Paper: ASAT-15-206-GU

15th International Conference on
AEROSPACE SCIENCES & AVIATION TECHNOLOGY,
ASAT - 15 – May 28 - 30, 2013, Email: asat@mtc.edu.eg ,
Military Technical College, Kobry Elkobbah, Cairo, Egypt,
Tel: +(202) 24025292 –24036138, Fax: +(202) 22621908

Performance Analysis of INS/GNSS Integration for Ballistic Missiles Applications

{I. I. Arafa, S. A. Shedied, A. El-Elboray}*

Abstract: Inertial Navigation System (INS) and Global Navigation Satellite System (GNSS) technologies have been widely used in a variety of positioning and navigation applications. Both systems have their unique features and shortcomings. Therefore, the integration of GNSSS with INS is now critical to overcome each of their drawbacks and to maximize each of their benefits. The integration of GPS/GNSS with INS can be implemented using a Kalman filter in such modes as loosely, tightly and ultra-tightly coupled. In all these integration modes the INS error states, together with any navigation state (position, velocity, attitude) and other unknown parameters of interest, are estimated using GPS/GNSS measurements. In this study, the analysis has been carried out using the tightly coupled GPS/SDINS integration. In addition, it has been demonstrated that vehicle dynamics affect the Kalman filter initialization time, especially for the heading component.

Keywords: Inertial navigation systems, and Kalman filtering, INS/GNSS integrated systems

1. Introduction

Integrated GPS/INS systems have been developed in order to overcome the inherent drawbacks of each system. Applications for such systems include airborne gravity surveying, mobile mapping, vehicle navigation, guidance and control [6]. Moreover, such a system is well suited for trajectory determination, as it can be easily described using position and attitude information. In such an integrated system, low data rate, high accuracy GNSS measurements can be used to estimate and to correct the error states of the INS within a dedicated Kalman filter.

The integration of GNSS with INS can be implemented using a Kalman filter in such modes as loosely, tightly and ultra-tightly coupled. In all these integration modes the INS error states, together with any navigation state (position, velocity, and attitude) and other unknown parameters of interest, are estimated using GNSS measurements. In a high performance system it is expected that all these unknown states will be precisely estimated. Although it has been noted that both the quality of the GNSS measurements and the trajectory and/or manoeuvre characteristics in a specific application will have a significant impact on system performance [6]. The performance analysis results are very relevant to system design and missile trajectory optimization.

The Onboard computer software program was developed to predict the full trajectory of the missile in order to improve the accuracy at impact. The primary purposes of missile launch surveillance are: 1) to provide a timely report of each occurrence of a missile launch, 2) to estimate launch/trajectory parameters, and 3) to estimate present and future missile

* Egyptian Armed Forces, Egypt.

trajectories as a function of time during flight. After the missile position and velocity vectors are obtained, the prediction starts with the time integration of equations of motion using a six parameter-state vector, which is composed of the position and velocity vectors, [1].

2. Strap Down INS Mechanization (SDINS)

This section describes techniques for initialization of inertial navigation system (INS) which is hosted on a missile. The INS initialization aims to determine initial values of the system, including position, velocity, and attitude. In case of alignment on stationary base, it is required to align an inertial navigation system to the local geographic co-ordinate frame defined by the directions of true north and the local vertical [3,13]. For this purposes, it is assumed that the navigation system is stationary w.r.t. the Earth. That is, the accelerometers measure three orthogonal components of the specific force needed to overcome gravity whilst the gyroscopes measure the components of the Earth's turn rate in the same directions. In a strap down system, attitude information may be stored as a direction cosine matrix. The objective of the angular alignment process is to determine the direction cosine matrix which defines the relationship between the inertial sensor axes and the local geographic frame. The measurements provided by the inertial sensors in body axes may be resolved into the local geographic frame using the current best estimate of the body attitude with respect to this frame [3].

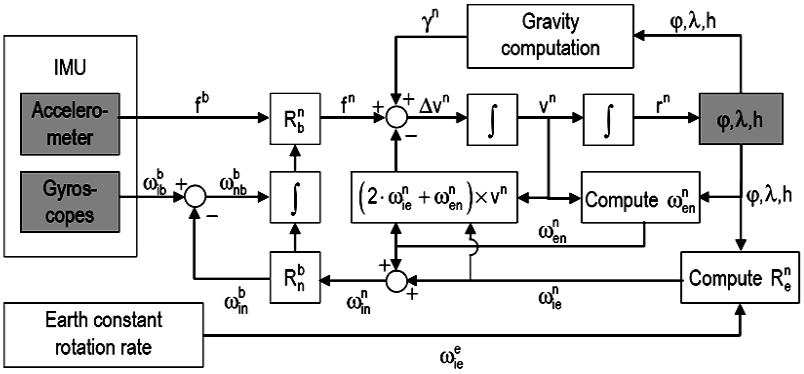

Figure (1) SDINS Mechanization Frame

3. On-Board Computer Software

The On-Board Software (OBCS) development was comprised of 3 major components: 1) the operating system, IMU digital signal processing code and ground alignment software and 2) ground checkout code developed uniquely for the program system and 3) the flight guidance, navigation, control and sequencing code [5].

After the final implementation integrates the GPS information into the flight navigation solution via the Kalman filter: There are two modes of operation for the system:

1) Ground test which assure that the navigation system is healthy before launch and 2) navigation in flight [5,10].

In the ground test we compute the system azimuth, level orientation and auxiliary sensor compensation through the ground alignment time. This creates the initial attitude reference and gyro and accelerometer biases and scale factors which are used at the time of (Go Inertial) for the flight navigation system [4, 5].

At "Go Inertial", the flight navigation routines, the GNSS Update Process and The Kalman filter are turned on and the system begins navigating.

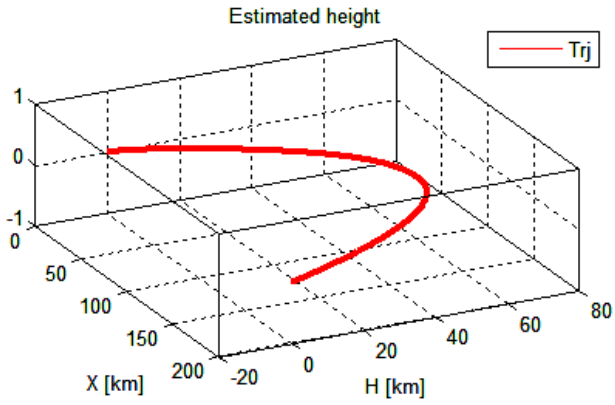

Figure (2): Mission profile

4. INS/GNSS Integration Approaches

In order to combine INS and GNSS data by using filtering techniques, we have two approaches as shown in figure (3). The 1st one is the standard Kalman filter, which processes the data from different systems in one step, is called the centralized Kalman filter. The 2nd one is the decentralized filtering technique, in which two Kalman filters are formulated [10, 11]. One is the GPS filter, which is used as a local filter and processes GPS data only. The other one is the INS filter which is considered as the master filter and estimates position, velocity, and attitude along the trajectory.

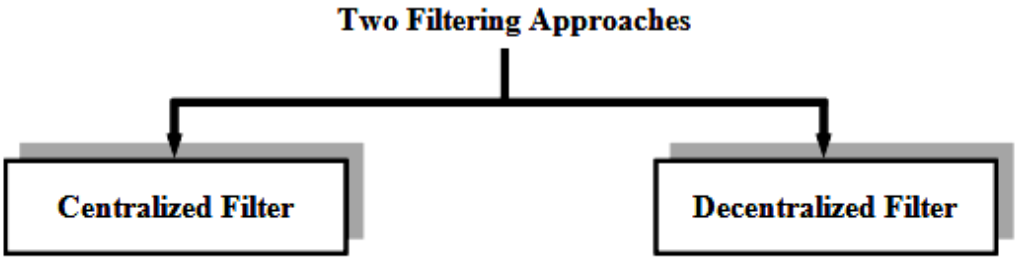

Figure (3): INS/GNSS integration approaches

In the loosely coupled integration, decentralized Kalman filtering is employed, but in the tightly coupled, centralized Kalman filtering is employed. One of the main advantages of the loosely coupled integration is that the dimension of the state vector is smaller as compared to other methods, on the other hand, in the tightly coupled large size of state vector which required more computation time [10,11].

5. Model of GNSS/INS Integrated System

A twenty seven state Kalman filter integration will be implemented in flight test mode to evaluate system performance. The flight test data consisted of measurement of IMU, GNSS, and trajectory generator data [5,12,13].

5.1. INS Error Model

5.1.1 Position error

The geographic implementation of the inertial navigation equations are as follows [5]:

$$\dot{\phi} = \frac{V_y^n}{R_M + h} \quad ; \quad \dot{\lambda} = \frac{V_x^n}{(R_N + h)} \sec\phi \quad ; \quad \dot{h} = V_z^n \tag{1}$$

The position error can be modeled as:

$$\delta\dot{\phi} = \frac{\delta V_y^n}{R_M + h} \quad ; \quad \delta\dot{\lambda} = \frac{\delta V_x^n}{(R_N + h)\cos\phi} + \frac{V_x^n \tan\phi \sec\phi}{R_N + h}\delta\phi \quad ; \quad \delta\dot{h} = \delta V_z^n \tag{2}$$

5.1.2 Attitude error model

The attitude error model using Euler angle can be written as [5]:

$$\dot{\varphi}^n = \varphi^n \times \omega_{in}^n + \delta\omega_{in}^n - C_b^n \delta\omega_{ib}^b \tag{3}$$

This equation can be rewrite as:

$$\dot{\varphi}^n = -\omega_{in}^n \times \varphi^n + \delta\omega_{in}^n - C_b^n \delta\omega_{ib}^b = -(\omega_{ie}^n + \omega_{en}^n) \times \varphi^n + \delta\omega_{ie}^n + \delta\omega_{en}^n + \Delta\omega^n \tag{4}$$

where $\delta\omega_{ib}^n$ is the error in measuring angular velocity vector due to gyro errors from body frame to inertial frame resolved in the navigation frame and can be define as:

$$\Delta\omega^n = C_b^n \delta\omega_{ib}^n \tag{5}$$

From the standard navigation mechanization in ENU frame we get:

$$\omega_{ie}^n = \begin{bmatrix} 0 & \Omega\cos\phi & \Omega\sin\phi \end{bmatrix}^T \tag{6}$$

$$\delta\omega_{ie}^n = \begin{bmatrix} 0 & -\Omega\sin\phi\delta\phi & \Omega\cos\phi\delta\phi \end{bmatrix}^T \tag{7}$$

$$\omega_{en}^n = \begin{bmatrix} \frac{-V_y^n}{R_M + h} & \frac{V_x^n}{R_N + h} & \frac{V_x^n \tan\phi}{R_N + h} \end{bmatrix}^T \tag{8}$$

5.1.3 Velocity error model

The SINS true velocity error in navigation frame is given by [13]:

$$\dot{V}^n = C_b^n f^b - (2\omega_{ie}^n + \omega_{en}^n) \times V^n + g^n \tag{9}$$

This equation can be rewrite as:

$$\dot{V}^n = f^n - (2\omega_{ie}^n + \omega_{en}^n) \times V^n + g^n \tag{10}$$

The velocity error equation can be written as:

$$\delta\dot{V}^n = \delta f^n - (2\delta\omega_{ie}^n + \delta\omega_{en}^n) \times V^n - (2\omega_{ie}^n + \omega_{en}^n) \times \delta V^n + \delta g^n \tag{11}$$

Then (11) can be rewrite as:

$$\delta\dot{V}^n = f^n \times \varphi^n - (2\delta\omega_{ie}^n + \delta\omega_{en}^n) \times V^n - (2\omega_{ie}^n + \omega_{en}^n) \times \delta V^n + \Delta f^n \tag{12}$$

5.2 Sensor Error Model

5.2.1 Gyro error model

Because of the complexity of the gyro error model so, we only focused on the two important factors which tends to gyro error measurements and we can consider all the above unknown parameters as a white noise (ε_g). The two main factors are the gyro constant drift vector (D_F) and the gyro scale factor vector (S_G). Then the gyro measuring error can be modeled as [5]:

$$\begin{bmatrix} \Delta\omega_x^p \\ \Delta\omega_y^p \\ \Delta\omega_z^p \end{bmatrix} = C_b^p \begin{bmatrix} \Delta\omega_x^b \\ \Delta\omega_y^b \\ \Delta\omega_z^b \end{bmatrix} = C_b^p \begin{bmatrix} D_{Fx} + S_{Gx}\omega_x^b + \varepsilon_{gx} \\ D_{Fy} + S_{Gy}\omega_y^b + \varepsilon_{gy} \\ D_{Fz} + S_{Gz}\omega_z^b + \varepsilon_{gz} \end{bmatrix} = C_b^p \begin{bmatrix} D_{Fx} \\ D_{Fy} \\ D_{Fz} \end{bmatrix} + D_\omega \begin{bmatrix} S_{Gx} \\ S_{Gy} \\ S_{Gz} \end{bmatrix} + C_b^p \begin{bmatrix} \varepsilon_{gx} \\ \varepsilon_{gy} \\ \varepsilon_{gz} \end{bmatrix} \tag{13}$$

where $\Delta\omega_x^p, \Delta\omega_y^p, \Delta\omega_z^p$ represents the error in measuring angular velocities in platform coordinates system due to gyro errors and needed for attitude error equation

5.2.2 Accelerometer error model

The two main factors are the accelerometer bias vector (K_o) and the accelerometer scale factor vector (K_1). Then the accelerometer measuring error can be modeled as:

$$\begin{bmatrix} \Delta f_x^p \\ \Delta f_y^p \\ \Delta f_z^p \end{bmatrix} = C_b^p \begin{bmatrix} \Delta f_x^b \\ \Delta f_y^b \\ \Delta f_z^b \end{bmatrix} = C_b^p \begin{bmatrix} K_{ox} + K_{1x} f_x^b + \nabla_x \\ K_{oy} + K_{1y} f_y^b + \nabla_y \\ K_{oz} + K_{1z} f_z^b + \nabla_z \end{bmatrix} = C_b^p \begin{bmatrix} K_{ox} \\ K_{oy} \\ K_{oz} \end{bmatrix} + D_f \begin{bmatrix} K_{1x} \\ K_{1y} \\ K_{1z} \end{bmatrix} + C_b^p \begin{bmatrix} \nabla_x \\ \nabla_y \\ \nabla_z \end{bmatrix}$$

where $\Delta f_x^n, \Delta f_y^n, \Delta f_z^n$ represents the error in specific acceleration measurement due to accelerometer errors and needed for velocity error equations

5.3 GPS/GLONASS Error Model

In many literatures the GPS/GLONASS have a lot of error sources which affects the accuracy of position and velocity. Even more, if most of these errors are corrected, still some random errors exist. For instance random errors are included in clock errors that are the first differential equation which describe the state errors can be written as [4,5,9]:

$$\dot{x}_{GPS} = F_{GPS} x_{GPS} + w_{GPS} \tag{14}$$

$$x_{GPS} = \begin{bmatrix} \delta\phi_{GPS} & \delta\lambda_{GPS} & \delta h_{GPS} & \delta V_{x_{GPS}} & \delta V_{y_{GPS}} & \delta V_{z_{GPS}} \end{bmatrix}^T \tag{15}$$

where $\delta\phi_{GPS}, \delta\lambda_{GPS}, \delta h_{GPS}$ are the position errors of GPS and $\delta V_{x_{GPS}}, \delta V_{y_{GPS}}, \delta V_{z_{GPS}}$ are the velocity errors of GPS. Then the GPS error system matrix can be define as:

$$F_{GPS} = diag\left(\begin{bmatrix} -\frac{1}{\tau_{\phi_{GPS}}} & -\frac{1}{\tau_{\lambda_{GPS}}} & -\frac{1}{\tau_{h_{GPS}}} & -\frac{1}{\tau_{Vx_{GPS}}} & -\frac{1}{\tau_{Vy_{GPS}}} & -\frac{1}{\tau_{Vz_{GPS}}} \end{bmatrix}\right) \tag{16}$$

The GPS noise vector can be define as:

$$w_{GPS} = \begin{bmatrix} \omega_{\phi_{GPS}} & \omega_{\lambda_{GPS}} & \omega_{h_{GPS}} & \omega_{Vx_{GPS}} & \omega_{Vy_{GPS}} & \omega_{Vz_{GPS}} \end{bmatrix}^T$$

where $\omega_{\phi_{GPS}}, \omega_{\lambda_{GPS}}, \omega_{h_{GPS}}$ and $\omega_{Vx_{GPS}}, \omega_{Vy_{GPS}}, \omega_{Vz_{GPS}}$ are the white noise in the GPS position and velocity respectively.

5.4 State and Measurement Equations of Integrated System

From the above 3-models (INS error model, sensors error model, and GPS/GLONASS error model) we can construct the integration navigation system error model as follows:

$$\dot{x}(\mathrm{t}) = F(t)x(\mathrm{t}) + G(\mathrm{t})w(\mathrm{t}) \tag{17}$$

The state vector of the INS/GPS/GLONASS integrated system can be written as [5,9]:

$$x(\mathrm{t}) = \begin{bmatrix} \varphi_x^p & \varphi_y^p & \varphi_z^p & \delta\phi & \delta\lambda & \delta h & \delta V_x^p & \delta V_y^p & \delta V_z^p \\ K_{ox} & K_{oy} & K_{oz} & K_{1x} & K_{1y} & K_{1z} & D_{Fx} & D_{Fy} & D_{Fz} \\ S_{Gx} & S_{Gy} & S_{Gz} & \delta_{\phi_{GPS}} & \delta_{\lambda_{GPS}} & \delta_{h_{GPS}} & \delta_{Vx_{GPS}} & \delta_{Vy_{GPS}} & \delta_{Vz_{GPS}} \end{bmatrix}^T \tag{18}$$

$$F(\mathrm{t}) = \begin{bmatrix} F_{INS} & F_{sen.} & 0_{9\times6} \\ 0_{12\times9} & 0_{12\times12} & 0_{12\times6} \\ 0_{6\times9} & 0_{6\times12} & F_{GPS} \end{bmatrix} \tag{19}$$

and the system noise vector $w(\mathrm{t})$ can be define as:

$$w(\mathrm{t}) = \begin{bmatrix} \varepsilon_{gx} & \varepsilon_{gy} & \varepsilon_{gz} & \nabla_x & \nabla_y & \nabla_z & \omega_{\phi_{GPS}} & \omega_{\lambda_{GPS}} & \omega_{h_{GPS}} & \omega_{Vx_{GPS}} & \omega_{Vy_{GPS}} & \omega_{Vz_{GPS}} \end{bmatrix}^T$$

where ε_g, ∇ are the gyro and accelerometer white noise vectors. The input system noise matrix can also be constructing as follows:

$$G(\mathrm{t}) = \begin{bmatrix} C_b^p & 0_{3\times3} & 0_{3\times6} \\ 0_{3\times3} & C_b^p & 0_{3\times6} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times6} \\ 0_{12\times3} & 0_{12\times3} & 0_{12\times6} \\ 0_{6\times3} & 0_{6\times3} & I_{6\times6} \end{bmatrix} \tag{20}$$

5.5 Kalman Filter Functionality and Mechanization

The flight System Error Estimate (SEE) Utilizes the Kalman filter to generate corrections to the navigation system using the difference between the GPS position, velocity and navigation position, velocity as the measurement [12]. Then we must discretize the continuous system matrix equation. The discretization of the state equation and measurement equation are as follows [4,5,6]:

$$x_k = \Phi_{k,k-1}\, x_{k-1} + \Gamma_{k-1}\, w_{k-1} \tag{21}$$

$$z_k = H_{k,} x_k + v_k \tag{22}$$

$$\begin{aligned} \hat{x}_{k/k-1} &= \Phi_{k,k-1}\, \hat{x}_{k-1} \\ \mathrm{P}_{k/k-1} &= \Phi_{k,k-1} \mathrm{P}_{k-1} \Phi_{k,k-1}^T + \Gamma_{k-1}\, Q_{k-1}\, \Gamma_{k-1}^T \end{aligned} \tag{23}$$

$$\begin{aligned} \mathrm{K}_k &= \mathrm{P}_{k/k-1}\, H_k^T (H_k\, \mathrm{P}_{k/k-1}\, H_k^T + R_k)^{-1} \\ \hat{x}_k &= \hat{x}_{k/k-1} + \mathrm{K}_k (z_k - H_k \hat{x}_{k/k-1}) \\ \mathrm{P}_k &= (I - \mathrm{K}_k\, H_k)\, \mathrm{P}_{k/k-1} \end{aligned} \tag{24}$$

5.6 Simulation Parameters

5.6.1 INS parameters

The initial attitude error angle is chosen as $500''$, initial velocity error $1.0\ m/s$, initial error of latitude and longitude angle 0.15 rad, initial altitude $10\ 00\ m$, initial accelerometer bias error $10^{-8}\ m/s^2$, initial accelerometer scale factor error 10^{-8}, initial gyro fixed drift error $10^{-8}\ ''/s^2$, initial gyro scale factor error 10^{-8}, random drift of each gyro 0.1^o/h, and random bias of each accelerometer $10\mu g$.

5.6.2 GPS/GLONASS parameters

Initial velocity error is chosen as $5\ m/s$, initial error of latitude and longitude angle 0.0015 rad, initial altitude $10\ m$, Markov time constant $\tau = 60$ sec, random altitude $5\ .0m$, and random in measuring velocity $1.0m/s$.

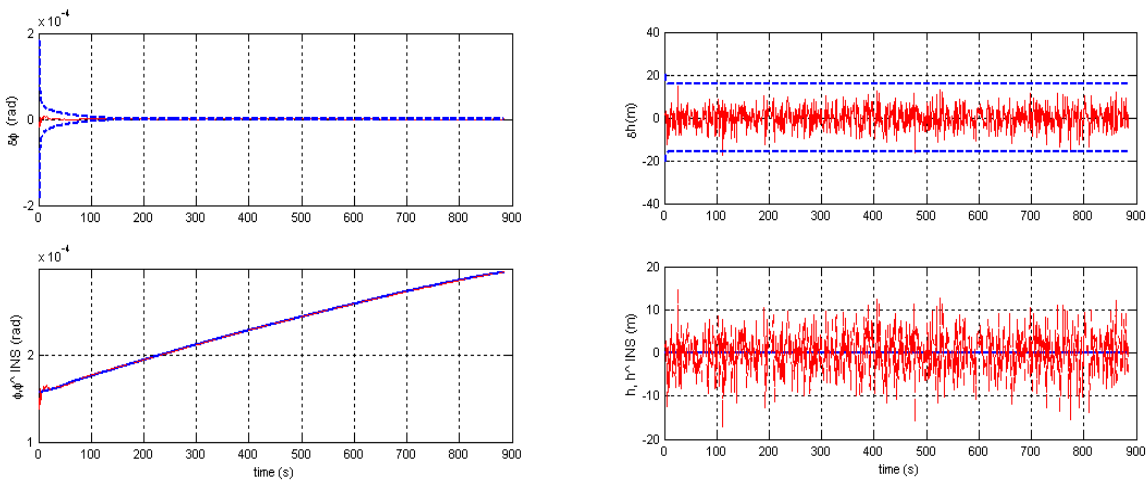

Figure (4) INS errors estimate

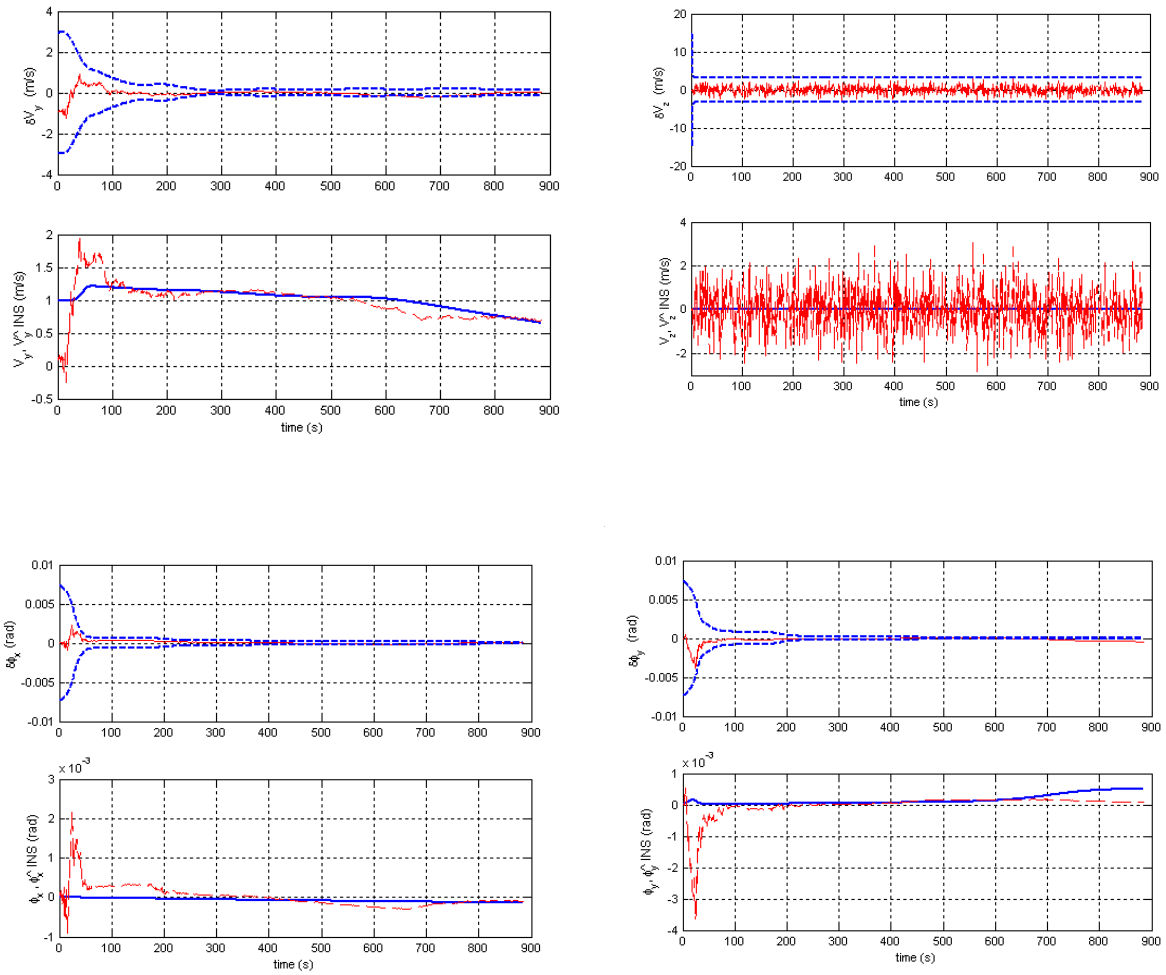

Figure (4) (Continued) INS errors estimate

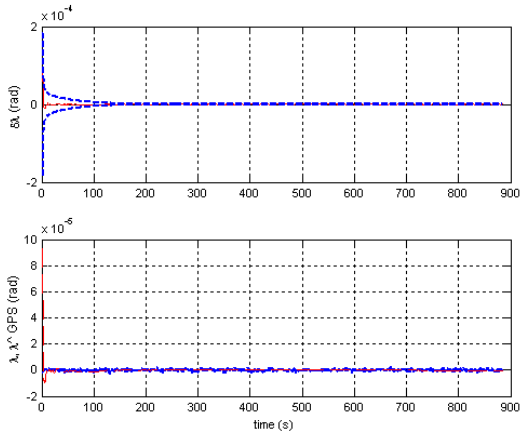

Figure (5): GNSS errors estimate

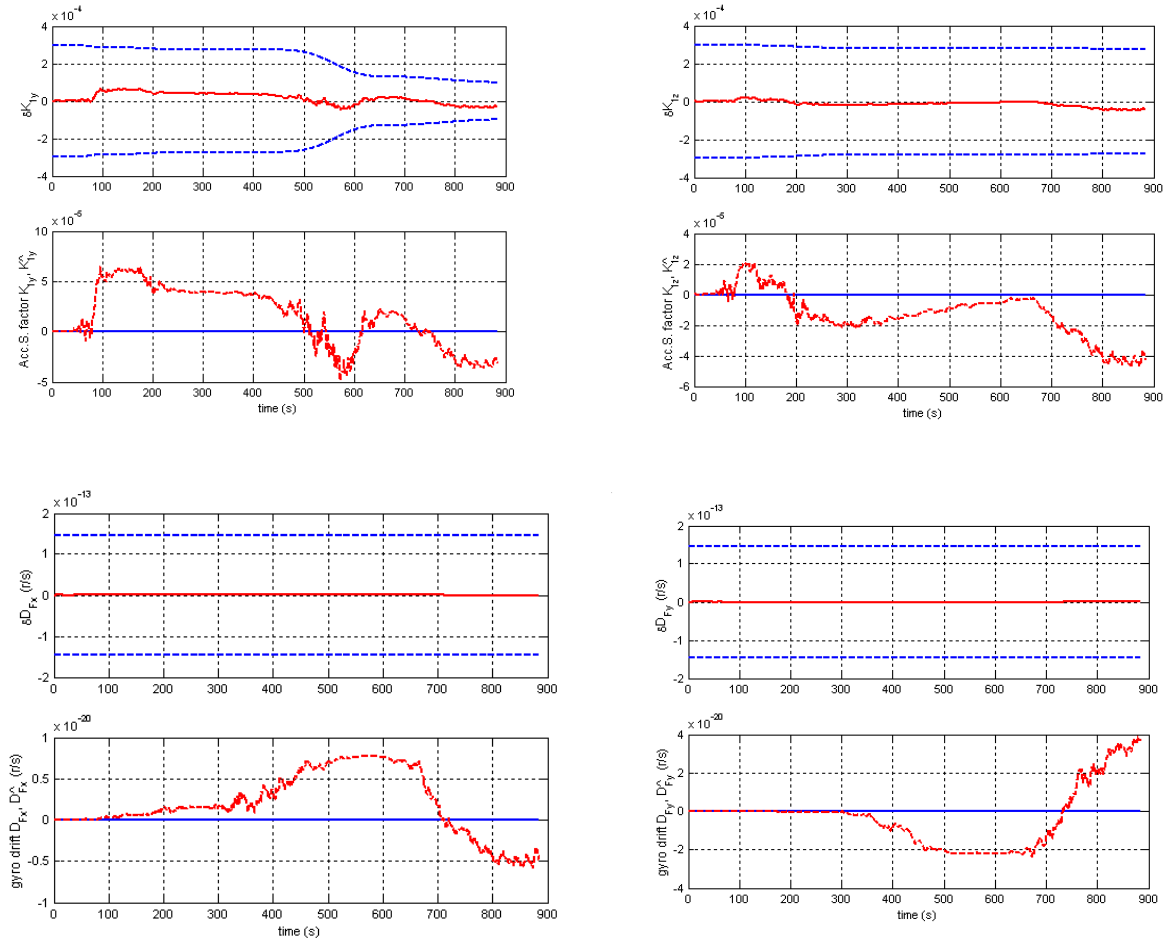

Figure (6): Sensor errors estimate

6. Conclusions

This paper demonstrated the importance of the INS/GPS/GLONASS integration into ballistic navigation solution. The SINS uses (ENU) frame and the integrated system uses position and velocity as measurements. The system model of the integrated system for Kalman filtering are derived and modeled as 27-states. This state estimation system shown clearly the application of fundamental modeling and filtering techniques. The simulation is built on the integrated system INS/GPS/GLONASS and the trajectory generator data. From the results we found that some parameters of estimated gyro errors such as vertical and east gyro drifts, and also estimated east accelerometer bias are not observables. The simulation shows that in the integrated system, the navigation errors in both the INS and GPS/GLONASS can be estimated with high accuracy. These results of this work will lead to the development of "IMU" state estimation system that supplies current motion information (position and attitude states) that can be used to carry out guidance and control strategies.

References

[1] Elleithy, M.A. , M.O Tantawy , M. E. Ghoniemy, "Development Of Six-Degrees of Freedom Model For Surface to Surface Strap down Ballistic Missile" GC-5, Third ASAT Conferences M.T.C. Cairo , Egypt 1989.

[2] George M. Siouris, "Aerospace Avionics Systems "A Modern Synthesis Air Force Institute of Technology 1993.

[3] El-sheikh, G.A.,"An Investigation into the performance of strap down Inertial Navigation Systems" Thesis, M.T.C., Cairo, Egypt 1990.

[4] Ibrahim I. Arafa, M. I. Latif, Zhong .H.Yue " Correction of the Flight Path Based on Error Estimation Of SINS Ground Alignment Using Modern Computer Techniques" Beihang University, Beijing 100083,Conferences second Asian workshop of foundations of software. Nanjing China DEC. 2003.

[5] Ibrahim Ismail Arafa,'' Analysis and Design of Guidance Navigation and Control Systems for Missile Applications'', Beihang University Sep. 2004

[6] Rodeny E. Stubbs, Whittak H. Huang and Eric Schimtz " Integration of GPS into Ballistic Navigation Solution" Lockheed Martin Astronautics By American Institute of aeronaumatic and Astronautics, Inc. 1997

[7] M. Napier, "Integration of Satellite and Inertial Positioning Systems" Institute of Engineering Surveying and Space Geodesy, University of Nottingham, Vol. 43, 1995

[8] Ye Fuh Jiang Chung Shan Institute of Science and technology, Taiwan "Error Analysis of Analytic Coarse Alignment methods" IEEE Transaction on Aerospace and Electronic, 2. May 1997.

[9] He. L. Shu" Ballistic Missile and Launch Vehicle Design" School of space technology, China, BUAA, 2009.

[10] Huddle, J. R. (1983) "Inertial navigation system error-model considerations in Kalman filtering applications". In C.T. Leondes (Ed.), Control and dynamics Systems. New York: Academic Press, 20 (1983)

[11] Kortum, W. (1976) "Design and Analysis of Low-Order Filters Applied to the Alignment of Inertial Platform". In practical aspect of Kalman filtering implementation, AGARD lecture series 82(AD-A024377), Mar. 1976.

[12] Yu Jiaxing, "Kalman Filter and Its Application in Inertial Navigation". Northwestern Poly technical University Express, Xian, 1984.

[13] Mostafa elzoghoby M.S. Thesis "Error Estimation of INS Alignment on Stationary and Moving Bases through Observability Analysis" M.T.C. JULE 2011. Cairo, Egypt