average of daily optimum tilt angles for that month.

Figure 4 shows the monthly variation of daily optimum tilt angle for solar collector. These monthly average tilt angles show that their variation during the periods May-July and November-January is not much, and therefore, the influence of keeping a fixed angle during these periods was also studied. As shown from Fig. 4 it is more advantageous to tilt the surfaces with the horizontal more during autumn and winter than summer.

Figure 5 shows the hourly variation of optimum surface azimuth angle for the average day for each month of the year. This variation is determined for a concentrating or a flat plate solar collector tracking the sun in Amman. It should be pointed out that a variety of orienting systems have been designed based on manual or mechanized operation. Manual systems are used in areas of low labour cost. Figure 5 shows that the optimum surface azimuth angle reaches maximum in June (\(-110.8 < \gamma_{opt} < 110.8\)), then decreases with time to a minimum value in December (-69.3 < \gamma_{opt} < 69.3) and increases with time to reach its maximum value again in June. Surface azimuth angle equals to zero for collectors facing south, east negative and west positive.

Figure 6 shows the hourly variation of optimum tilt angle for the average day for each month of the optimum orientation. This optimum tilt is calculated for the corresponding optimum surface azimuth angle at the same time. The figure shows that optimum tilt angle decreases with the day time, reaching a minimum value at solar noon and then increases during all months of the year. The minimum and maximum variations of the optimum tilt angle during the year occur in December and June, respectively.

Figure 7 shows the hourly variation of maximum total radiation on collector surface with optimum orientation and tilt angle for the average day of the months. It is clear from the figure that minimum and maximum radiations occur in December and June with maximum values of 3.04 and 3.37 MJ/h.m², respectively. Also, the figure shows the difference between the maximum total radiation at optimum orientation and tilt angle, and that maximum total radiation calculated at optimum tilt angle for a collector facing south.

Figure 8 shows the variations of daily average optimum tilt angle and maximum total radiation throughout the year for a collector tracking the sun and tilted with the daily average optimum tilt angle.
Figure 9 shows the monthly variation of daily optimum tilt angle. It shows that the variation of tilt angles during the periods May-July and November-January is not much, and therefore, a fixed tilt angle during these periods can be used without much error. For average (autumn/winter and spring/summer) performance, the optimum tilts are latitude +33.3° and latitude +20.7% respectively.

It is also observed that the loss in total radiation using fixed tilt angles during the periods April-August and November-January is 3.14% compared to that yearly total radiation calculated at the monthly average optimum tilt angles.

CONCLUSIONS

The optimum values of tilt angles and orientation for solar collectors in Amman were determined by developing a mathematical model and using a computer program. The mathematical model and computer program developed can be used to calculate the optimum tilt, surface azimuth angles and maximum total radiation on the solar collector at any time, altitude, climate and latitude.

The following conclusions have been drawn:

1. The optimum slope is found to depend on several parameters, namely $\beta_{opt} = f(\phi, \gamma, \rho, C, Z, A, W_p)$ the first three of these parameters are the most important; the effect of the remaining parameters is not so strong. Since diffuse radiation decreases $\beta_{opt}$ and reflected radiation increases $\beta_{opt}$ combined they tend to neutralize each other's effect, resulting in surprisingly good estimates for $\beta_{opt}$.

2. The daily optimum tilt angle, for a flat plate collector facing south, changes throughout the year with its minimum value in June and maximum value in December.

3. The daily optimum tilt angle, for a concentrating or a flat plate solar collector with optimum orientation, changes throughout the year with its minimum value in June and maximum value in December. Its value at any day of the year can be determined.

4. The yearly average of the daily optimum tilt angles is found to be 32.63° and 54.19° for a flat plate collector facing south and a collector with the optimum orientation, respectively.

REFERENCES

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Fig. 1 Hourly Variation of maximum solar radiation on tilted surface with optimum tilt angle for the average day for selective month and facing south in

- Dec
- Oct, Feb
- Jan

Maximum total radiation

Day time (h)
Fig. 2 Hourly Variation of the optimum tilt angle for the average day for each month in the year

Fig. 3 Variation of daily average optimum tilt angle and maximum total radiation throughout the year for a flat plate solar collector facing south with optimum tilt angle
Fig. 4 Monthly variation of daily optimum tilt angle for solar collector facing south

Fig. 5 shows the hourly variation of optimum surface azimuth angle for the average day for each month of the year.
Fig. 6 The hourly variation of optimum tilt angle for the average day for each month of the year of the optimum orientation.

Fig. 7 The hourly variation of maximum total radiation on the collector surface with optimum orientation and tilt angle for the average day of different months.
Fig. 8 The Variation of daily average optimum tilt angle and maximum total radiation throughout the year for a solar collector with optimum tilt angle and orientation.

Fig. 9 The monthly variation of daily optimum tilt angle for a solar collector with optimum orientation.