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Design optimization for a three-layer shrink-fitted pressure vessel exposed to very high pressure

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Abstract. In this paper, design optimization for a different materials three layers shrink-fitted cylinder subjected to very high pressure has been investigated. The optimum design of the cylinder has been brought into being by upsurge the advantageous compressive residual stresses, thus increasing the carrying load capacity of the cylinder as well as its fatigue life. A finite element model has been established to estimate the residual hoop stress, the equivalent von-Mises stress and the equivalent plastic strain. In this optimization problem, the design parameters have been taken as the thicknesses of the layers and the diametral interference between them. Also, the major constraint is that the equivalent von-mises stress for each layer does not exceed the yield strength of each layer material during the shrink-fitting process and when subjected to 750 MPa inner working pressure. Design of Experiment (DOE) and the Response Surface Method (RSM) have been joined together to obtain an effective objective function to be used in the optimization formulation. Two optimization techniques have been sequentially used to obtain accurate global optimum parameters, Multi Objective Genetic Algorithm (MOGA) and Lagrange's Multiplier (LM). The hoop and von-Mises stresses distributions along the thickness as well as the mechanical fatigue life have been calculated for the cylinder before and after optimization. It was found that the residual hoop stress has been enhanced at the near bore area by 31 %, also the fatigue life for the cylinder is better than that before optimization by 10900 cycles.

Keywords: Optimization, Multilayers cylinder, Shrink-fit, Residual stresses.

1. Introduction

Cylindrical shell is one of the most well-known machineries used in several applications which need very high internal pressure such as gun barrels, food reserving, nuclear and chemical plants.

In order to increase fatigue life, imperishability, pressure load capacity or even reduce the mass of these cylindrical shells, researchers have tried to cope with these margins by designing multilayer shrink-fitted cylinders. The shrink-fitted process and its resultant residual stresses based on Tresca yield conditions have been studied in Ref.s. [1-4]. Kumar et al. [5] examined the effect of layers number on maximizing the residual stress for a multilayer cylinder. They found that the increase of layers number decreases the residual stresses at the near bore area. Also, they obtained optimum dimensions using G.A method. O. R. Abedelsalam et al. [6, 7] studied the optimum design of a



two-layer compound shrink-fit and autofrettage cylinders to maximize the beneficial residual hoop stress. They combined MOGA with Sequential Quadratic Program (SQP) techniques to enhance the optimized design parameters. Gennaro et al. [8] proposed a solution for a multilayer cylinder includes pre-stresses and three kinds of interface transmission conditions: perfect, membrane-type and spring-type. The method was adopted for studying the uniform cooling of multilayer cylinders whose composing materials start to deform together below their joining temperature. Lyudmyla et al. [9] presented a closed-form solution for cylinders consisting of a number of arbitrarily nonhomogeneous layers. This solution makes it possible to provide a passable analysis of the stress distributions in the cylinder including the zones near the layer interfaces for different arrangements of the properties of neighboring layers. Rahman [10] examined the combined effects of the shrink-fit and autofrettage processes on the increasing the working pressure of the three-layer compound cylinder. He found that in a three-layer compound cylinder with proper interferences in suitable positions, the working pressure can be increased to about the double for a simple cylinder.

In this paper, ANSYS WORKBENCH 18 has been used to develop an accurate finite element model for a three layer cylinder to calculate the residual and von-Mises stresses during the shrink fit process and when subjected to high inner pressure up to 750 MPa.

Moreover in the optimization problem, the design parameters have been taken as the thickness of each layer and diametral interference for shrink-fitting between layers. However, constrains considered as fixed inner diameter of the cylinder and the elastic limit of each material used for the different layers. The goal is to increase the advantageous compressive residual hoop stress at the near bore area as well as to minimize the detrimental tensile residual hoop. That leads to minimizing the resultant equivalent stress which is compared with the yield strength of each layer material for no plastic deformation takes place during the shrink fit process and also when subjected to the inner working pressure. To create the objective function formula, (DOE) has been utilized to find the best possible arrangements for the prementioned design parameters. Then using the finite element model, the values of the responses (residual hoop stresses and the equivalent von-Mises stresses) have been calculated for each arrangement to complete the DOE matrix. Far ahead, the changes of these responses with the different design parameters have been illustrated using the RSM. Later, this objective functions have been utilized to maximize the compressive residual hoop stress and minimize the equivalent von-Mises stress by expending two sequential optimization techniques. MOGA has been firstly used, and then the obtained optimum design parameters have been enhanced by importing them as initial values for the LM optimization technique. Finally, fatigue life for the cylinder after optimization has been calculated and compared with that before optimization.

2. Finite Element Model

A finite element model of the three-layer cylinder is created in the atmosphere of ANSYS 18 WORKBENCH, as shown in Figure 1, to calculate the residual and von-Mises stresses for the three-layer cylinder when subjected to different shrink-fit arrangements. A 3-D solid structure element (SOLID 186) has been here. This element has 20 nodes in brick geometry; each node has four degrees of freedom (three displacements –temperature). Different outputs can be provided from this element as deformations, strains, stresses, temperature, equivalent Von-Mises stresses and total plastic strain [11]. However the number of elements and nodes differs regarding the change of the thickness of each layer through the optimization problem, but in general for the original dimensions of the cylinder which are 3.5 m long 10, 95 and 200 mm thicknesses for inner, middle and outer layers respectively, 7740 elements and 43086 nodes have been obtained for this finite element model.

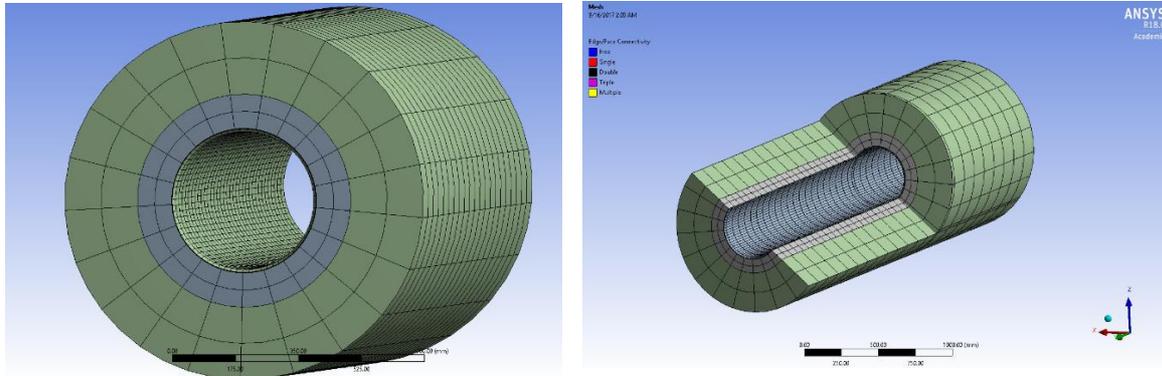


Figure 1. Finite element model of a three-layered cylinder.

The Material used for middle and outer layers is SA 723 GR.2 CL.2a; however, for the inner layer, the material used is SA 705 M GR XM -12. Table (1) displays the mechanical and physical properties for the prementioned materials.

Table 1: The mechanical and physical properties for the used materials for different layers of the cylinder.

Material	Density [kg mm ⁻³]	Coefficient of Thermal Expansion [C ⁻¹]	Young's Modulus [MPa]	Poisson's Ratio	Yield Strength [MPa]	Tensile Ultimate Strength [MPa]
<u>SA 723 GR.2 CL.2a</u>	<u>7.85e-006</u>	<u>1.1e-005</u>	<u>1.91e+005</u>	<u>0.3</u>	<u>895</u>	<u>1000</u>
<u>SA 705 M GR XM -12</u>	<u>7.75e-006</u>	<u>1.08e-005</u>	<u>1.91e+005</u>	<u>0.3</u>	<u>1000</u>	<u>1170</u>

3. Verification of Finite Element Model

To verify the F.E.M, its results have been compared with an analytical model's results for the residual stresses obtained for a two-layer cylinder after the shrink-fit process. The interference pressure P_{sh} which is established at the boundary radius between the two layers can be calculated [12] as:

$$P_{sh} = \frac{0.5 \delta}{\frac{c}{E_o} \left(\frac{b^2 + c^2}{b^2 - c^2} + \nu_o \right) + \frac{c}{E_i} \left(\frac{a^2 + c^2}{a^2 - c^2} - \nu_i \right)} \quad (1)$$

Where δ is the total diametral interference, a and b are inner and outer radii, c is the interfering radius, E_i , E_o and ν_i , ν_o are the modulus of elasticity and the Poisson's ratio associated to inner and outer layers, respectively. In this shrink-fitted process, the interference pressure acts as an external load for the inner layer and internal load for the outer one. That causes residual stresses in the hoop (σ_θ) and radial (σ_r) directions of the cylinder. These stresses through the radial position r for the inner and outer layers can be calculated as [12]:

$$\sigma_{\theta_{inner}} = -P_{sh} \frac{c^2}{c^2 - a^2} \left(1 + \frac{a^2}{r^2} \right) \quad (2)$$

$$\sigma_{r_{inner}} = -P_{sh} \frac{c^2}{c^2 - a^2} \left(1 - \frac{a^2}{r^2} \right) \quad (3)$$

$$\sigma_{\theta_{outer}} = P_{sh} \frac{c^2}{b^2 - c^2} \left(1 + \frac{b^2}{r^2} \right) \quad (4)$$

$$\sigma_{r_{outer}} = P_{sh} \frac{b^2}{b^2 - c^2} \left(1 - \frac{b^2}{r^2} \right) \quad (5)$$

Both layers have the same mechanical properties as:

$E_i, E_o = 190$ GPa, $\nu_i, \nu_o = 0.3$, $\sigma_y = 895$ MPa, and the diametral interference is taken as 0.4 mm. The residual hoop stresses obtained from the analytical solution of the equations 1, 2 and 4, have been compared with the FEM results, as shown in Fig.2 which shows a good agreement between the results obtained from the two models.

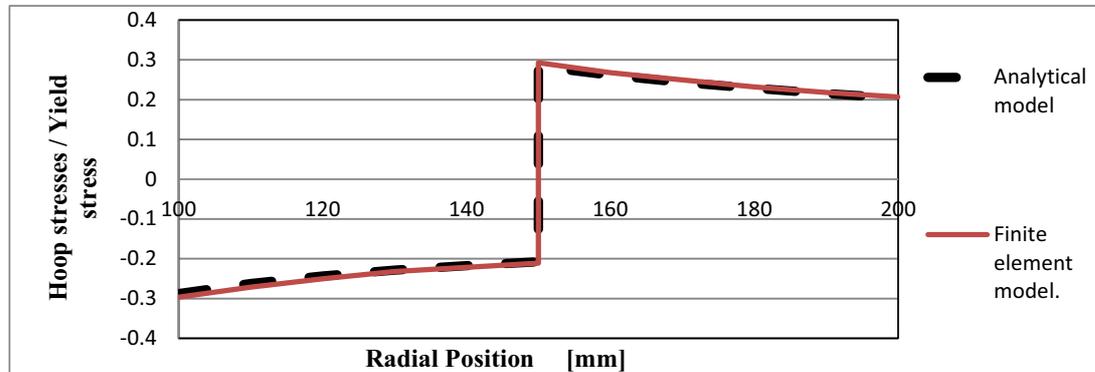


Figure 2. Comparison between residual hoop stresses obtained from the analytical and F.E. models.

4. Design Optimization Structure

4.1 Design Parameters

The design parameters are the thickness of inner layer t_1 , the thickness of middle layer t_2 , the thickness of outer layer t_3 , the shrink-fitting diametral interference δ_1 between the inner and middle layers, and the shrink-fitting diametral interference δ_2 between the middle and outer layers.

4.2 Constraints

The margins of the design parameters are considered in the inequality equation 6.

$$\text{Lower bound} \leq \text{design variable} \leq \text{Upper bound} \quad (6)$$

The design parameters margins are defined as shown in Table2.

Table2. Design parameters boundaries.

DVS	t_1 , mm	t_2 , mm	t_3 , mm	δ_1 , mm	δ_2 , mm
Lower bound	9.14	81.45	301.88	0.104	0.48
Upper bound	11.17	99.55	412.96	0.151	1.012

Also, the major constraint is that the equivalent von-mises stress for each layer does not exceed the yield strength of each layer material for no plastic deformation takes place during the shrink fit process and also when subjected to the inner working pressure.

4.3 Creation of Objective Function.

The goal of the optimization process is to find the parameters values that maximizing or minimizing the objective function which is a mathematical formula can describe the relationship between the design parameters and the output responses which are the prementioned stresses in this case.

To create this objective function, the statistical technique DOE has been employed to find the superlative arrangements of design parameters which can cover the whole design space. To complete the DOE matrix the values of the outputs (residual hoop stresses, equivalent von-Mises stresses, equivalent plastic strain) have been calculated

consuming the FEM for each row in the DOE matrix. Then, RSM has been used to illustrate fully quadratic response surfaces to show the variation of the outputs with the change of the design parameters. Finally, objective function of this optimization problem has been considered as these response surfaces.

4.3.1 Design of experiments (DOE).

DOE is an approach to improve an experimentation tactic that maximizes gain using minimum resources. In many applications, resources and time constrain the scientist while investigating complex processes have numerous factors. As an alternative, (DOE) certificates multiple input parameters to be employed to determine these parameters effect on an expected output (response) [13].

In this study, the DOE matrix has been created using central composite design technique. Appendix A shows the DOE Matrix after calculating the maximum obtained compressive residual, equivalent von-Mises stresses and equivalent plastic strain values through the whole thickness of the cylinder.

4.3.2 Response surface method (RSM).

RSM is utilized to illustrate the relationship (surface) between each response with the prementioned design parameters. Generally, RSM used to formulate a polynomial function which can be obtained using regression analysis. A fully quadratic fitting model has been used here [14]. For this case with five design parameters, the fully quadratic response surface equation can be written as shown in Eq. 7.

$$f = a_0 + a_1t_1 + a_2t_2 + a_3t_3 + a_4\delta_1 + a_5\delta_2 + a_6t_1t_2 + a_7t_1t_3 + a_8t_1\delta_1 + a_9t_1\delta_2 + a_{10}t_2t_3 + a_{11}t_2\delta_1 + a_{12}t_2\delta_2 + a_{13}t_3\delta_1 + a_{14}t_3\delta_2 + a_{15}\delta_1\delta_2 + a_{16}t_1^2 + a_{17}t_2^2 + a_{18}t_3^2 + a_{19}\delta_1^2 + a_{20}\delta_2^2 \quad (7)$$

Using the information from the DOE matrix, the constants in Eq.7 can be determined and the response surfaces for residual compressive hoop stress, equivalent von-Mises stresses and equivalent plastic strain with the design parameters can be illustrated in Figures 3-8.

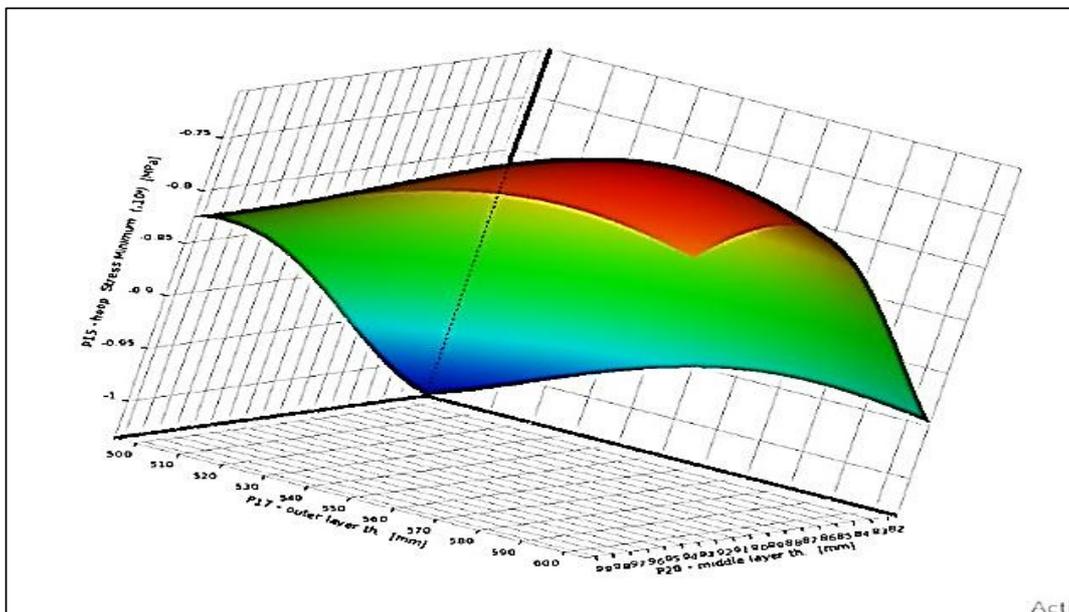


Figure 3. Compressive residual hoop stress change with the inner and middle layers thicknesses.

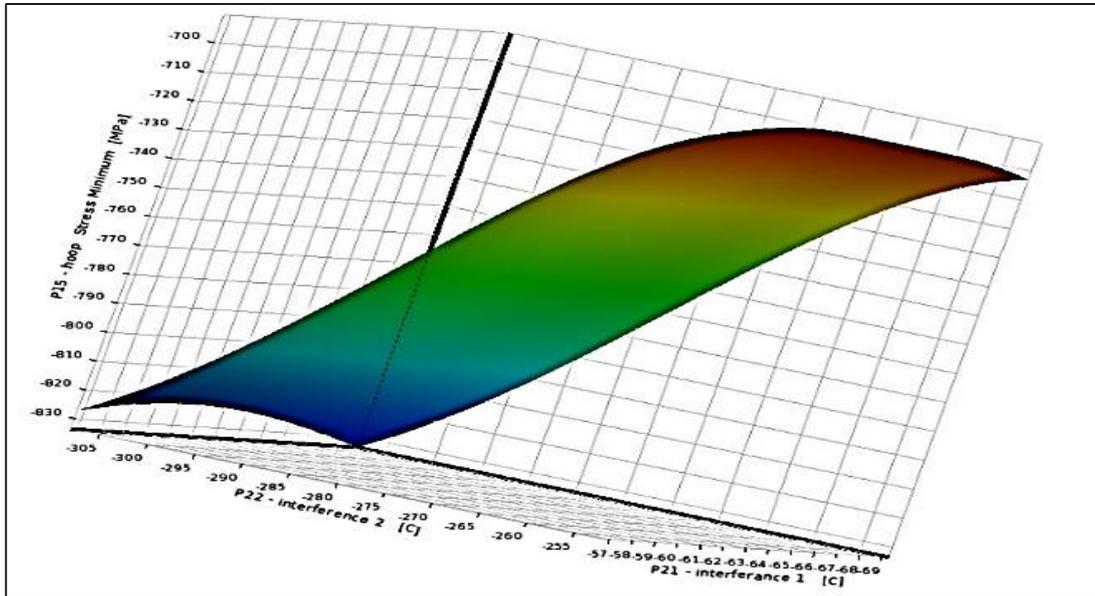


Figure 4. Compressive residual hoop stress change with the radial interferences.

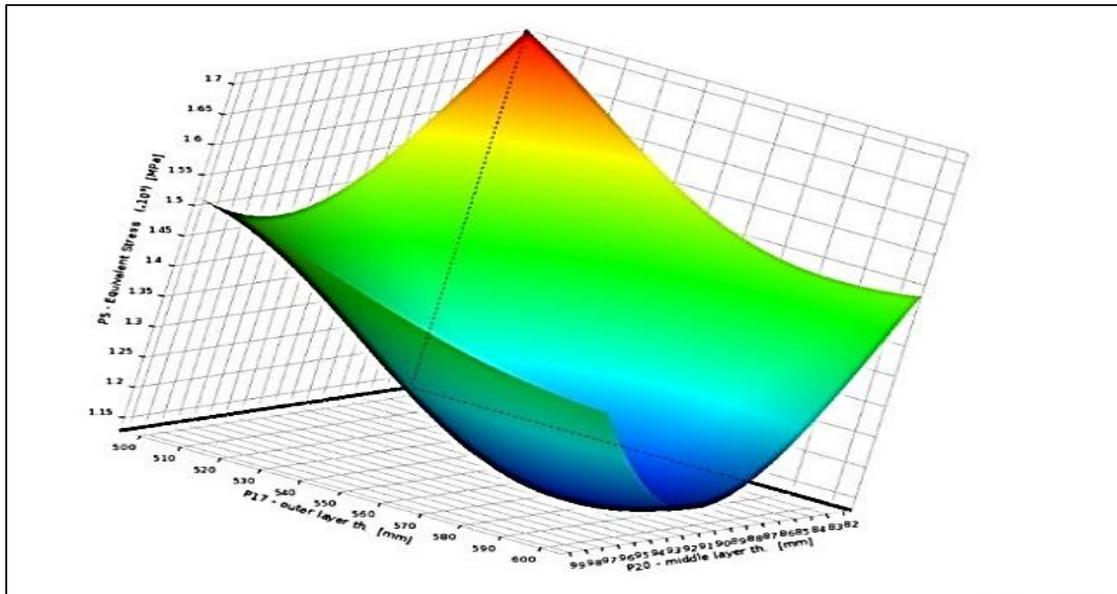


Figure 5. Equivalent von-Mises stress change with the middle and outer layers thicknesses.

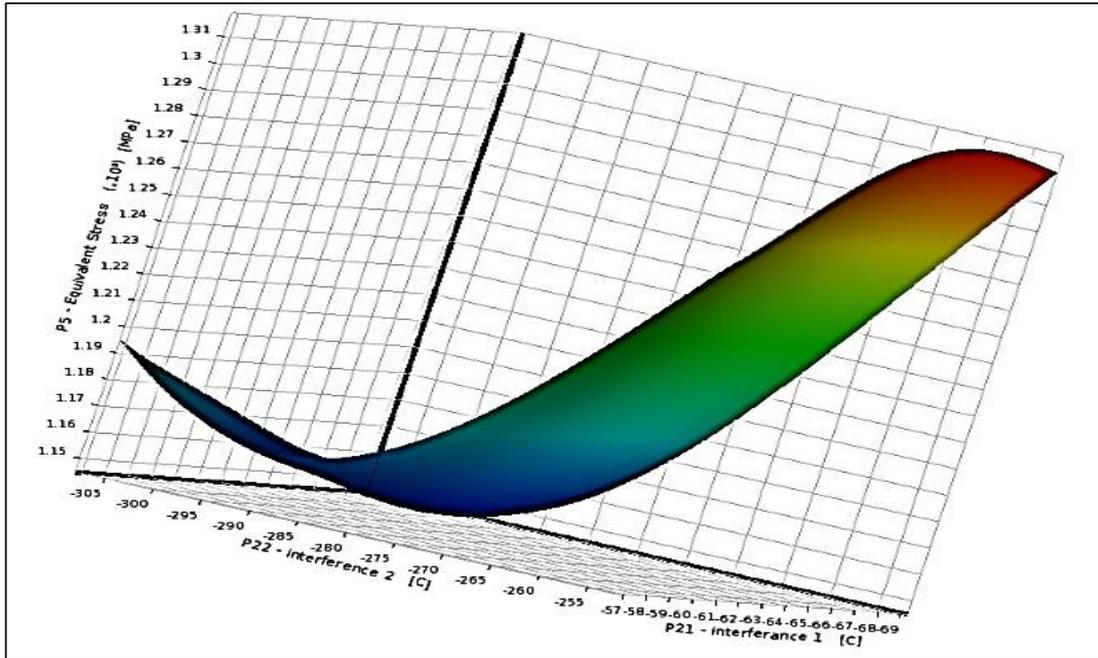


Figure 6. Equivalent von-Mises stress change with the radial interferences.

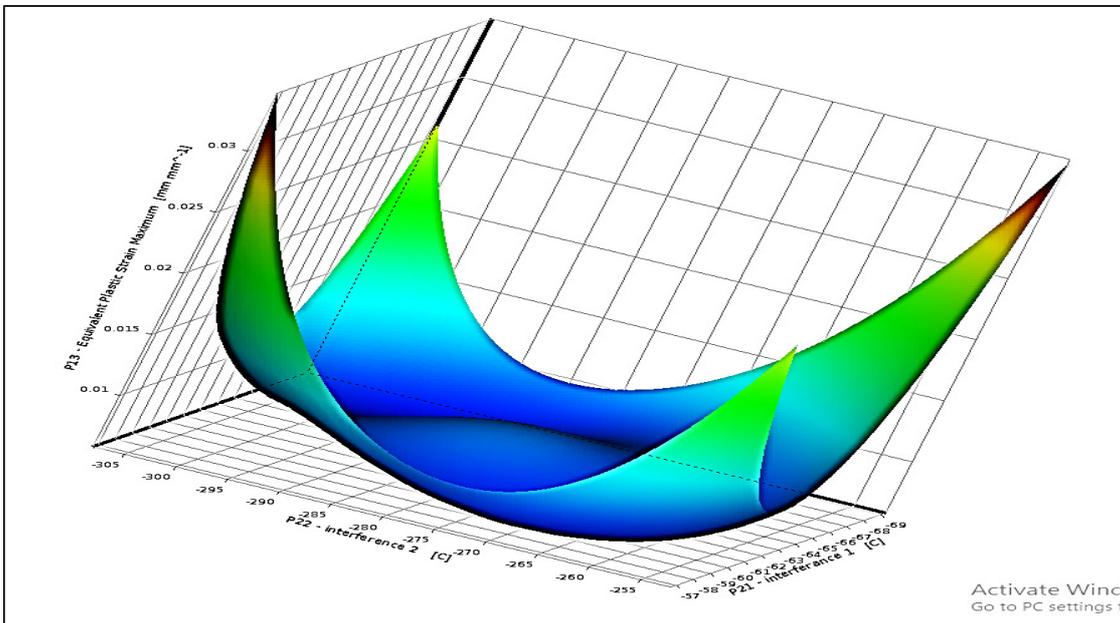


Figure 7. Equivalent plastic strain change with the radial interferences.

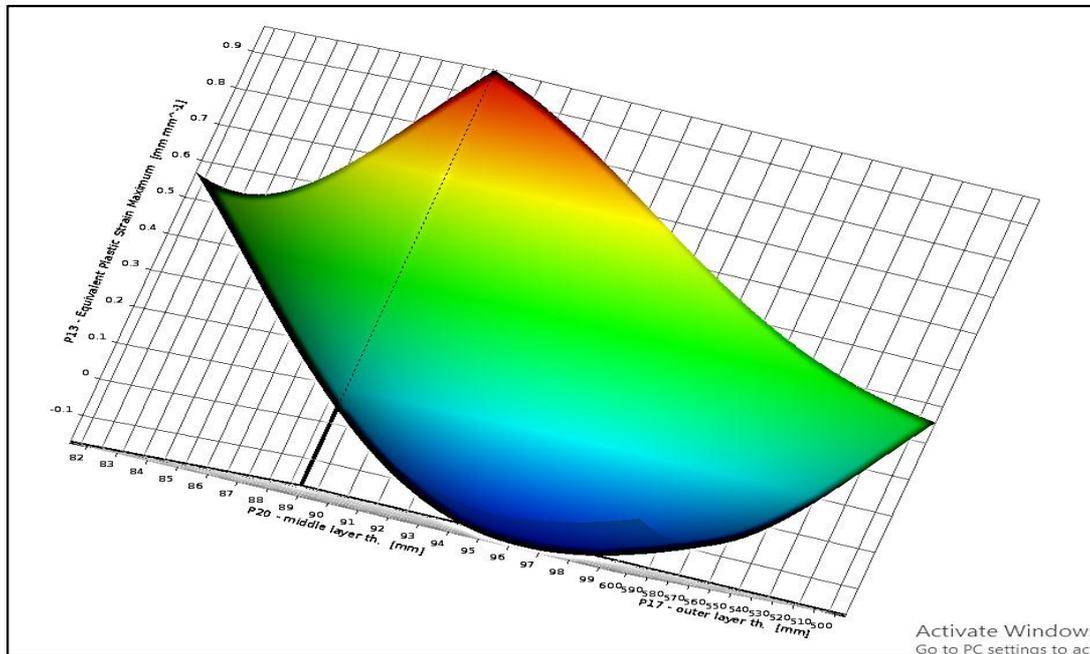


Figure 8. Equivalent plastic strain change with the middle and outer layers thickness.

Figures 3-8 show the change of the responses (hoop residual stress, equivalent von-Mises stress and equivalent plastic strain) with the different design parameters. It is clear that one can estimate from these response surfaces, where the minimum values of the objective function are, regarding each design parameters. Also, the sensitivity of each design variable with the output parameters is exposed in Figure 9.

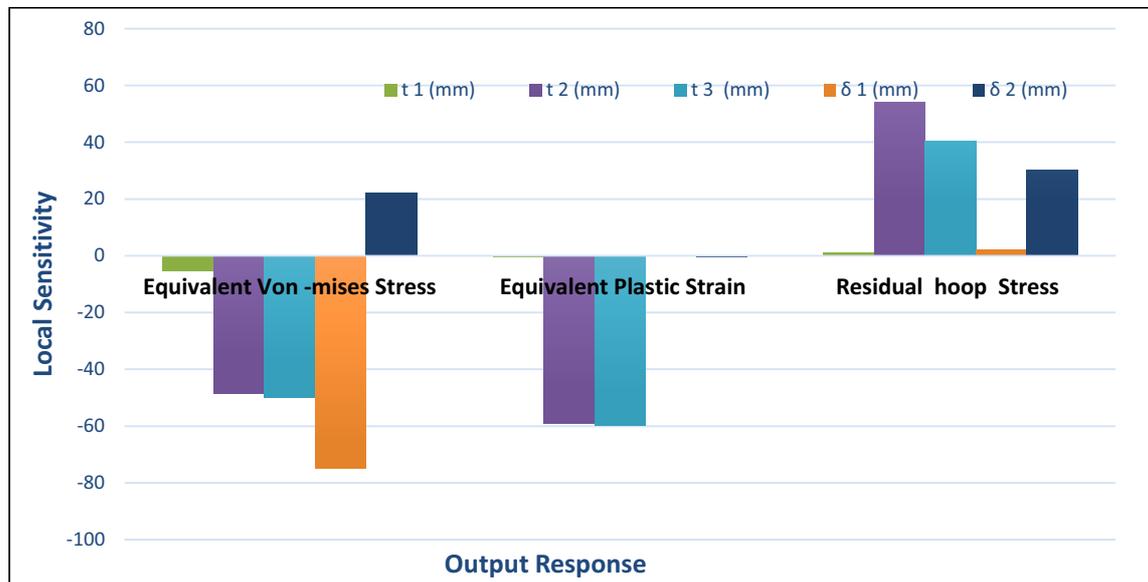


Figure 9. Objective functions sensitivity with different design parameters

Figure 9 reveals that the thicknesses of the outer and middle layers have a significant effect on all output responses. Also, it displays that the thickness of the inner layer, in the studied region, has less effect in all design parameters on the output responses.

5. Optimization Techniques and optimum design parameters

Generally, the objective function has only one output response. But in this case the following requirements have to be satisfied: (a) Maximizing the advantageous compressive residual hoop stress at the cylinder near bore area, (b) Minimizing the maximum equivalent von-Mises stresses during the shrink-fit process and when applying the working internal pressure, (c) Tending to zero the equivalent plastic strain during the shrink-fit process and when applying the inner working pressure. To solve this optimization problem which has three objective functions, two consecutive optimization techniques have been used here, the (MOGA) [15] which its optimum design parameters values are then imported as initial values for (LM) optimization technique [16]. LM has been utilized to enhance the design parameters values of MOGA and to be much closer to the global minimum / maximum objective functions and their related best optimum values. It is important to mention that the MOGA and LM have been done using the ANSYS 18 optimization toolbox. Table 3 shows the optimum design parameters after using MOGA and LM consequently.

Table 3. Optimum design parameters after using the two optimization techniques

Design Parameter	Optimum value
t_1 , mm	10.8
t_2 , mm	89.5
t_3 , mm	397
δ_1 , mm	0.15
δ_2 , mm	0.9

6. Results and discussions

6.1. Residual hoop and equivalent von-Mises stresses.

For more understanding of the effect of the optimization techniques, a comparison between the distributions through the cylinder thickness of the residual hoop stresses after the shrink fit process and before applying the working inner pressure has been illustrated, as shown in Figure 10. Also, a comparison between the distributions through the cylinder thickness of equivalent von-Mises stress after applying the inner working pressure (750MPa) has been illustrated in Figure 11. This figure is mainly to show that there is no plastic deformation takes place after applying the working pressure inside the cylinder.

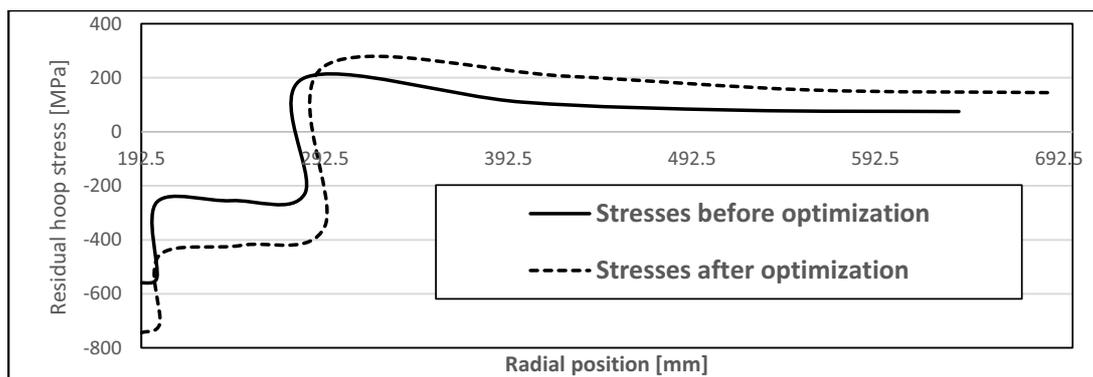


Figure 10. The distribution of residual hoop stress through the cylinder thickness.

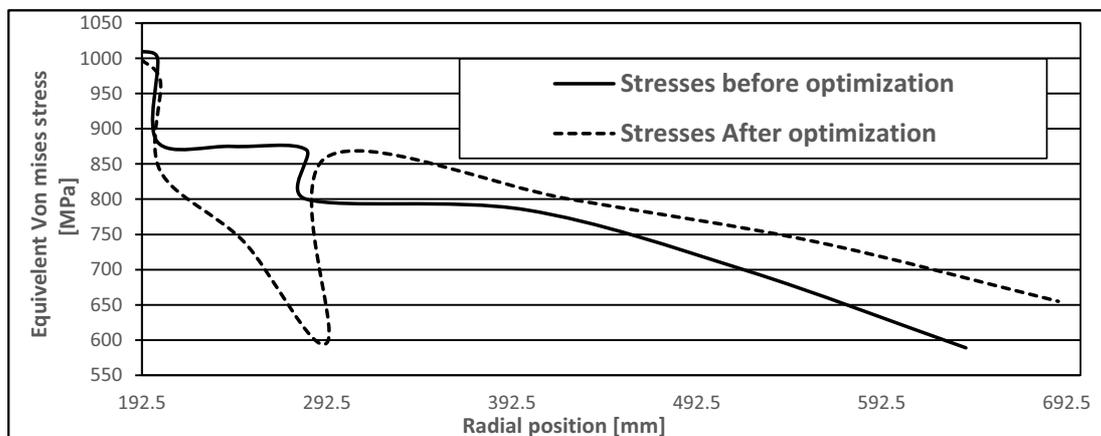


Figure 11. Equivalent von-Mises stress distribution through the cylinder thickness.

Figure 10 reveals that the reached optimum parameters can improve the residual hoop stresses. These optimum parameters increase the advantageous compressive hoop stress at the inner working area as well as reduce the increase in the detrimental tensile stress which considered as the main drawback in the shrink-fit process. On the other hand, Figure 11 shows that the equivalent von-Mises stresses have been decreased through the whole thickness of the cylinder using the reached optimum parameters, also it is clear that these stress values are less than the yield strength of the different layers materials, and consequently no plastic deformation has taken place when applying the inner working pressure.

6.2. Fatigue life principles

The aims of using multilayers cylinders are not only increase the load carrying capacity but also to enhance the fatigue life of the cylinder. Considering this and for more indulgent of the effect of the optimization technique, the fatigue life for each layer is calculated before and after optimization.

Hereinafter, and using the alternative rules for construction in ASME's of high-pressure vessels code [17], the fatigue life of the three layers cylinder has been calculated when exposed to inner cyclic pressure before and after optimization takes place. This has been evaluated by calculating the fatigue life for each layer under its loading condition, and the whole cylinder fatigue life is considered as same as the layer which has the smallest lifetime of the different layers in the cylinder.

Lifetime calculation has been done according to ASME VIII Codes Div. 3 article KD3 [17] for fatigue evaluation. Mean and alternating stresses have been calculated for the loading conditions for each layer. The loading conditions are the variation of the working inner pressure at the inner layer surface and their effect on different layers as follows:

- 1- For the inner layer, the inner pressure varies from zero to the maximum working pressure of the cylinder (750MPa)
- 2- For the middle and the outer layers, the variation radial stresses at the inner interface surface of each layer are considered as the inner working pressures variation for each layer respectively.
- 3- Stresses in z-direction is always zero for plane stress problem.

There are three principal stresses in the different directions have been calculated under loading condition; radial (σ_r), tangential (σ_t) and longitudinal (σ_z) which is considered as zero for this axisymmetric problem. Using Von Mises criteria, the mean and equivalent stresses have been provided. Then using Goodman theory, the equivalent stress intensity has been evaluated then corrected. Numbers of operating cycles have been determined according to the corrected equivalent stress intensity using ASME Figures KD 320. (1.4) M.

Table 4 shows the principal stresses for the different layers of the cylinder under different loading conditions before and after optimization.

Table 4. Principal stresses for different layer before optimization

Layer	Stresses	P= 0 [MPa]		P= 750 [MPa]		Alternating stress σ_a		Mean stress σ_m	
		Before	After	Before	After	Before	After	Before	After
Inner Layer	σ_r	0	0	-750	-750	350	325	-350	-325
	σ_t	-800	-750	507	405	-653.5	577.5	-146.5	-172.5
Middle Layer	σ_r	-130	-44	-650	-625	260	290.5	-390	-334.5
	σ_t	-460	-453	300	323	-380	388	-80	-65
Outer Layer	σ_r	-130	-128	-295	-285	82.5	78.5	-212.5	-206.5
	σ_t	210	243	470	528	-130	142.5	340	385.5

The equivalent amplitude and mean stresses for each layer have been then calculated using equations (8, 9), respectively.

$$\sigma_{eq} = \sqrt{\sigma_{ra}^2 + \sigma_{ta}^2 - \sigma_{ra}\sigma_{ta}} \quad (8)$$

$$\sigma_{meq} = \sigma_m + \sigma_{tm} \quad (9)$$

Using Goodman theory, convert equivalent amplitude stress and equivalent mean stress to equivalent stress under the symmetry circulating state, as shown in equation (10).

$$S_{eq} = \sigma_{eq} \left(\frac{1}{1 - \frac{\sigma_{meq}}{\sigma_{ult.}}} \right) \quad (10)$$

Where $\sigma_{ult.}$ is the ultimate strength of the material of each layer.

Correcting the equivalent stress is to include many environmental and production factors such as surface roughness K_R , temperature K_T . Based on the average surface roughness R_a 3.2 μm of cylinder body's inner bore, from figure KD320.5M(a) [17], it finds coefficient $K_R=1.16$. For the region of working temperature less than 230 degrees, $K_T=1$. Figure KD-320.4M used elastic modulus $E_{curve}=200$ GPa, Analysis and calculation used the elastic modulus $E_{analyse}=191$ GPa. Therefore the stress value used for lookup can be calculated as shown in equation (11).

$$S_a = K_R K_T S_{eq} \frac{E_{curve}}{E_{analyse}} \quad (11)$$

The design cycle times (N) are concluded using Figure KD-320.4M [17] for all layers before and after optimization according to each relative corrected equivalent stress (S_a) values. The values of (S_a) for different layers before and after optimization and the related number of fatigue life number of cycles (N) are as shown in Table (5).

Table 5. The values of (S_a) and (N) for different layers before and after optimization.

Layer	Before optimization		After optimization	
	S_a [MPa]	N (cycles)	S_a [MPa]	N (cycles)
Inner	650	1100	410	12000
Middle	360	110000	230	10^7
Outer	270	10^7	90	10^7

Table (5) shows that the lifetimes for the whole cylinder are 1100 and 12000 cycles before and after optimization, respectively. This reveals the importance and the effect of optimizing the prementioned design parameters in section (4).

7. Conclusion

This paper addresses the outcome of shrink-fitting multilayers cylinder on improving the residual stresses. In order to find the optimum design parameters for this cylinder, the objective function first has been created using DOE and RMS for three different responses which are the residual hoop stress, the equivalent von Mises stress and the equivalent plastic strain. Then GA and LM optimization techniques have been utilized consequently to maximize the advantageous compressive residual stress, minimize the equivalent von Mises stress and to tending to zero the equivalent plastic strain for the whole design process, after shrink fitting the three-layers and when applying the inner working pressure. On the other hand, fatigue lifetime in terms of the number of working cycles has been calculated for the different layers to deduce the lifetime of the cylinder. And therefore the following conclusions can be drawn as:

- 1- Shrink fit process for a three-layer cylinder could produce high advantageous residual hoop stress at the near bore area and consequently increase the carrying load capacity of the cylinder.
- 2- Using DOE with RSM could distinguish the change and sensitivity of each design parameter with the different responses and hence create an accurate objective function to be optimized.
- 3- MOGA combined with LM optimization techniques could deal with multi-objective optimization problem and detect the optimum design parameters. The effect of using these optimization techniques on the design of the considered cylinder is clear when analyzing the following results:
 - a- The residual hoop stress has been enhanced at the near bore area by 31 %.

- b- The equivalent von Mises stress has been reduced at the near bore area when applying the inner working pressure by 4 %.
- c- No plastic strain has been detected during the whole design and working processes.
- d- The lifetime has been increased of the cylinder by 10900 cycles.

Appendix A

DOE Matrix.

number of configurations	t_1 , mm	t_2 , mm	t_3 , mm	δ_1 , mm	δ_2 , mm	Max. hoop stress [MPa]	Max. von-mises stress [MPa]	Equiv. plastic strain [mm/mm]
1	10.16	90.5	357.55	0.127	0.72	-754.657	1172.438	0.003
2	10.16	90.5	302.545	0.127	0.57	-914.168	1491.83	0.614
3	10.16	90.5	412.555	0.127	0.88	-766.959	1183.598	0.003
4	9.144	90.5	357.55	0.127	0.73	-756.664	1186.378	0.003
5	11.176	90.5	357.55	0.127	0.72	-752.439	1160.115	0.003
6	10.16	81.45	357.55	0.123	0.75	-956.145	1524.616	0.616
7	10.16	99.55	357.55	0.131	0.70	-748.205	1427.909	0.003
8	10.16	90.5	357.55	0.147	0.72	-758.798	1174.028	0.003
9	10.16	90.5	357.55	0.108	0.72	-750.516	1169.817	0.003
10	10.16	90.5	357.55	0.127	0.80	-819.347	1176.874	0.004
11	10.16	90.5	357.55	0.127	0.64	-689.797	1320.188	0.003
12	9.144	81.45	302.545	0.142	0.53	-874.817	1757.671	0.728
13	9.144	81.45	412.555	0.142	1.012	-1068.46	1636.745	0.693
14	11.176	81.45	302.545	0.142	0.66	-994.011	1326.916	0.526
15	11.176	81.45	412.555	0.142	0.809	-871.216	1355.67	0.527
16	9.144	99.55	302.545	0.151	0.609	-787.267	1173.936	0.003
17	9.144	99.55	412.555	0.151	0.768	-705.62	1352.73	0.003
18	11.176	99.55	302.545	0.151	0.48	-660.95	1261.335	0.003
19	11.176	99.55	412.555	0.151	0.94	-834.064	1155.697	0.004
20	9.144	81.45	302.545	0.104	0.66	-1026.7	1758.913	0.728
21	9.144	81.45	412.555	0.104	0.81	-889.48	1636.093	0.693
22	11.176	81.45	302.545	0.104	0.53	-827.547	1319.018	0.526
23	11.176	81.45	412.555	0.104	1.006	-1027.21	1395.041	0.526
24	9.144	99.55	302.545	0.111	0.490	-655.768	1257.6	0.003
25	9.144	99.55	412.555	0.111	0.955	-830.472	1176.261	0.004
26	11.176	99.55	302.545	0.111	0.603	-771.073	1147.589	0.003
27	11.176	99.55	412.555	0.111	0.603	-694.402	1347.846	0.003

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