Paper: ASAT-13-CM-21

13th International Conference on
AEROSPACE SCIENCES & AVIATION TECHNOLOGY,
ASAT- 13, May 26 – 28, 2009, E-Mail: asat@mtc.edu.eg
Military Technical College, Kobry Elkobbah, Cairo, Egypt
Tel : +(202) 24025292 – 24036138, Fax: +(202) 22621908

Performance Evaluation of Adaptive MMSE Receivers in Frequency Selective Fading Channels Using Aided Tentative Coefficients

H. E. Abou Bakr[*], Hany A. A. Mansour[*]

Abstract: Recently, the code division multiple access (CDMA) technique has become the most widely communication technique, because of its varity of applications. The well-known matched filter (MF) is considered as the optimum filter that recovers the CDMA signals. However; its performance recently becomes unsatisfying. This is due to increasing the multiple access interference (MAI), in addition to the other channel impairments such as the fading or the multi path interference (MPI). In this paper, an adaptive minimum mean square error-maximum likelihood (MMSE-ML) receiver is presented as an alternative solution to overcome the drawbacks of the MF receiver. This adaptive MMSE-ML receiver uses adaptive aided tentative coefficients in addition to the basic adaptive coefficients to increase the tracking ability of the channel impulse response variations. The performance of the adaptive MMSE-ML receiver is compared with the traditional adaptive MMSE receiver over a frequency selective fading channel. Moreover, the performances of both the presented and traditional adaptive receivers are compared with the performance of the MF under the same condition. It is found that the performance of the adaptive MMSE-ML receiver is much improved using the proposed technique.

Keywords: code division multiple access (CDMA), minimum mean square error-maximum likelihood (MMSE-ML), matched filter (MF), multiple access interference (MAI), least mean square (LMS), single user matched filter (SUMF).

1. Introduction:

The code division multiple access (CDMA) is the technique, that enables more than one user to access the same channel on the same frequency at the same time. This process is performed by assigning a specific code to each user. The main problems facing the demodulation process of the CDMA signals are the increase of the MAI, and other channel impairments such as noise and fading. To overcome these problems, it is required to have a perfect synchronization between the received signal and the stored signatures, in addition to perfect knowledge for the channel impulse response. These requirements are too difficult to be realized, moreover it will affect on the system performance and the system capacity.

---

[*] Egyptian Armed Forces

Many researches are concentrated on other alternative solutions [1]-[8]. A comparison between different types of receiver structures based on a bank of matched filters has been derived, analyzed, and compared in [9]. These structures include the MF receiver, CDMA decorrelator receiver (MF-DEC), MMSE receiver (MF-MMSE), and CDMA decision feedback receiver (MF-DFE). The comparison includes the signature waveform and the timing of the desired and interfering users, the received amplitudes, and the training sequence of the desired user.

The adaptive MMSE-ML receiver provides an alternative solution to the problems facing the CDMA signals recovery. Its concept of operation based on using adaptive coefficients with initial values, and a training sequence known to the receiver. The algorithm begins by transmitting a training sequence through the real channel. The receiver uses the basic coefficients as the recovered signature. The receiver compares the actual output with ta version of the training sequence stored in the receiver. The error between the actual output and the desired output for the duration of each bit is used to adapt the coefficients. The adaptation process is continuitied until the error between the actual and desired outputs becomes minimum and constant (error convergence). Then, the coefficients are kept fixed and are used to recover the real signal. The adaptation process is performed using the least mean square (LMS) Algorithm.

In this paper, a model of an adaptive MMSE-ML receiver is presented. This adaptive MMSE-ML receiver uses adaptive aided tentative coefficients in addition to the main adaptive coefficients. The performance of this receiver is compared with the traditional adaptive MMSE receiver, which uses only the main adaptive coefficients. Moreover, the performances of both adaptive receivers are compared to the single user matched filter (SUMF), which is taken as a performance reference, in addition to the MF performance in presence of different number of MAI. These comparisons are performed in frequency selective fading channel. The adaptation process is performed using the LMS algorithm. The simulation including also the upper and the lower bounds of the adaptive MMSE-ML receiver. These bounds are derived from the bit error rate (BER) analysis of the adaptive MMSE-ML receiver. It is shown that, the performance of the adaptive MMSE-ML receiver is better than the performances of the traditional adaptive MMSE receiver and the MF in all cases. Moreover, it is shown that the performance of the adaptive MMSE-ML receiver approaches the performance of the SUMF performance without any need to make any estimation of the channel parameters.

This paper is organized as follows. Section 2 presents the structure of the adaptive MMSE-ML receiver, discussing the adaptive solution of the MMSE-ML. The BER analysis of the adaptive MMSE-ML receiver is presented in section 3. In Section 4, the numerical results are presented showing the performance of the adaptive MMSE-ML receiver and compared with the traditional adaptive MMSE, and the MF receiver. Last, Section 5 summarizes our study.

2. Adaptive MMSE-ML Receiver

In this section, the adaptive MMSE-ML receiver is inrtoduced. First, the problem of the MF in CDMA signal detection is presented. The asynchronous CDMA signal in additive white Gaussian noise (AWGN) can be expressed as

$$y(t) = \sum_{k=1}^{K+K_1} \sum_{i=0}^{M-1} b_k(i) s_k(t - iT - \tau_k) + n(t) \tag{1}$$

where K is the number of users in the cell of interest, K_1 is number of non negligible interferers from other cells, $s_k(t)$ is the received signature waveform of the k^{th} user. $b_k(i)$ is the i^{th} symbol of the k^{th} user, τ_k is the transmission delay of the k^{th} user, T is the inverse data rate, and $n(t) = \sigma w(t)$,where $w(t)$ is normalized white Gaussian noise and σ is the noise variance. In the absence of intercellular interference $K_1 = 0$, the optimum MF receiver passes the signal $y(t)$ through a bank of matched filters and samples the outputs as

$$r_k(i) = \int_{iT+\tau_k}^{(i+1)T+\tau_k} y(t) s_k(t - iT - \tau) dt, \qquad 0 \le i \le M-1 \tag{2}$$

The next step is to find the K sequences of length M from the samples $r_k(i)$ that correspond to the largest correlation metric [10]

$$\Lambda(\mathbf{b}^{'}) = 2\sum_{k=1}^{K}\sum_{i=0}^{M-1} b_k(i) r_k(i) - \sum_{k,l=1}^{K}\sum_{i,j=0}^{M-1} b_k(i) b_l(i) . \int s_k(t - iT - \tau_k) s_l(t - iT - \tau_l) dt \tag{3}$$

It is clear from (2) that $r_k(i)$ requires a perfect synchronization of $s_k(t)$ and τ_k of each user. In other words, equation (2) shows the difficulty of recovering the data from the asynchronous CDMA system using the MF. This is because it need to have a perfect synchronization and knowledge about the assigned code and the transmission delay of each user, especially with the difference between the received user signature $s_k(t)$, and transmitted used signature $f_k(t)$, where the relation between $s_k(t)$ & $f_k(t)$ is the convolution relation which is $s_k(t) = f_k(t) * \gamma_k(t)$, and $\gamma_k(t)$ is the channel impulse response. In case of frequency selective fading channel $\gamma_k(t) = \sum_{l=0}^{L-1} \gamma_{Kl} \delta(t - \tau_{kl})$ where τ_{kl} are the path delay, and γ_{Kl} are the path gain. Therefore, the performance degradation of MF receiver is due to the estimation errors of $\gamma_k(t)$.

The adaptive MMSE-ML receiver consists of a bank of K adaptive fractionally spaced MMSE finite impulse response (FIR) filters along with the ML detector. The number of intercellular interference K_1 is unknown and only K input MMSE filters interferers are used in the presented receiver. The output of the MMSE filter (actual output) at the n^{th} symbol interval for the k^{th} user is

$$a_k(n) = \sum_{m=-P}^{P} c_k(m) y(nT - mT_f) \tag{4}$$

where $c_k(m)$ are the adaptive filter coefficients, $T_f = T_c / p$ with, $p > 1$,T_c being the chip interval. The total number of adaptive filter coefficients is $(2P+1) > pN$ where N is the spreading gain. The symbol estimate is obtained as[11]

$$\hat{b}_k(n) = a_k(n) - \mathbf{d}_k^H \mathbf{b}_k \tag{5}$$

where $\mathbf{d}_k$ is a vector of tentative decision aided coefficient sequences, defined as

$$\mathbf{d}_k = [\mathbf{d}_{k1}^T, \mathbf{d}_{k2}^T, \mathbf{d}_{k3}^T, \mathbf{........} \mathbf{d}_{kK}^T]^T \tag{6}$$

with $\mathbf{d}_{\mathbf{Km}} = [d_{km}(0), \ldots\ldots d_{km}(M-1)]^T$. The elements of $\mathbf{d}_{\mathbf{Km}}$ represent the aided tentative weighting coefficients, which multiply the respective known symbols coming from the m^{th} user. The vector $\mathbf{b}_k$ that contains known symbols to the receiver is defined as

$$\mathbf{b}_k = [\mathbf{b}_{k1}^T, \mathbf{b}_{k2}^T, \mathbf{b}_{k3}^T, \mathbf{........} \mathbf{b}_{kK}^T]^T \tag{7}$$

with $\mathbf{b}_{\mathbf{Km}} = [b_{km}(0), \ldots\ldots b_{km}(M-1)]^T$. The coefficients $\mathbf{d}_k$ and $\{c_k\}$ are obtained adaptively during a training period. After the training period, the coefficients $\mathbf{d}_k$ and $\{c_k\}$ can be kept fixed during data detection. Alternatively, in a decision directed mode, these coefficients can be updated by tentative decisions.

The filter coefficients are obtained by minimizing the MSE ($E[|e_k(n)|^2]$), where $e_k(n) = b_k(n) - \hat{b}_k(n)$ is the error between the desired and actual outputs. There are many adaptive algorithms that are used to obtain the optimal coefficients $\mathbf{d}_k(n)$ and $\mathbf{c}_k(n)$ [11]. These coefficients are evaluated using LMS algorithm according to the following steps

$$\mathbf{c}_k(n+1) = \mathbf{c}_k(n) + \alpha_1 e_k^*(n)\mathbf{y}(n) \tag{8}$$
$$\mathbf{d}_k(n+1) = \mathbf{d}_k(n) - \alpha_2 e_k^*(n)\mathbf{b}_D(n) \tag{9}$$

for $n = 1, 2, 3, \ldots..$ and α_1 , α_2 are the step size parameters of the algorithm.

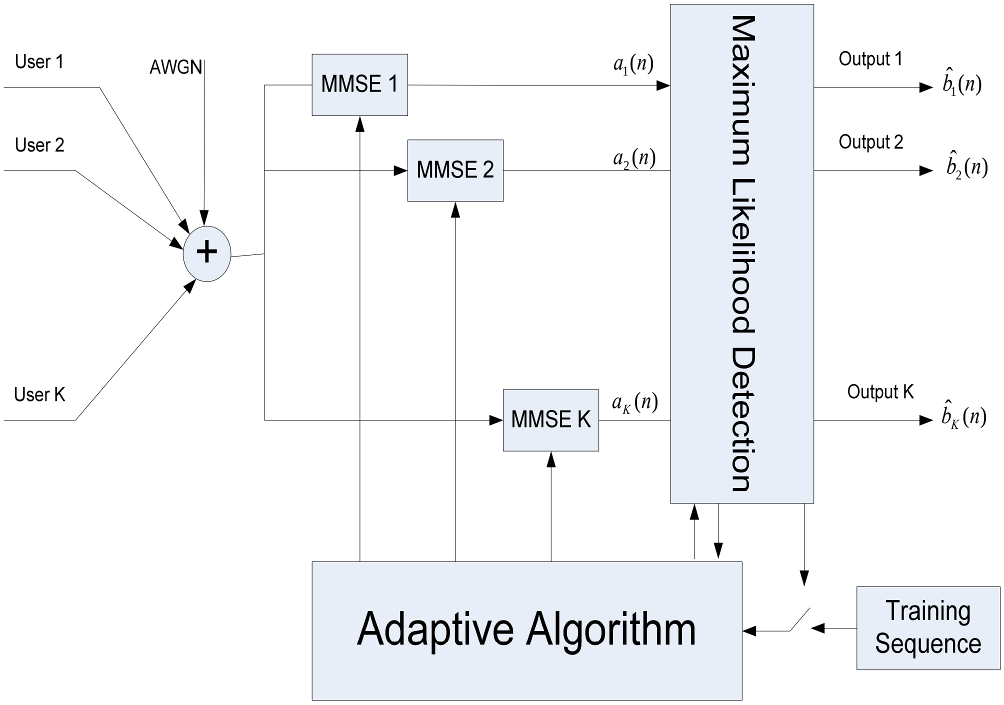

Fig. 1. Block Diagram of the Adaptive MMSE-ML Receiver.

3. Bit Error Rate Analysis of Adaptive MMSE-ML Receiver

Let $\mathbf{e}$ denote an error event whose element $e_k(i)$ denotes an error in the i^{th} symbol of the k^{th} user. For BPSK, $e_k(i) \in \{-2,0,2\}$, this error event occurs if the metric $\mathbf{\Lambda}$ represented in (3) is smaller for the data sequence $(\mathbf{b}+\mathbf{e})$ than the correct sequence $\mathbf{b}$. Actually $\mathbf{\Lambda}(\mathbf{b}+\mathbf{e})$ has to be the minimum metric for the data sequence $(\mathbf{b}+\mathbf{e})$ to be detected. Therefore, an upper bound on the probability of the error event $\mathbf{e}$ is [12]

$$P\{\mathbf{e}\} \le P\{\mathbf{\Lambda}(\mathbf{b}+\mathbf{e}) > \mathbf{\Lambda}(\mathbf{b})/A(\mathbf{e})\}P\{A(\mathbf{e})\} \tag{10}$$

where $A(\mathbf{e})$ is the event that the transmitted sequence $\mathbf{b}$ is compatible with the occurrence of $\mathbf{e}$. Using (3) with appropriate changes for the MMSE-ML receiver, the inequality $\mathbf{\Lambda}(\mathbf{b}+\mathbf{e}) > \mathbf{\Lambda}(\mathbf{b})$ can be expressed after some algebraic manipulations as

$$\sum_{k,l=1}^{K}\sum_{i,j=0}^{M-1} e_k(i)e_l(j)d_{kl}(i,j) \le \sum_{k=1}^{K}\sum_{i=0}^{M-1} e_k(i)\varepsilon_k(i) \tag{11}$$

where $\varepsilon_k(i) = 2r_k(i) - \sum_{l=1}^{K}\sum_{j=0}^{M-1} b_l(j)\big(d_{kl}(i,j) + d_{l,k}(j,i)\big)$. Denoting the left-hand side of (11) as $\mathbf{e}^H\mathbf{D}\mathbf{e}$, where the matrix $\mathbf{D}$ contains the decision-aided coefficients $\{d_{kl}(i,j)\}$ (11) can be written as

$$\mathbf{e}^H\mathbf{D}\mathbf{e} \le \sum_{k=1}^{K}\sum_{i=0}^{M-1} e_k(i)\varepsilon_k(i) \tag{12}$$

At low SNR, the additive Gaussian noise is dominant in $\varepsilon_k(i)$ Therefore, assuming $\{\varepsilon_k(i)\}$ to be Gaussian, the variance of the right-hand side (RHS) of (12) will be

$$\sigma_\varepsilon^2 = E\left[\sum_{k,l=1}^{K}\sum_{i,j=0}^{M-1} e_k(i)e_l(j)\varepsilon_k(i)\varepsilon_l(j)\right] = \sum_{k,l=1}^{K}\sum_{i,j=0}^{M-1} e_k(i)e_l(j)E\left[\varepsilon_k(i)\varepsilon_l(j)\right] \tag{13}$$

In our numerical results, the decision-aided coefficients $d_{kl}(i,j)$ for user k, $1 \le k \le K$ are only kept limited to $d_{kl}(i,i)$, $1 \le l \le K$. In other words, quasi-synchronous systems are only considered where the intersymbol interference is ignored. Hence, to keep the analysis simple, the time index can be removed. Then, assuming uncorrelated symbols between users, the correlation $E[\varepsilon_k(i)\varepsilon_l(j)]$ becomes

$$\begin{aligned} E[\varepsilon_k\varepsilon_l] = {} & 4\mathbf{c}_k^H\left[\mathbf{SS}^H + \sigma^2\mathbf{I}\right]\mathbf{c}_l - 2\mathbf{c}_k^H\sum_m \mathbf{s}_m(d_{ml}^* + d_{lm}^*) \\ & - 2\mathbf{c}_l^T\sum_m \mathbf{s}_m^*(d_{mk} + d_{km}) + \sum_m (d_{mk} + d_{km})(d_{ml}^* + d_{lm}^*) \end{aligned} \tag{14}$$

Let $d^2(\mathbf{e}) = \mathbf{e}^H\mathbf{De}$ and assuming the RHS of (12) to be Gaussian, then

$$P\{\mathbf{\Lambda}(\mathbf{b}+\mathbf{e}) > \mathbf{\Lambda}(\mathbf{b})/A(\mathbf{e})\} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{erfc}\left(\frac{\mathbf{d}^2(\mathbf{e})}{2\sqrt{\sigma_\varepsilon^2}}\right) \tag{15}$$

Then an upper bound on BER will be

$$\mathbf{BER} \le \frac{\mathbf{1}}{\mathbf{2}}\sum_{\mathbf{e}} w(\mathbf{e})P\{A(\mathbf{e})\}\mathbf{erfc}\left(\frac{\mathbf{d}^2(\mathbf{e})}{2\sqrt{\sigma_\varepsilon^2}}\right) \tag{16}$$

where $w(\mathbf{e})$ denotes the number of bit errors associated with the error event $\mathbf{e}$. Similarly, a lower bound on the BER will be

$$\mathbf{BER} \ge \sum_{\mathbf{e}_{\min}} P\{A(\mathbf{e})\}\mathbf{erfc}\left(\frac{\mathbf{d}^2(\mathbf{e})}{2\sqrt{\sigma_\varepsilon^2}}\right) \tag{17}$$

4. Numerical Results

In this section, the performance of the adaptive MMSE-ML receiver is evaluated and compared with the performance of the traditional adaptive MMSE receiver. In addition, the performances of the adaptive receivers are compared with the performance of the matched filter of a single user (SUMF) in AWGN channel, which is considered as a reference for the other receivers. Moreover, the performance of the matched filter in presence of different number of MAI in AWGN and frequency selective fading channel in addition to the upper and the lower bounds of the adaptive MMSE-ML receiver is presented.

All the simulations are performed under nearly identical conditions to make the comparison fair as much as possible. The simulations are performed using 5000 random transmitted symbol for each user and averaged over 100 independent trials to make the complexity of the program visible. The Gold code with code length 15 and sampling rate $p = 2$ sample per chip is used as spreading code. The convergence simulations are taken over 500 iterations for the LMS algorithm. The coefficients tap chosen to be $(2N)$ in case of traditional adaptive MMSE receiver and $(2N+1)$ in case of adaptive MMSE-ML receiver, where N is the code length. The step size parameters are adjusted according to the other simulation parameters (i.e. the MAI, the code length… etc).

Figs (2) and (3) show the mean square error against the number of iterations for the adaptive MMSE receiver and the adaptive MMSE-ML receiver respectively in presence of 5 MAI. Fig (2) shows that the MSE decreased to be close to 10^{-4} at iteration 200, while Fig (3) shows that the error decreased to be lower than 10^{-4} at iteration 150. Therefore, we can conclude that the adaptive MMSE-ML receiver needs less number of iterations to reach a MSE of 10^{-4}. Moreover, the value of MSE in case of adaptive MMSE-ML reaches to a lower value than the MSE of the adaptive MMSE receiver.

Fig (4) plots the BER against the SNR. The figure illustrates the performance comparison of the adaptive MMSE-ML receiver, the traditional adaptive MMSE receiver, the SUMF, MF in presence of 5 MAI in AWGN channel, and the MF in presence of 5 MAI in frequency selective fading channel. The upper and lower bounds of the adaptive MMSE-ML receiver are also included. The reason of including the SUMF is that its performance is considered as the lower bound (a reference performance) of the performances of the other CDMA receivers. The figure shows that the performances of the adaptive receivers are much better than the performance of the MF over the frequency selective fading channel. It is also shown that the performance of the MF over the frequency selective fading channel is extensively degraded due to the fading effect. From the figure, we can see that the performance of the adaptive MMSE-ML receiver has the same performance as the SUMF. This means that the adaptive MMSE-ML receiver eliminates most of the effect of the MAI and the fading. The figure also shows that the performance of the adaptive MMSE-ML receiver is better than the performance of the traditional adaptive MMSE receiver by 2 dB at BER10^{-4}.

Figs (5) and (6) show the performance comparison of the receivers when the number of the MAI increased to 10 and 14 respectively. It is shown that the performance of the MF receiver over the frequency selective fading channel is completely failed, since the BER exhibit an error floor of about 10^{-1} over all the SNR range. Fig (5) shows that the performance of the adaptive MMSE-ML receiver in presence of 10 MAI over frequency selective fading is better than the performance of the MF in presence of the same number of the MAI over the AWGN channel. The figure also shows that the performance of the adaptive MMSE-ML receiver is much better than the performance of the adaptive MMSE receiver. This is clear from the gap between the performance of the SUMF and the performance of the adaptive MMSE-ML receiver, which is equal to 0.5 dB at BER of10^{-3}. This gap is increased to 3 dB in case of the adaptive MMSE receiver at the same BER level. The performance in the case of full system capacity is illustrated in Fig (6). Full system capacity means that the max number of MAI in the system is applied. Here, the number of MAI in case of using Gold 15 is 14 interfering user in addition to the desired user. The figure shows that the performance of the adaptive MMSE-ML receiver achieves performance gain of 3 dB over the performance of the adaptive MMSE receiver at BER10^{-3}.

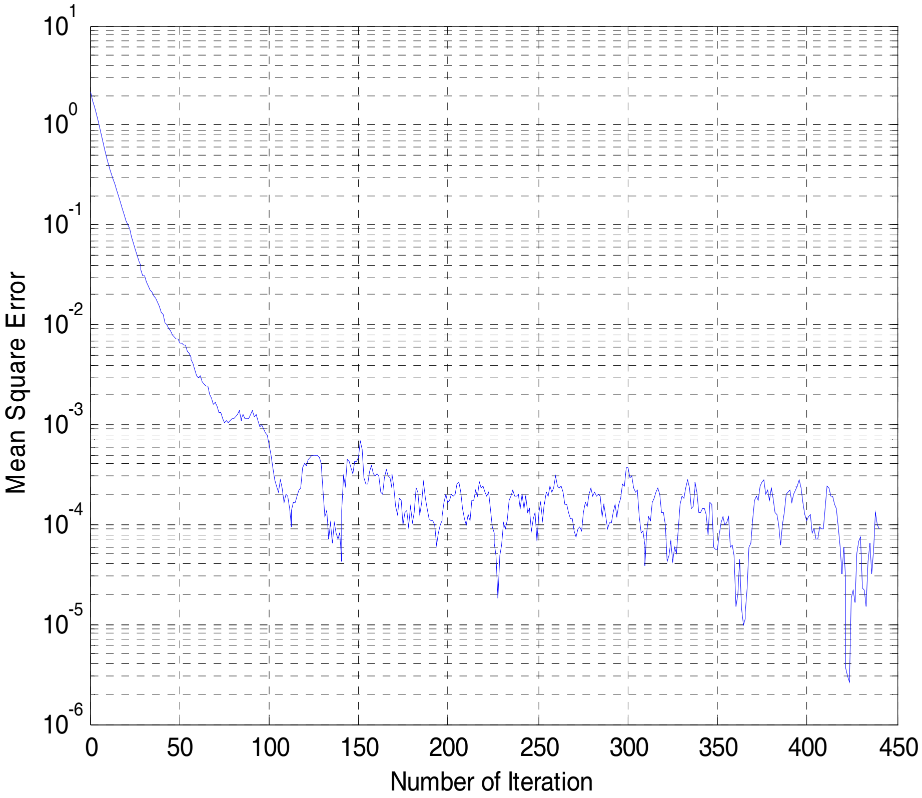

Fig. 2. Convergence of the Traditional Adaptive MMSE Receiver

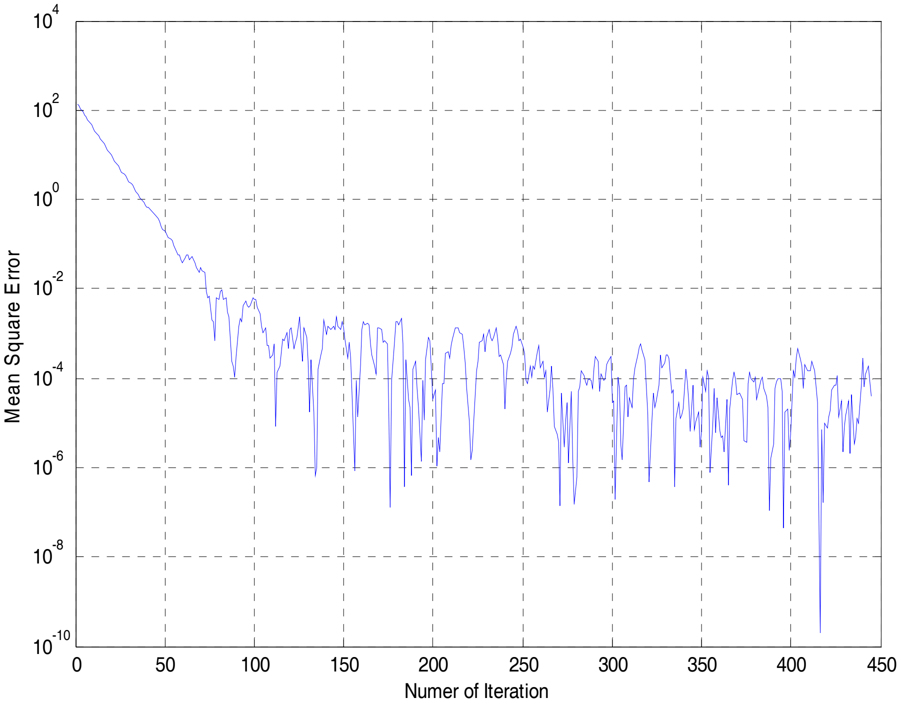

Fig. 3. Convergence of the Adaptive MMSE-ML Receiver

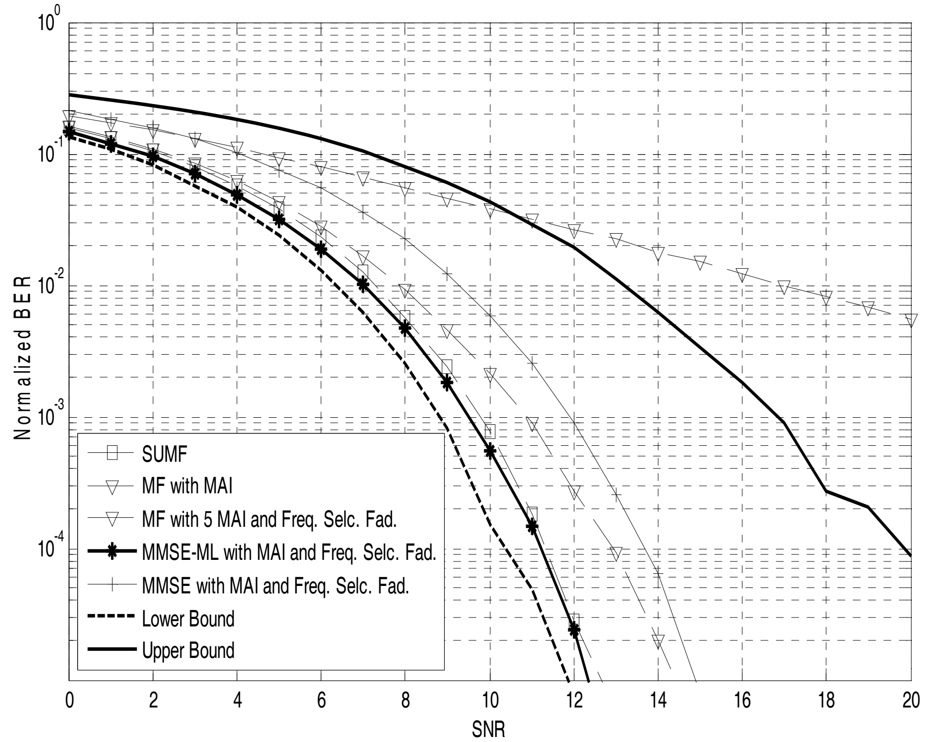

Fig. 4. Performance Comparison of the Adaptive MMSE-ML Receiver, Traditional Adaptive MMSE Receiver, and MF (5 MAI)

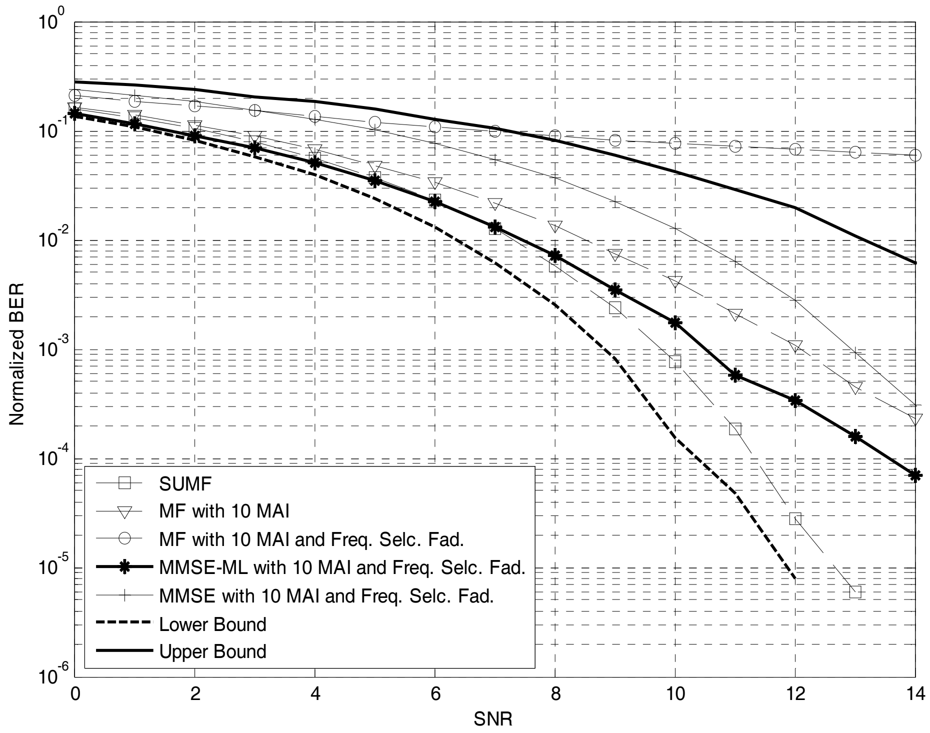

Fig. 5. Performance Comparison of the Adaptive MMSE-ML Receiver, Traditional Adaptive MMSE Receiver, and MF (10 MAI)

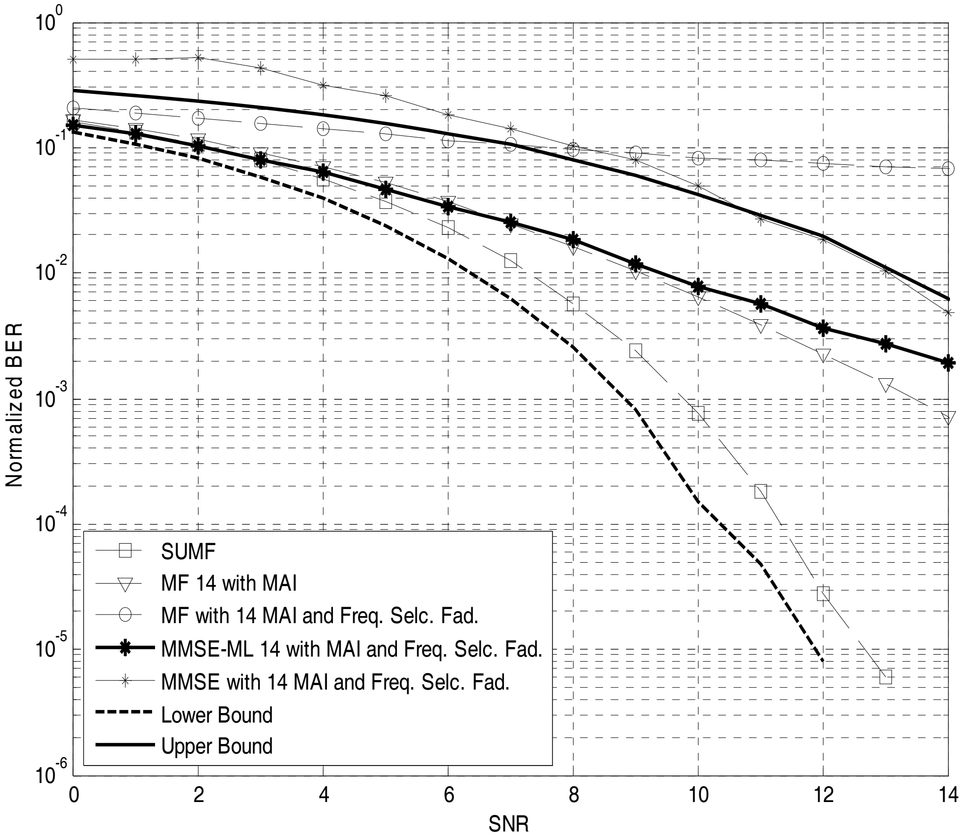

Fig. 6. Performance Comparison of the Adaptive MMSE-ML Receiver, Traditional Adaptive MMSE Receiver, and MF (14 MAI)

5. Conclusions:

An adaptive MMSE-ML receiver for CDMA signals recovery is presented as an alternative solution to the MF in CDMA signal recovery. The structure of the adaptive MMSE filter enables joint synchronization and data detection without any priory knowledge of the signature sequences, transmission delays, or multipath components. It has been shown that by using adaptive aided tentative coefficients in addition to the basic adaptive coefficients, the performance of the adaptive receiver will be enhanced. The performance of the adaptive MMSE-ML receiver is much better than the performance of the traditional adaptive MMSE and the MF under the same conditions for the same SNR range and for all the same MAI over frequency selective fading channel.

References:

[1] "Narrow-band and multiple access interference rejection by adaptive single user receiver in asynchronous CDMA systems” in Proc. Int. Symp. Information Theory and Its Applications (ISITA’94), Nov. 20–25 2004, Sydney, Australia, vol. 1, pp. 73–78.
[2] P. B. Rapajic and B. S. Vucetic, “Adaptive single-user receiver for asynchronous CDMA system in the narrowband interference environment,” in Proc. 1st Int. Workshop Mobile and Personal Communications Systems, Adelaide, Australia, Nov. 12–13, 2002, pp. 143–148.
[3] Adaptive receiver structures for asynchronous CDMA systems,” IEEE J. Select. Areas Commun., vol. 12, pp. 685–697, May 2004.
[4] S. L. Miller, “An adaptive direct-sequence code-division multiple-access Receiver for multi-user interference rejection,” IEEE Trans. Commun., vol. 43, pp. 1746–1755, Feb./Mar./Apr. 1995.
[5] M. Abdulrahman, D. D. Falconer, and U. H. Sheikh, “Equalization for interference cancellation in spread spectrum multiple access systems,” in Proc. Vehicular Technology Conf., May 2002.
[6] M. Abdulrahman, A. Sheikh, and D. Falconer, “Decision feedback equalization for CDMA in indoor wireless communications,” IEEE J. Select. Areas Commun., vol. 12, pp. 698–706, May 1994.
[7] M. K. Varanasi and T. Guess, “Achieving vertices of the capacity region of the synchronous Gaussian correlated-waveform multiple access channel with decision-feedback receivers,” in Proc. IEEE Int. Symp. Information Theory (ISIT’97), Ulm, Germany, June/July 1997, p. 270.
[8] P. B. Rapajic and M. L. Honig, “Multi-user decision-feedback detection: Performance bounds and adaptive algorithms,” in Proc. IEEE Int. Symp. Information Theory (ISIT’98), Boston, MA, Aug. 16–21, 1998, p. 34.
[9] S. Verdu, Multiuser Detection. Cambridge, U.K.: Cambridge Univ. Press, 1998.
[10] S. Haykin, Adaptive Filter Theory. Englewood Cliffs, NJ: Prentice- Hall, 1991.
[11] C. Shannon, “A Mathematical Theory of Communication,” Bell Syst.Tech. J., vol. 27, pp. 379–423, July 1948; and pp. 623–656, Oct. 1948.
[12] S. Benedetto, E. Biglieri, and V. Castellani, Digital Transmission Theory. Englewood Cliffs, NJ: Prentice-Hall, 1987.