



Statistical-Based Models for Behavior of Flexible Pavements

A. El-Desouky*

Abstract: The main objective of this paper was to provide some useful statistical-based models for flexible pavement behavior. The results of a theoretical analysis, using the finite element (FE) program SAP was utilized in this statistical analysis study. The maximum surface deflection, the maximum horizontal tensile strain (ϵ_t) at the bottom of AC layer and the maximum compressive strain (ϵ_c) at the top of subgrade are the most commonly used criteria for flexible pavement design. They were used in this study as the basis for measuring the flexible pavement response. The Statistical Package for Social Sciences (SPSS) version 11.0 was used to relate these parameters to the basic flexible pavement layer properties. Clearly, the properties of the top layer, asphalt concrete (AC) layer, are the key to control the behavior of flexible pavements. Based on that, the models developed in this paper concerned with the thickness and modulus of AC layer. Three different AC thicknesses and eight different AC moduli were used to investigate the response of flexible pavement. Based on the theoretical and statistical work presented in this study, simple statistical-based models for the response of flexible pavements were developed. These models could be easily used by pavement engineers to predict the flexible pavement indicators; maximum surface deflection, maximum horizontal tensile strain at the bottom of asphalt concrete layer and maximum vertical compressive strain at the top of subgrade layer. The presented models emphasized that the thickness of asphalt concrete layer was the most important independent variable for the response of flexible pavement followed by the modulus of AC layer.

Keywords: Flexible pavements, finite elements, and statistical analysis.

1. Introduction

More and more flexible pavement designs are being based on a mechanistic approach. In a mechanistic design procedure, structural analysis tools or computer programs are required to predict the stress-strain and displacement response of pavements. A number of computer programs based on the FE or the multilayered elasticity (MLE) method have been developed and used for structural analysis of flexible pavement. The results may vary among analysts because of the assumptions made in each procedure and the different input assigned by individual analysts. Thus, selection of an appropriate computer program for structural analysis of flexible pavements is a challenge for the pavement engineers.

* Egyptian Armed Forces, Egypt.

The flexible pavement structure was assumed to have three layers (AC, untreated base, and subgrade). The interface between any two consecutive layers was assumed to be perfectly bonded as recommended by the asphalt institute [1]. The main objective of this study was to relate the parameters measuring the flexible pavement behavior to the AC layer properties. Based on these parameters, pavement life in terms of allowable number of load repetitions based on fatigue cracking (N_f) and allowable number of load repetitions to control permanent deformation (N_d) could be calculated. The FE analysis of flexible pavement response was carried out using the FE computer program SAP. SAP is a FE structural analysis program deals with linear elastic materials [2]. All necessary geometric and loading options associated with the elements have been incorporated. The sensitivity analysis was focused on the effect of AC layer thickness (h_1) and AC layer modulus (E_1) on the maximum surface deflection, the maximum tensile strain at the bottom of the AC layer, and the maximum compressive strain at the top of subgrade. They are the most commonly used criteria for flexible pavement design [3, 4, 5], and were used in this paper as the basis for measuring the flexible pavement behavior. Stresses and surface deflections were obtained directly from the FE solution phase. Strains were calculated from the relationships of isotropic materials that assumed in this study. A number of researchers studied the effect of E_1 on the flexible pavements behavior. For example, Yue et al. [6] studied the flexible pavement response under different tire-pavement contact pressures. They found that, ϵ_t decreased as E_1 increased. However, in cases of high-strength bases there was a critical E_1 value below which ϵ_t also decreased.

2. Finite Element Models

The flexible pavement structure was assumed to have three layers (asphalt concrete, untreated base, and subgrade). The interface between any two consecutive layers was assumed to be perfectly bonded as recommended by the asphalt institute [1]. The properties required for each layer were the thickness (h), Poisson's ratio (ν), and modulus of elasticity (E). The layer properties data are; E_1 ranged from 500 to 3000 MPa, $E_2 = 250$ MPa, and $E_3 = 50$ MPa. The Poisson's ratios were $\nu_1 = 0.35$, $\nu_2 = 0.30$, and $\nu_3 = 0.40$. The thicknesses of the layers were h_1 varied from 5 to 15 cm, $h_2 = 25$ cm, and $h_3 = 90$ cm such that the depth of the finite element model was taken \geq four times the diameter of the wheel-pavement contact area ($d = 30$ cm as mentioned later), so the stresses at this depth were very small. The analysis model was established with a fixed boundary at the bottom and roller supports on sides. A single wheel load of 40 kN was applied over a circular contact area of 30 cm diameter [7, 8, 9].

3. Results of Finite Element Analysis

3.1. Maximum Surface Deflection

The maximum surface deflection for AC layer thickness of 5, 10, and 15 cm are presented in Table 1. It was found that the maximum surface deflection decreased by increasing E_1 . The percentage decreases due to increasing E_1 from 500 to 3000 MPa were 11.07%, 22.37%, and 30.82% for $h_1 = 5, 10,$ and 15 cm, respectively. The thickness h_1 has a significant effect on the maximum surface deflection, with increasing h_1 the maximum surface deflection decreased considerably. The maximum surface deflection decreased by 13.12% and 22.97% due to increasing h_1 from 5 cm to 10 cm and 15 cm, respectively for $E_1 = 500$ MPa, and by 24.16% and 40.08% for $E_1 = 3000$ MPa. The percentage of reduction in the maximum surface deflection increased for higher E_1 values.

Table 1. Maximum Surface Deflection for Different E_1 and h_1 Values

E_1 (MPa)	AC layer thickness h_1 (cm)							
	5		10			15		
	(1)	(2)	(1)	(2)	(3)	(1)	(2)	(3)
500	1.251	---	1.087	---	13.12	0.964	---	22.97
750	1.220	2.55	1.031	5.19	15.48	0.887	8.03	27.30
1000	1.198	4.28	0.993	8.69	17.13	0.837	13.17	30.13
1250	1.181	5.59	0.964	11.37	18.44	0.800	16.96	32.25
1500	1.168	6.67	0.940	13.59	19.56	0.771	19.97	33.04
2000	1.146	8.42	0.901	17.14	21.40	0.727	24.58	36.94
2500	1.128	9.85	0.870	19.99	22.89	0.693	28.06	38.53
3000	1.113	11.07	0.844	22.37	24.16	0.667	30.82	40.08

- (1) maximum surface deflection (mm), downward.
(2) %-age decrease in maximum surface deflection by increasing E_1 from 500 MPa.
(3) %-age decrease in maximum surface deflection by increasing h_1 from 5 cm.

3.2. Maximum Horizontal Tensile Strain at the Bottom of the AC Layer

Table 2 shows the effect of h_1 and E_1 on maximum tensile strain at the bottom of AC layer. Generally, ε_t decreased when h_1 increased. In addition, when applying a vertical wheel load to the pavement system, there was a so-called critical modulus E_1 at which maximum ε_t value existed. Below this critical modulus, the lower the asphalt concrete modulus, the smaller the value of ε_t . This confirms the findings of Yue et al. [6]. The critical modulus E_1 was 2000 MPa for $h_1 = 5$ cm, and decreased to 1500 MPa, and 750 MPa for $h_1 = 10$ cm and 15 cm, respectively.

Table 2. Maximum Horizontal Tensile Strain ($\times 10^{-4}$) at the Bottom of AC Layer

E_1 (MPa)	AC layer thickness h_1 (cm)		
	5	10	15
500	1.681	3.298	3.558
750	2.033	3.110	3.714
1000	2.196	3.070	3.612
1250	2.279	3.190	3.451
1500	2.321	3.216	3.283
2000	2.346	3.153	2.973
2500	2.335	3.036	2.709
3000	2.308	2.908	2.486

3.3. Maximum Vertical Compressive Strain at the Top of the Subgrade Layer

The effect of E_1 on ε_c is presented in Table 3 for the different h_1 values. It was found that, ε_c decreased by increasing E_1 . The thickness h_1 had a pronounced effect on the maximum compressive strain at the top of the subgrade layer. The table also shows that ε_c decreased considerably by increasing h_1 , this confirms the findings of Yue et al. [6].

Generally, Table 1 and Table 3 show the advantages of using higher asphalt concrete modulus regarding to maximum surface deflection and maximum vertical compressive strain at the top of the subgrade layer.

Table 3. Variation of Maximum Vertical Compressive Strain at the top of the Subgrade

E ₁ (MPa)	AC layer thickness h ₁ (cm)								
	5		10			15			
	(1)	(2)	(1)	(2)	(3)	(1)	(2)	(3)	
500	66.44	--	50.81	--	23.53	39.00	--	41.30	
750	64.28	3.26	47.70	6.12	25.79	35.39	9.27	44.95	
1000	62.74	5.57	45.40	10.64	27.64	32.73	16.07	47.83	
1250	61.56	7.34	43.55	14.28	29.25	30.62	21.48	50.26	
1500	60.58	8.82	41.98	17.37	30.70	28.87	25.99	52.35	
2000	58.95	11.28	39.39	22.48	33.18	26.04	33.23	55.82	
2500	57.63	13.27	37.25	26.68	35.36	23.83	38.90	58.65	
3000	56.49	14.98	35.44	30.25	37.26	22.02	43.54	61.02	

- (1) maximum vertical compressive strain ($\times 10^{-4}$) at the top of subgrade.
(2) % age decrease in maximum vertical compressive strain by increasing E₁ from 500 MPa.
(3) % age decrease in maximum vertical compressive strain by increasing h₁ from 5 cm.

4. Statistical-Based Models

This section includes the statistical analysis of FE results. Linear regression analysis was used utilizing the Statistical Package for Social Sciences (SPSS) version 11.0 [10]. The regression analysis was performed assuming that; the random errors have normal distribution with mean equal to 0.0, any two random errors are independent, and the random errors have equal variances. Many transformations and combinations of the independent variables were performed and included in the statistical study. The models were developed using the stepwise method.

The first set of developed models was to relate maximum surface deflection (SD) to the AC layer properties as follows;

$$SD = 1.331 - 3.28 \times 10^{-2} h_1 - 3.37 \times 10^{-5} E_1, \quad R^2 = 0.74 \quad (1)$$

$$SD = 1.293 - 3.31 \times 10^{-2} h_1, \quad R^2 = 0.72 \quad (2)$$

$$SD = 1.171 - 1.75 \times 10^{-3} h_1^2, \quad R^2 = 0.74 \quad (3)$$

The second set of developed models was to predict the maximum horizontal tensile strain (TS) at the bottom of AC layer from the basic layer properties of AC. Many transformations and combinations of independent variables were used to develop the following models;

$$TS = 0.226 + 0.4h_1 - 1.7 \times 10^{-3} E_1 + 1.9 \times 10^{-4} h_1 E_1 - 1.9 \times 10^{-2} h_1^2 + 6.8 \times 10^{-8} E_1 - 2.9 \times 10^{-9} h_1^2 E_1^2 + 4.8 \times 10^{-3} \frac{E_1}{h_1}, \quad R^2 = 0.87 \quad (4)$$

$$TS = 1.943 + 9.732 \times 10^{-2} h_1 + 1.017 \times 10^{-4} E_1 - 1.33 \times 10^{-5} h_1 E_1, \quad R^2 = 0.46 \quad (5)$$

$$TS = 2.091 + 8.201 \times 10^{-2} h_1 - 2.97 \times 10^{-5} E_1, \quad R^2 = 0.46 \quad (6)$$

The last set of equations was to relate the layer properties of AC layer to the maximum vertical compressive strain (CS) at the top of subgrade. Equations (7-8) present the developed models;

$$CS = 83.050 - 3.433h_1 - 3.08 \times 10^{-2} (E_1/h_1), \quad R^2 = 0.92 \quad (7)$$

$$CS = 70.311 - 2.679h_1, \quad R^2 = 0.88 \quad (8)$$

where:

SD = maximum surface deflection (mm)

TS = maximum horizontal tensile strain at the bottom of the AC layer ($\times 10^{-4}$)

CS = maximum vertical compressive strain at the top of the subgrade ($\times 10^{-4}$)

h_1 = thickness of AC layer (cm)

E_1 = modulus of AC layer (MPa)

The assumptions of the linear regression analysis for all developed models in this study were checked by plotting the residuals versus the fitted values. All plots showed random scatter of points about zero and they did not show any specific pattern. Fig. (1-3) show examples of the residual plots for each set of developed models.

Based on their R^2 values, Equations (5-6) are not recommended to be used to estimate tensile strains at the bottom of AC layers of flexible pavements. Equations (1, 4 and 7) were chosen to predict the flexible pavement indicators (SD, TS and CS). These equations have the highest R^2 values of 0.74, 0.87 and 0.92, which indicated that a large amount of the variation in the predicted variables could be explained by the independent variables included in these models. In addition, the selected equations have more independent variables compared to the ignored equations. Based on the significance of each independent variable of the developed models, it was found that h_1 was more significant than E_1 . In other words, the flexible pavement indicators could be estimated as functions of h_1 only which could explain a large amount of variations in these indicators. More variation in the flexible pavement indicators could be explained by the modulus of AC layer E_1 .

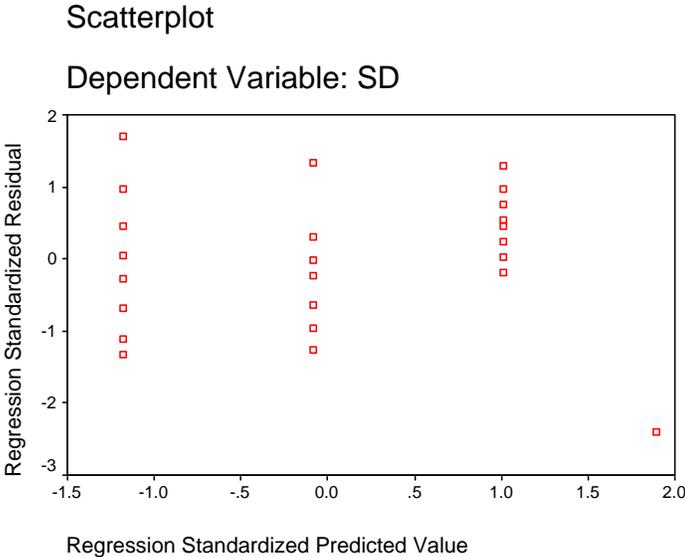


Fig. 1. Scatter plot of residuals for statistical-based model presented in Equation (2)

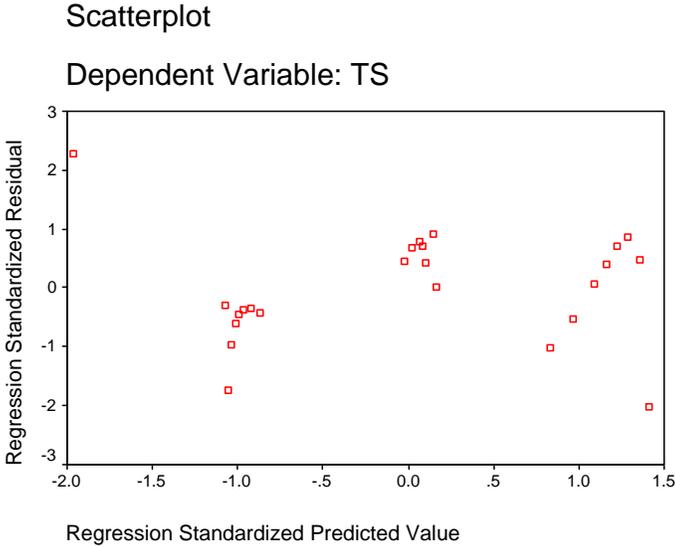


Fig. 2. Scatter plot of residuals for statistical-based model presented in Equation (5)

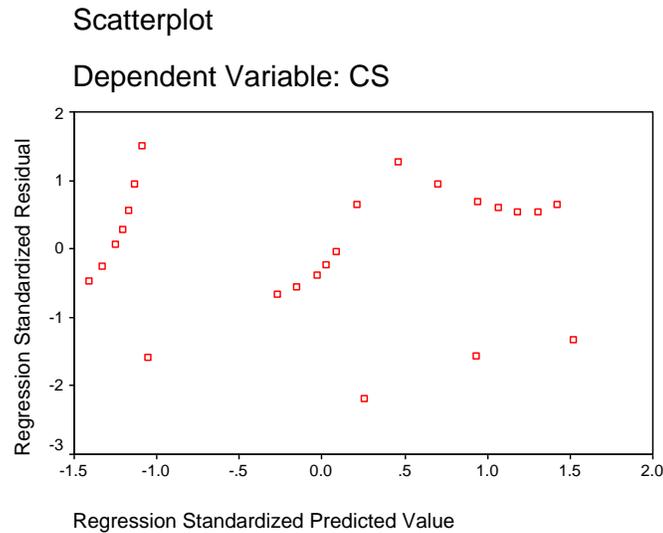


Fig. 3. Scatter plot of residuals for statistical-based model presented in Equation (7)

5. Conclusions

Based on the theoretical and statistical work presented in this study, the following could be concluded;

The paper presented simple statistical-based models for the response of flexible pavement. These models could be easily used by pavement engineers to predict; maximum surface deflection, maximum horizontal tensile strain at the bottom of asphalt concrete layer and maximum vertical compressive strain at top of the subgrade layer, which are the most commonly used criteria for flexible pavement design and analysis.

The presented models emphasized that the thickness of asphalt concrete layer was the most important independent variable for the response of flexible pavement followed by the modulus of AC layer.

Large amount in the variation of the flexible pavement indicators could be explained by the asphalt concrete thickness (h_1).

6. References

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