13th International Conference on
AEROSPACE SCIENCES & AVIATION TECHNOLOGY,
ASAT- 13, May 26 – 28, 2009, E-Mail: asat@mtc.edu.eg
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Peristaltic Pumping of Fluid in Cylindrical Tube and its Applications in the Field of Aerospace

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Abstract: This work is concerned with the peristaltic transport of a Newtonian and non-Newtonian Maxwellian fluid in an axisymetric cylindrical tube filled with a homogenous porous medium, in which the flow is induced by traveling transversal waves on the tube wall. ***Like in peristaltic pumping,*** the traveling transversal waves induce a net flow of the liquid inside the tube. The viscosity as well as the compressibility of the liquid is taken into account. Modified Darcy's law has been used to model the governing equation. The present theoretical model may be considered as mathematical representation to the case of gall bladder and bile duct with stones and dynamics of blood flow in living creatures. The Navier-Stokes equations for an axisymmetric cylindrical tube are solved by means of a perturbation analysis, in which the ratio of the wave amplitude to the radius of the tube is small parameter. In the second order approximation, a net flow induced by the traveling wave has been computed through numerical integration for various values of the compressibility of the liquid, relaxation time and the permeability parameter of porous medium. The calculations disclose that the compressibility of the liquid, the permeability parameter of porous medium and non-Newtonian effects in presence of peristaltic transport have a strong influence of the net flow rate. Finally, the graphical results are reported and discussed for various values of the physical parameters of interest.

Keywords: Peristaltic Pumping, Maxwell fluid, Compressible liquid, Traveling Waves, Porous medium, Modified Darcy's law, Non-Newtonian effects.

Nomenclature

R	Radius of the tube.
L	Length of the tube.
a	Amplitude of the traveling wave.
λ	Wave length.
c	Wave velocity.
(r,ϕ,z)	The cylindrical coordinate system.
$W(z,t)$	The vertical displacement of the tube.

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k	The permeability of porous medium.
λ_v	Retardation time.
t_m	Relaxation time.
μ	Dynamic viscosity.
p	Hydrostatic pressure.
$\vec{v}$	Velocity vector.
φ	Porosity of the porous medium.
ρ	Liquid density.
$\tilde{\tau}$	Viscous stress tensor.
k^*	Compressibility of the liquid.
ρ_o	Constant density at the reference pressure.
p_o	Reference pressure.
ε	Amplitude ratio.
α	Wave number.
Re	Reynolds number.
χ	Compressibility number.
I_0	Modified Bessel function of the first kind of order 0.
I_1	Modified Bessel function of the first kind of order 1.
$\langle Q \rangle$	Net flow rate.
De	Deborah number.
Dec	Critical value of Deborah number.

1. Introduction

Peristaltic transport is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of an extensible tube containing a liquid. It appears to be major mechanism for urine transport in ureter, food mixing and chyme movement in intestinal, transport of spermatozoa in cervical canal, transport of bile in bile ducts and so on. Technical roller and finger pumps also operate according to this rule.

Peristalsis can inspire many potential solutions in modern space robotic systems. Available state-of-the-art technologies for systems applied to in situ sample positioning and transport, particle conveyance from soi excavation, or distributed peristaltic pumps for active fluid transport and thermal control could gain in functionality efficiency without added complexity. To understand peristaltic action in different situations, several theoretical and experimental attempts have been made since the first investigation of Latham [1]. The literature on peristalsis is by now quite extensive. Important recent contributions to the topic include the works of Hayat et al. [2-6], Elshehawey and El-Sebaei [7], Mekheimer [8, 9].

Recently, Kothandapani and Srinivas [10,11] studied the effect of elasticity of the flexible walls on the MHD peristaltic flow of a Newtonian fluid in a two-dimensional porous channel with heat transfer under the assumptions of long wavelength and low-Reynolds number. Vajravelu et al. [12] studied the interaction of peristalsis with heat transfer for the flow of a viscous fluid in a vertical porous annular region between two concentric tubes. Manoranjan and Ramachandra [13] studied the peristaltic transport in a two dimensional channel, filled with a porous medium in the peripheral region and a Newtonian fluid in the core region. Elshehawey et al. [14] studied Peristaltic transport in an asymmetric channel through a porous medium. Hayat et al. [15,16] studied a mathematical model of peristalsis in tubes through a porous medium and Hall effects.

Recently, the motion of non-Newtonian fluids has been an important subject in the field of chemical, biomedical and environmental engineering and science. Undoubtedly the mechanics of non-Newtonian fluids presents special challenges to engineers, physicists, modelers, numerical simulates and mathematicians. This is due to the fact that nonlinearity manifests itself in a variety of ways. Recently, Abd Elnaby and Haroun [17] discussed the peristaltic flow of a viscous fluid in a channel having compliant walls. A recent few papers have been made in the filed of compressible fluid. Aarts and Ooms [18], Tsiklauri and Bresenev [19], Elshehawey et al. [20,21]

To the best of the author's knowledge, no attempt has been made yet to discuss the peristalsis of non-Newtonian Maxwellian fluid through a porous medium taking the compressibility of the liquid into account. Therefore the main purpose of the present paper is to present a theoretical analysis of the peristaltic flow of a non-Newtonian Maxwellian fluid through porous medium with constant permeability in a cylindrical tube. Modified Darcy's law for a Maxwell fluid has been used for the modeling. Actually we extend the analysis of Yin and Fung [22] by taken the permeability of porous medium into account. It is also an extension of the later work, Aarts and Ooms [18], in which the compressibility has been taken into account and of Tsiklauri and Bresenev [19], where the non-Newtonian effects have been incorporated.

2. Basic Equations and Problem Formulation

We consider an axisymmetric cylindrical tube of radius R and length L. We assume that an elastic wave induces a traveling wave on the wall (boundary) of the tube with the displacement of the following form:

$$W(z,t)=R+a\cos\left(\frac{2\pi}{\lambda}(z-ct)\right), \tag{1}$$

where a is the amplitude of the traveling wave, while λ and c are its wave length and velocity, respectively. We note that the z-axis of the (r,ϕ,z) cylindrical coordinate system is directed along the axis of the tube.

It is well known that in an unbounded porous medium, Darcy's law holds for a Newtonian fluid at low speed. This law provides a relation between pressure drop and velocity. According to this law the pressure drop induced by the frictional drag is directly proportional to the velocity. There are some studies [23,24] on Stock's problem involving viscoelastic fluid through a porous medium. There is no study on peristalsis dealing viscoelastic fluid in a porous medium. On the basis of Oldroyd's model [25-29] the following law has been suggested

$$\left(1+t_m\frac{\partial}{\partial t}\right)\nabla p=-\frac{\mu\varphi}{k}(1+\lambda_v)\vec{v}, \tag{2}$$

where k is the permeability, λ_v is the retardation time, t_m is the relaxation time, μ is the dynamic viscosity, p is the hydrostatic pressure, $\vec{v}$ is the velocity vector and φ is the porosity of the porous medium.

It is known that constitutive equation for Maxwell fluid can be obtained from the constitutive equation of an Oldroyd-B fluid by letting $\lambda_v = 0$. Since we have an interest in Maxwell fluid in this paper, the filtration law for Maxwell fluid can be inferred from Eq. (2) as follows:

$$\left(1+t_m \frac{\partial}{\partial t}\right)\nabla p = -\frac{\mu\varphi}{k}\vec{v}, \tag{3}$$

Note that for $t_m = 0$, the above equation yields Darcy's law. Since the pressure gradient in Eq. (3) can also be interpreted as a measure of the resistance to flow in the bulk of the porous medium and $\vec{r}$ is a measure of the flow resistance offered by the solid matrix. Therefore $\vec{r}$ can be inferred from Eq. (3) to satisfy the following equation [16]

$$\left(1+t_m \frac{\partial}{\partial t}\right)\vec{r} = -\frac{\mu\varphi}{k}\vec{v}, \tag{4}$$

Under the assumptions mentioned above the continuity and momentum equations governing the flow of a non-Newtonian Maxwellian fluid (taking its compressibility into account) through a porous medium in a cylindrical tube can be written as:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho\, \vec{v}) = 0, \tag{5}$$

$$\rho\frac{\partial \vec{v}}{\partial t} + \rho(\vec{v}.\vec{\nabla})\vec{v} = -\vec{\nabla}p - \vec{\nabla}\,\tilde{\tau} + \vec{r}, \tag{6}$$

where ρ is the liquid density, p the pressure, $\vec{v}$ the velocity vector and $\tilde{\tau}$ represents the viscous stress tensor . We describe the viscoelastic properties of the fluid using Maxwell's model [30], which assumes that

$$t_m \frac{\partial \tilde{\tau}}{\partial t} = -\mu\vec{\nabla}\vec{v} - \frac{\mu}{3}\vec{\nabla}\vec{v} - \tilde{\tau}. \tag{7}$$

We further, assume that the following equation of state holds [31]:

$$\frac{1}{\rho}\frac{\partial \rho}{\partial p} = k^*, \tag{8}$$

where k^* is the compressibility of the liquid. We also assume that the fluid's velocity has only r and z components. The solution of this equation for the density as a function in the pressure is given by

$$\rho = \rho_o e^{[k^*(p-p_o)]}. \tag{9}$$

where ρ_o is the constant density at the reference pressure p_o .
We make use of "no-slip" boundary condition at the boundary of the tube i.e.,

$$v_r(W,z,t)=\frac{\partial W}{\partial t},\qquad v_z(W,z,t)=0. \tag{10}$$

Eq. (7) can be rewritten in the following form:

$$(1+t_m\frac{\partial}{\partial t})\tilde{\tau}=-\mu\vec{\nabla}\,\vec{v}-\frac{\mu}{3}\vec{\nabla}.\vec{v}, \tag{11}$$

Further, we apply the operator $(1+t_m\frac{\partial}{\partial t})$ to the momentum Eq. (6) and eliminate $\tilde{\tau}$ in it using Eq. (11)

$$(1+t_m\frac{\partial}{\partial t})\left(\rho\frac{\partial\vec{v}}{\partial t}+\rho(\vec{v}.\vec{\nabla})\vec{v}\right)=-(1+t_m\frac{\partial}{\partial t})\vec{\nabla}p+\mu\vec{\nabla}^2\vec{v}+\frac{\mu}{3}\vec{\nabla}(\vec{\nabla}\vec{v})+(1+t_m\frac{\partial}{\partial t})\vec{r}, \tag{12}$$

In cylindrical coordinates, the balance of mass Eq. (5) reads

$$\frac{\partial\rho}{\partial t}+v_r\frac{\partial\rho}{\partial r}+v_z\frac{\partial\rho}{\partial z}+\rho\left(\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}\right)=0. \tag{13}$$

while the Navier-Stokes Eq. (12) becomes

$$\left(1+t_m\frac{\partial}{\partial t}\right)\left\{\rho\frac{\partial v_r}{\partial t}+\rho\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)\right\}=-\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial r}+ \\ \mu\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}-\frac{v_r}{r^2}+\frac{\partial^2 v_r}{\partial z^2}\right)+\frac{\mu}{3}\frac{\partial}{\partial r}\left(\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}\right)-\frac{\mu\varphi}{k}v_r, \tag{14}$$

$$\left(1+t_m\frac{\partial}{\partial t}\right)\left\{\rho\frac{\partial v_z}{\partial t}+\rho\left(v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right)\right\}=-\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z}+ \\ \mu\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial z}+\frac{\partial^2 v_z}{\partial z^2}\right)+\frac{\mu}{3}\frac{\partial}{\partial z}\left(\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}\right)-\frac{\mu\varphi}{k}v_z\quad. \tag{15}$$

It would be expedient to simplify these equations by introducing non-dimensional variables. We have a characteristic velocity c and have characteristic lengths a, λ, and R. The following variables based on c and R could this be introduced.

$$\overline{W}=\frac{W}{R},\ ,\ \overline{v_r}=\frac{v_r}{c},\quad \overline{v_z}=\frac{v_z}{c},\quad \overline{\rho}=\frac{\rho_o}{\rho},\quad \overline{p}=\frac{p}{\rho_o c^2},$$

$$\overline{p_o}=\frac{p_o}{\rho_o c^2},\quad \bar{t}=\frac{ct}{R},\ \overline{Q}=\frac{Q}{cR^2},\ \bar{k}=\frac{k}{\varphi R^2}$$

Here the over par denotes the dimensionless quantity.

The amplitude ratio ε , the wave number α , the Reynolds number *Re*, and the compressibility number χ are defined by: $\varepsilon = \dfrac{a}{R}$, $\alpha = \dfrac{2\pi R}{\lambda}$, $\mathrm{Re} = \dfrac{\rho_O c R}{\mu}$ and $\chi = k^{*} \rho_O c^2$.

Under the above assumptions the Eqs. (9),(13)-(15) can be rewritten in the non-dimensional form after dropping the bars, as:

$$\frac{\partial \rho}{\partial t} + v_r \frac{\partial \rho}{\partial r} + v_z \frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z}\right) = 0, \tag{16}$$

$$\begin{gathered}\left(1+t_m \frac{\partial}{\partial\ t}\right)\left\{\rho\frac{\partial v_r}{\partial t}+\rho\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)\right\}=-\left(1+t_m\frac{\partial}{\partial t}\right)\ \frac{\partial p}{\partial r}+ \\ \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}-\frac{v_r}{r^2}+\frac{\partial^2 v_r}{\partial z^2}\right)+\frac{1}{3\mathrm{Re}}\frac{\partial}{\partial r}\left(\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}\right)-\frac{1}{\mathrm{Re}\,k}v_r,\end{gathered} \tag{17}$$

$$\begin{gathered}\left(1+t_m \frac{\partial}{\partial\ t}\right)\left\{\rho\frac{\partial v_z}{\partial t}+\rho\left(v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right)\right\}=-\ \left(1+t_m\frac{\partial}{\partial\ t}\right)\ \frac{\partial p}{\partial z}+ \\ \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial z}+\frac{\partial^2 v_z}{\partial z^2}\right)+\frac{1}{3\ \ \mathrm{Re}}\frac{\partial}{\partial z}\left(\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}\right)-\frac{1}{\mathrm{Re}\,k}v_z,\end{gathered} \tag{18}$$

$$\rho = e^{\chi(p - p_O)}. \tag{19}$$

Also the boundary conditions Eq. (10) becomes

$$v_r((1+\eta), z, t) = \frac{\partial \eta(z,t)}{\partial t}, \qquad v_z((1+\eta), z, t) = 0 \ , \tag{20}$$

where $\qquad \eta(z,t) = \varepsilon \cos\ \alpha(z - t).$ (21)

3. Perturbation Solution

To illustrate the nature of the solution we shall consider the important case of no flow in absence of the peristaltic wave. Following Ref. [18], we seek the solution of the governing equations in a form

$$\begin{aligned} p &= p_0 + \varepsilon\, p_1(r,z,t) + \varepsilon^2\, p_2(r,z,t) + \ldots, \\ v_r &= \varepsilon\, u_1(r,z,t) + \varepsilon^2\, u_2(r,z,t) + \ldots, \\ v_z &= \varepsilon\, v_1(r,z,t) + \varepsilon^2\, v_2(r,z,t) + \ldots, \\ \rho &= 1 + \varepsilon\, \rho_1(r,z,t) + \varepsilon^2\, \rho_2(r,z,t) + \ldots. \end{aligned} \tag{22}$$

Then, doing a usual perturbative analysis using the latter expansions, we can obtain a closed set of governing equations for the first (ε) and second (ε^2) order as the following:

$$\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial u_1}{\partial t}=-\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial p_1}{\partial r}+$$
$$\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial r^2}+\frac{1}{r}\frac{\partial u_1}{\partial r}-\frac{u_1}{r^2}+\frac{\partial^2 u_1}{\partial z^2}\right)+\frac{1}{3\mathrm{Re}}\frac{\partial}{\partial r}\left(\frac{\partial u_1}{\partial r}+\frac{u_1}{r}+\frac{\partial v_1}{\partial z}\right)-\frac{u_1}{\mathrm{Re}\,k},\tag{23.1}$$

$$\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial t}=-\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial p_1}{\partial z}+$$
$$\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1}{\partial r^2}+\frac{1}{r}\frac{\partial v_1}{\partial r}+\frac{\partial^2 v_1}{\partial z^2}\right)+\frac{1}{3\mathrm{Re}}\frac{\partial}{\partial z}\left(\frac{\partial u_1}{\partial r}+\frac{u_1}{r}+\frac{\partial v_1}{\partial z}\right)-\frac{v_1}{\mathrm{Re}\,k},\tag{23.2}$$

$$\frac{\partial \rho_1}{\partial t}+\frac{\partial u_1}{\partial r}+\frac{u_1}{r}+\frac{\partial v_1}{\partial z}=0,\tag{23.3}$$

$$\rho_1=\chi p_1.\tag{23.4}$$

$$\left(1+t_m\frac{\partial}{\partial t}\right)\left\{\frac{\partial u_2}{\partial t}+\rho_1\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial r}+v_1\frac{\partial u_1}{\partial z}\right\}=-\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial p_2}{\partial r}+$$
$$\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_2}{\partial r^2}+\frac{1}{r}\frac{\partial u_2}{\partial r}-\frac{u_2}{r^2}+\frac{\partial^2 u_2}{\partial z^2}\right)+\frac{1}{3\mathrm{Re}}\frac{\partial}{\partial r}\left(\frac{\partial u_2}{\partial r}+\frac{u_2}{r}+\frac{\partial v_2}{\partial z}\right)-\frac{u_2}{\mathrm{Re}\,k},\tag{24.1}$$

$$\left(1+t_m\frac{\partial}{\partial t}\right)\left\{\frac{\partial v_2}{\partial t}+\rho_1\frac{\partial v_1}{\partial t}+u_1\frac{\partial v_1}{\partial r}+v_1\frac{\partial v_1}{\partial z}\right\}=-\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial p_2}{\partial z}+$$
$$\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_2}{\partial r^2}+\frac{1}{r}\frac{\partial v_2}{\partial r}+\frac{\partial^2 v_2}{\partial z^2}\right)+\frac{1}{3\mathrm{Re}}\frac{\partial}{\partial z}\left(\frac{\partial u_2}{\partial r}+\frac{u_2}{r}+\frac{\partial v_2}{\partial z}\right)-\frac{v_2}{\mathrm{Re}\,k},\tag{24.2}$$

$$\frac{\partial \rho_2}{\partial t}+u_1\frac{\partial \rho_1}{\partial r}+v_1\frac{\partial \rho_1}{\partial z}+\frac{u_2}{r}+\frac{\partial u_2}{\partial r}+\frac{\partial v_2}{\partial z}+\rho_1\left(\frac{\partial u_1}{\partial r}+\frac{u_1}{r}+\frac{\partial v_1}{\partial z}\right)=0,\tag{24.3}$$

$$\rho_2=\chi p_2+\frac{1}{2}\chi^2 p_1^2.\tag{24.4}$$

Expanding Eq. (20) by Taylor expansion around $r=1$ and substituting from Eq. (22) we get the following boundary conditions:

$$u_1(1,\ z,\ t)=-\frac{i\alpha}{2}\left(e^{i\alpha(z-t)}-e^{-i\alpha(z-t)}\right),\tag{25.1}$$

$$u_2(1,z,t)+\frac{1}{2}\left(e^{i\alpha(z-t)}+e^{-i\alpha(z-t)}\right)\frac{\partial u_1}{\partial r}(1,z,t)=0,\tag{25.2}$$

$$v_1(1,z,t)=0,\tag{25.3}$$

$$v_2(1,z,t)+\frac{1}{2}\left(e^{i\alpha(z-t)}+e^{-i\alpha(z-t)}\right)\frac{\partial v_1}{\partial r}(1,z,t)=0.. \tag{25.4}$$

Further, following the authors of ref. [18,19], we seek the solution of the liner problem in the form

$$\begin{aligned}
u_1(r,z,t) &= U_1(r)e^{i\alpha(z-t)}+\overline{U_1}(r)e^{-i\alpha(z-t)},\\
v_1(r,z,t) &= V_1(r)e^{i\alpha(z-t)}+\overline{V_1}(r)e^{-i\alpha(z-t)},\\
p_1(r,z,t) &= P_1(r)e^{i\alpha(z-t)}+\overline{P_1}(r)e^{-i\alpha(z-t)},\\
\rho_1(r,z,t) &= \chi P_1(r)e^{i\alpha(z-t)}+\chi\,\overline{P_1}(r)e^{-i\alpha(z-t)}.
\end{aligned} \tag{26}$$

Here and in the following equations, the bar denotes a complex conjugate.

On the other hand, we seek the second (ε^2) order solution in the form

$$\begin{aligned}
u_2(r,z,t) &= U_{20}(r)+U_2(r)e^{2i\alpha(z-t)}+\overline{U_2}(r)e^{-2i\alpha(z-t)},\\
v_2(r,z,t) &= V_{20}(r)+V_2(r)e^{2i\alpha(z-t)}+\overline{V_1}(r)e^{-2i\alpha(z-t)},\\
p_2(r,z,t) &= P_{20}(r)+P_2(r)e^{2i\alpha(z-t)}+\overline{P_2}(r)e^{-2i\alpha(z-t)},\\
\rho_2(r,z,t) &= D_{20}(r)+D_2(r)e^{2i\alpha(z-t)}+\overline{D_2}(r)e^{-2i\alpha(z-t)}.
\end{aligned} \tag{27}$$

The latter choice of solution is motivated by the fact that the peristaltic flow is essentially a nonlinear (second-order) effect [18], and adding a nonoscillatory term in the first order gives only a trivial solution. Thus, we can add nonoscillatory terms, such as $U_{20}(r), V_{20}(r), P_{20}(r),\ and\ \ D_{20}(r).$, which do not cancel out in the solution after the time averaging over the period, only in the second and higher orders. Substituting from Eq. (26) in Eqs. (23.1-23.4) and Eqs. (25.1-25.4) and after some mathematical operations we obtain the solution of the system of equations as the following:

$$U_1(r)=C_1\,I_1(\upsilon\,r)+C_2\,I_1(\beta\,r), \tag{28}$$

$$V_1(r)=\frac{i\alpha\,C_1}{\upsilon}I_0(\upsilon\,r)+\frac{i\beta\,C_2}{\alpha}I_0(\beta\,r), \tag{29}$$

$$P_1(r)=\frac{C_1(\upsilon^2-\beta^2)}{\gamma\,\upsilon}I_0(\upsilon\,r), \tag{30}$$

where the complex parameters γ and β are given by

$$\gamma=(1-i\alpha t_m)\mathrm{Re}-\frac{i\alpha\chi}{3},\qquad \beta^2=\alpha^2+\frac{1}{k}-i\alpha\,(1-i\alpha t_m)\mathrm{Re}. \tag{31}$$

and I_0 is the modified Bessel function of the first kind of order 0. I_1 is the modified Bessel function of the first kind of order 1,

$$C_1 = \frac{i\alpha\beta\upsilon I_0(\beta)}{2\left[\alpha^2 I_1(\beta)I_0(\upsilon) - \beta\upsilon I_1(\upsilon)I_0(\beta)\right]},$$
$$C_2 = \frac{-i\alpha^3 I_0(\upsilon)}{2\left[\alpha^2 I_1(\beta)I_0(\upsilon) - \beta\upsilon I_1(\upsilon)I_0(\beta)\right]}. \tag{32}$$

and $$\upsilon^2 = \alpha^2 \frac{(1-\chi)(1-i\alpha t_m)\mathrm{Re} - (4/3)i\alpha\chi - \frac{i\chi}{\alpha k}}{(1-i\alpha t_m)\mathrm{Re} - (4/3)i\alpha\chi}. \tag{33}$$

To solve the system of second order of ε, inserting Eq. (27) into Eqs. (24.1-24.4) and Eqs. (25.1-25.4). It will be seen that, as far as the net flow is considered only the functions U_{20}, V_{20}, P_{20} *and* D_{20} contribute to the net flow as long as terms up to O (ε^2) are retained. Thus, the functions U_2, V_2, P_2 *and* D_2 don't contribute to the net flow, and therefore, we shall not write down the equations that these functions satisfy or solve them. We continue with the solutions for U_{20}, V_{20}, P_{20} *and* D_{20} [18]. To that end, the second-order solution can also be found in a way similar to the one used in the first order as follows:

$$U_{20}(r) = -\chi \left[P_1(r)\ \overline{U_1}(r) + \overline{P_1}(r)\ U_1(r)\right], \tag{34}$$

$$V_{20}(r) = D_2 - (1 - i\alpha\ t_m)\ \mathrm{Re} \int_r^1 \left[V_1(\zeta)\overline{U_1}(\zeta) + \overline{V_1}(\zeta)U_1(\zeta)\right]d\zeta\ , \tag{35}$$

where D_2 is a complex constant defined by

$$D_2 = \frac{-1}{2}\left(V_1'(1) + \overline{V_1'}(1)\right), \tag{36}$$

The dimensionless fluid flow rate Q can be calculated as [18]

$$Q(z,t) = 2\pi\left[\varepsilon\int_0^1 v_1(r,z,t)rdr + \varepsilon^2\int_0^1 v_2(r,z,t)rdr + O(\varepsilon^3)\right].$$

Next, the net flow is considered over one period of time. The average of a variable G over one period T of time t as $\langle G\rangle = \frac{1}{T}\int_0^T G(r,z,t)dt$.

At $T = \frac{2\pi}{\alpha}$, Consequently, the mean net axial velocity $\langle V_z\rangle$ reads

$$\langle V_z\rangle = \varepsilon^2 V_{20}(r).$$

Under neglecting of O (ε^3)- terms, while the net flow rate $\langle Q\rangle$ is given by

$$\langle Q \rangle = 2\pi\,\varepsilon^2 \int_0^1 V_{20}(r)\, r\, dr\ . \tag{37}$$

Under neglecting of O (ε^3)- terms. Thus, the traveling wave induces a net flow of the liquid, of which the (dimensionless) rate is expressed by Eq. (48). Hence, The net flow and the mean net axial velocity are an effect of order ε^2 .

4. Numerical Results and Discussion

To study the behavior of the net flow rate, numerical calculations for several values of α, k, t_m and χ are carried out. We concentrate on the solution of the dimensionless problem as described in the previous section. It is clear that we have to choose $\varepsilon << 1$ because we used the perturbation method with the amplitude ratio ε as a parameter [32]. Also, for to be perturbation method valid and accurate we must have ($\varepsilon\,\alpha^2$ Re $<< 1$) accordingly to Takabatake [33].

We consider the net flow rate $\langle Q \rangle$ given by Eq. (37). After one integration by parts $\langle Q \rangle$ can be expressed as

$$\langle Q \rangle = \pi\,\varepsilon^2 \left(D_2 - (1 - i\,\alpha\, t_m)\,\mathrm{Re} \int_0^1 r^2 \left[V_1(r)\overline{U_1}(r) + \overline{V_1}(r)\,U_1(r) \right] dr \right), \tag{38}$$

where the solution of Eq. (35) for $V_{20}(r)$ is used.

A numerical code has been written to calculate $\langle Q \rangle$ according to Eq. (38). In order to check the validity of our code, we run it for the parameters similar to the ones used by other authors. For instance, for $\varepsilon = 0.15$, Re = 10000.0, $\alpha = 0.20$, $\chi = 0.0$, $t_m = 0.0$ and k $\to \infty$ we obtain $\langle Q \rangle$ = 0.2709, which equal (if we keep four digits after the decimal point) to the result of the authors of Ref. [18] who actually obtained $\langle Q \rangle$ = 0.2709. We note again that k and t_m enter the equations because we have included the flow through porous medium into non-Newtonian Maxwell's model.

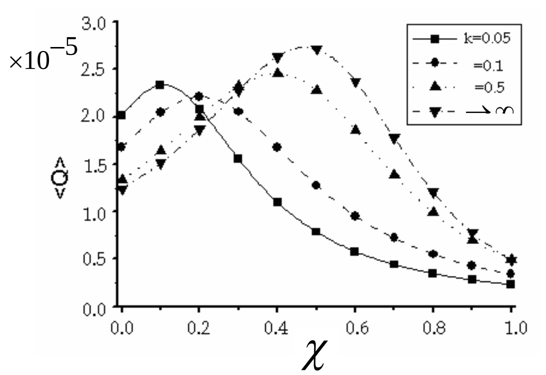

Fig. 1. The dimensionless flow rate $\langle Q \rangle$ versus χ at $\varepsilon = 0.001$, Re=10000. 0, $t_m = 0.0$ and $\alpha = 0.001$.

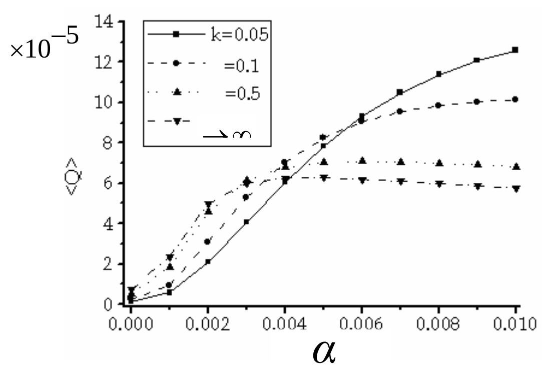

Fig. 2. The dimensionless net flow rate $\langle Q \rangle$ versus α at $\varepsilon = 0.001$, Re=10000.0, $t_m = 1000$ and $\chi = 0.6$.

First, we investigate the effect of the permeability of porous medium k in the case of a Newtonian ($t_m = 0.0$) Maxwellian fluid. The results of our calculations are presented in Fig. 1, where we investigate the dependence of $\langle Q \rangle$ on the compressibility parameter χ for various values of k. To investigate the dependence of the flow rate $\langle Q \rangle$ on k, we perform the calculation for a few values of k. When $\mathrm{k} = 0.05$ we observe that the net flow rate $\langle Q \rangle$ reaches to a maximum value at $\chi = 0.1$. Further, when $\mathrm{k} = 0.1$ we notice that the maximum value of $\langle Q \rangle$ occurs at $\chi = 0.2$. Moreover, the maximum value of $\langle Q \rangle$ occurs at $\chi = 0.4$ when $\mathrm{k} = 0.5$.

In Fig. 2 we investigate the behavior of the net flow rate $\langle Q \rangle$ on the parameter α, which is the tube radius measured in wavelengths. We set k to various values and investigate the changes induced by porous medium effects. We note that the net flow rate $\langle Q \rangle$ increases with increasing the wave number α. Moreover, we notice that at any value of α within the range of α between 0.0 and 0.004 (approximately) the net flow rate increases with increasing k while at any value of α within the range of α between 0.004 (approximately) and 0.01 the net flow rate $\langle Q \rangle$ decreases with increasing k .

Second, we investigate the effect of the permeability of porous medium in the case of non-Newtonian Maxwellian fluid. It is known that the viscoelastic fluids, described by the Maxwellian fluid, have different flow regimes depending on the value of the parameter $De = t_v / t_m$, which is called the Deborah number. In effect, the Deborah number is a ratio of the characteristic time of viscous effects $t_v = \rho R^2 / \mu$ to the relaxation time t_m. As noted in Ref. [30], the value of the parameter De (which the authors of Ref. [30] actually call α) determines in which regime the system resides. Beyond a certain critical value ($De = 11.64$), the system is dissipative, and conventional viscous effects dominate. On the other hand, for small De ($De < Dec$) the system exhibits viscoelastic behavior.

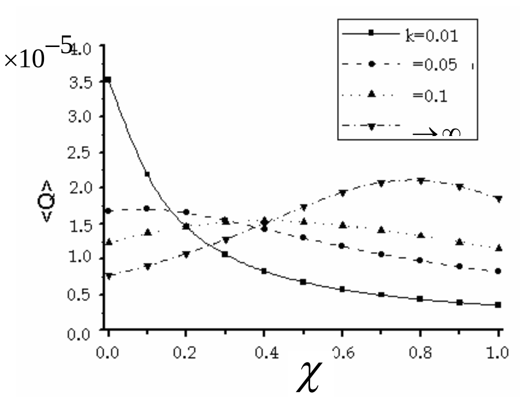

Fig. 3. The dimensionless net flow rate $\langle Q \rangle$ versus χ at $\varepsilon = 0.001$, Re=10000, $t_m = 1000$ and $\alpha = 0.001$.

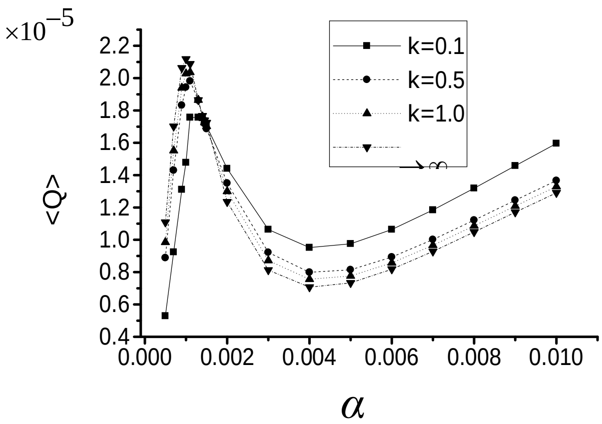

Fig. 4. The dimensionless net flow rate $\langle Q \rangle$ versus α at $\varepsilon = 0.001$, Re=10000.0, $t_m = 1000$ and $\chi = 0.6$.

The results of our calculations are presented in Fig. 3, where we investigate the dependence of $\langle Q \rangle$ on the compressibility parameter χ for various values of k. When k=0.01 we can observe that the flow rate decreases with increasing the compressibility parameter χ.

Furthermore, when $k = 0.05$ we observe that the net flow rate $\langle Q \rangle$ reaches to a maximum value at $\chi = 0.1$ .Moreover, the maximum value of $\langle Q \rangle$ occurs at $\chi = 0.8$ when $k \to \infty$.

In Fig. 4 the dimensionless net flow rate $\langle Q \rangle$ is plotted versus α at the following set of parameters: $\varepsilon = 0.001$, Re=10000.0, $\chi = 0.6$ and $t_m = 1000$. We investigate the behavior of the net flow rate $\langle Q \rangle$ on the parameter α at various values of k within the range of $0.0 \le \alpha \le 0.01$. The net flow rate $\langle Q \rangle$ is weakly affected by k when $\alpha < 0.0015$. At any value of α within the range $0.0015 < \alpha < 0.01$, the net flow rate $\langle Q \rangle$ decreases with increasing k.

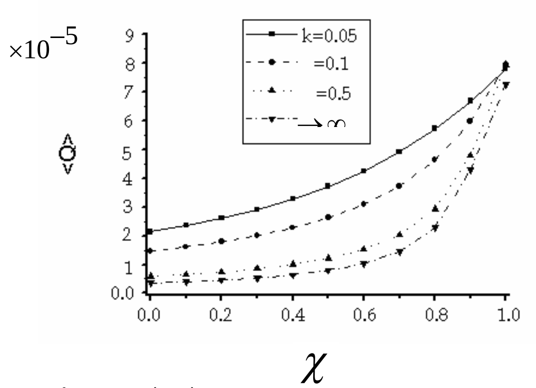

Fig. 5. $\langle Q \rangle$ **versus** χ **at** $\varepsilon = 0.001$ **,** Re=10000**,** $t_m = 10000.0$ **and** $\alpha = 0.001$ **.**

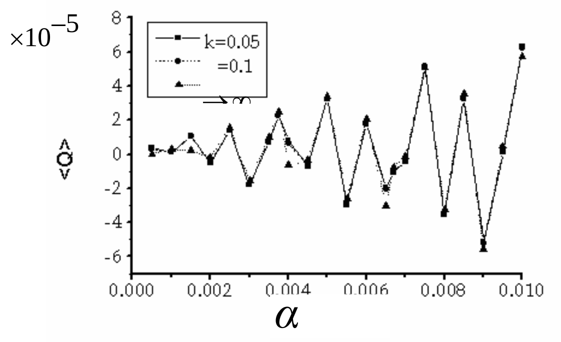

Fig. 6. $\langle Q \rangle$ **versus** α **at** $\varepsilon = 0.001$ **, Re=10000.0,** $t_m = 10000.0$ **and** $\chi = 0.6$

In Fig. 5 we investigate the behavior of the flow rate $\langle Q \rangle$ on the compressibility parameter χ at $t_m = 10000$ (deeply non-Newtonian regime). We set k to various values and investigate the effect of k at deeply non-Newtonian regime. In this deeply non-Newtonian regime we notice that, the net flow rate $\langle Q \rangle$ increases with increasing χ . Also, at any value of χ , the net flow rate $\langle Q \rangle$ decreases with increasing k. Furthermore, at high compressibility ($\chi = 1.0$) the net flow rate is weakly affected by k since the values of $\langle Q \rangle$ lie within the range of (7.2595 $\times 10^{-5}$ - 7.9463$\times 10^{-5}$).

In Fig. 6 the net flow rate $\langle Q \rangle$ is plotted versus α , for the following set of parameters: $\varepsilon = 0.001$, Re = 10000.0, $\chi = 0.6$ and $t_m = 10000$ (deeply non-Newtonian regime) and various values of *Kn* within the range of $0.0 \le \alpha \le 0.01$. We note that at $k \to \infty$ the curve in our Fig. 6 is the same curve in Fig. 2 from Ref. [19]. We note from this Fig. that in this deeply non-Newtonian regime, $\langle Q \rangle$ becomes high oscillatory, but what is unusual again is that we observe the negative flow rates for certain values of α . Oscillatory behavior (appearance of numerous of maxima in the behavior of a physical value) in the deeply non-Newtonian regime is not new [19,21]. Furthermore, we can observe that there is no effect (approximately) of k on the net flow rate $\langle Q \rangle$ in the deeply non-Newtonian regime.

5. Conclusions

For Newtonian fluid, the net flow rate $\langle Q \rangle$ attains a maximum for a certain value of χ and there is shifting the maximum value of $\langle Q \rangle$ towards higher values of χ 's with increasing k and the net flow rate $\langle Q \rangle$ increases with increasing the wave number α .

For non-Newtonian fluid, the net flow rate $\langle Q \rangle$ attains a maximum for a certain value of χ and there is shifting the maximum value of $\langle Q \rangle$ towards higher values of χ 's with increasing k. also, the net flow rate $\langle Q \rangle$ is weakly affected by k when the wave number $\alpha < 0.0015$.

In deeply non-Newtonian regime, the net flow rate $\langle Q \rangle$ increases with increasing χ . Also, the net flow rate $\langle Q \rangle$ decreases with increasing k. Furthermore, at high compressibility ($\chi = 1.0$) the net flow rate is weakly affected by k. Also, $\langle Q \rangle$ becomes high oscillatory, but what is unusual again is that we observe the negative flow rates for certain values of α .

The results of Tsiklauri and Beresnev [7,18,19 and 22] have been recovered .

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