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Fuzzy estimation of the mean for electric load consumption based on different algorithms

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Abstract. Five new fuzzy algorithms for estimating the mean have been presented. First, we provide the estimation of the mean with considering the given data is crisp and standard deviation parameter is known and defined as triangular fuzzy number (TFN). Second, the algorithm will be defined by considering the data is a TFN and standard deviation parameter is known and crisp. Third, by combining first and second algorithms and considering both the data and standard deviation parameter are TFNs. Fourth, the same of the third algorithm but considering standard deviation parameter is unknown. Fifth, an estimation for the mean has been provided by considering both data and standard deviation parameter are crisp, and the membership function of the mean has been calculated by the means of confidence interval for different significance levels. Then, the advantages and disadvantage of using fuzzy algorithms have been presented. Finally, the calculation for the mean power consumed for an electric load in a certain compound inside Cairo for one month will be calculated based on fuzzy data and parameter.

1. Introduction

Over the last few years there has been an interest given to the problem of the estimation of unknown parameters of statistical models in fuzzy situations (e.g., with fuzzy data, with fuzzy parameters,...). In fact, in many practical studies, it is necessary to take a procedure for constructing the estimation based on the fuzzy elements [1]. The interval estimation is a main part of statistical inference and having precise data is one of the assumptions on which the common methods in interval estimations are based. But, in our world, sometimes recording or collecting the data precisely is difficult. Therefore, the fuzzy sets theory is found to be a suitable method in dealing with the vague data.

The concept of fuzzy sets was presented by Zadeh [2]. Klir and Yuan [3], and Zimmermann [4] may introduced the concise theory and applications of fuzzy sets. Then, the fuzzy data has been noticed and the concept of fuzzy random variables should be considered. Kwakernaak [5], Puri and Ralescu [6] introduced the concept of fuzzy random variables.

In this paper, fuzzy sets theory is applied to the statistical confidence interval for unknown parameter by considering a random sample of fuzzy data. So a fuzzy confidence interval for an unknown parameter is constructed. Also, it is applied for known fuzzy and crisp parameter by assuming a random sample of crisp and fuzzy data consequently.

There are two sources of uncertainties. The first one comes from choosing the random sample instead of considering the full population. The second is related to the fuzzy data and/or the fuzzy parameters. In the following, a new approach to construct a fuzzy confidence interval is proposed considering the above conditions.



Corral and Gil [7] constructed confidence interval (for crisp parameters) using fuzzy data, but without considering any fuzzy random variables. Statistical inference is studied about an unknown parameter, based on fuzzy observations by Viertl [8]. Using the extension principle for crisp parameter with crisp data, Viertl obtained generalized estimators based on fuzzy data and he developed some other statistical inference. A method of describing a fuzzy confidence interval that combines a fuzzy identification methodology with some ideas from applied statistics is proposed by Skrjanc [9]. The method is to find the confidence interval defined by the lower and upper fuzzy bounds, [10].

2. Crisp data with known fuzzy population standard deviation

Suppose that $x_1, x_2, x_3, \dots, x_n$ are the data of a crisp random sample of size n from a population normally distributed with unknown mean μ and known fuzzy standard deviation σ .

Since data is crisp, then the sample mean \bar{x} will be crisp [11]

$$\bar{X} = \frac{\sum_{i=1}^n x_i}{n} \quad (1)$$

Assuming population standard deviation σ is triangular fuzzy number (TFN) [12]

$$\sigma_F = [\sigma_1, \sigma_2, \sigma_3] \quad (2)$$

With triangular membership function $\mu_{\sigma_F}(x)$ [12]

$$\mu_{\sigma_F}(x) = \begin{cases} 0 & x \leq \sigma_1, x \geq \sigma_3 \\ \frac{x-\sigma_1}{\sigma_2-\sigma_1} & \sigma_1 \leq x \leq \sigma_2 \\ \frac{\sigma_3-x}{\sigma_3-\sigma_2} & \sigma_2 \leq x \leq \sigma_3 \end{cases} \quad (3)$$

Then $(1-\alpha)$ % confidence interval for the population mean (μ) can be calculated as [13]

$$\mu_F = [\bar{x} - z_{\alpha/2} \frac{\sigma_F}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{\sigma_F}{\sqrt{n}}] \quad (4)$$

$$\mu_F = [[\bar{x} - \frac{z_{\alpha/2}}{\sqrt{n}} [\sigma_1, \sigma_2, \sigma_3], \bar{x} + \frac{z_{\alpha/2}}{\sqrt{n}} [\sigma_1, \sigma_2, \sigma_3]] \quad (5)$$

$$\mu_F = [[\bar{x} - \frac{z_{\alpha/2}}{\sqrt{n}} \sigma_3, \bar{x} - \frac{z_{\alpha/2}}{\sqrt{n}} \sigma_2, \bar{x} - \frac{z_{\alpha/2}}{\sqrt{n}} \sigma_1], [\bar{x} + \frac{z_{\alpha/2}}{\sqrt{n}} \sigma_1, \bar{x} + \frac{z_{\alpha/2}}{\sqrt{n}} \sigma_2, \bar{x} + \frac{z_{\alpha/2}}{\sqrt{n}} \sigma_3]] \quad (6)$$

$$\mu_F = [\mu_{F_1}, \mu_{F_2}] = [[\mu_{l1}, \mu_{l2}, \mu_{l3}], [\mu_{h1}, \mu_{h2}, \mu_{h3}]] \quad (7)$$

3. Fuzzy data with known crisp population standard deviation

Suppose that $x_1, x_2, x_3, \dots, x_n$ are the data of a fuzzy random sample of size n from a population normally distributed with unknown mean μ and known crisp population standard deviation σ .

Assuming $x_1, x_2, x_3, \dots, x_n$ are TFNs with triangular membership functions $\mu_{x_1}(x), \mu_{x_2}(x), \mu_{x_3}(x), \dots, \mu_{x_n}(x)$.

Since the sample mean can be calculated as [12]

$$\bar{X}_F = \frac{\sum_{i=1}^n x_i}{n} \quad (8)$$

Then \bar{x} will be TFN

$$\bar{X}_F = [\bar{x}_1, \bar{x}_2, \bar{x}_3] \quad (9)$$

With triangular membership function $\mu_{\bar{X}_F}(x)$ [14]

$$\mu_{\bar{X}_F}(x) = \begin{cases} 0 & x \leq \bar{x}_1, x \geq \bar{x}_3 \\ \frac{x-\bar{x}_1}{\bar{x}_2-\bar{x}_1} & \bar{x}_1 \leq x \leq \bar{x}_2 \\ \frac{\bar{x}_3-x}{\bar{x}_3-\bar{x}_2} & \bar{x}_2 \leq x \leq \bar{x}_3 \end{cases} \quad (10)$$

Then (1- α) % confidence interval for the mean of population (μ) can be calculated as

$$\mu_F = [\bar{X}_F - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{X}_F + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}] \quad (11)$$

$$\mu_F = [[\bar{x}_1, \bar{x}_2, \bar{x}_3] - \frac{z_{\alpha/2}}{\sqrt{n}} \sigma, [\bar{x}_1, \bar{x}_2, \bar{x}_3] + \frac{z_{\alpha/2}}{\sqrt{n}} \sigma] \quad (12)$$

$$\mu_F = [[\bar{x}_1 - \frac{z_{\alpha/2}}{\sqrt{n}} \sigma, \bar{x}_2 - \frac{z_{\alpha/2}}{\sqrt{n}} \sigma, \bar{x}_3 - \frac{z_{\alpha/2}}{\sqrt{n}} \sigma], [\bar{x}_1 + \frac{z_{\alpha/2}}{\sqrt{n}} \sigma, \bar{x}_2 + \frac{z_{\alpha/2}}{\sqrt{n}} \sigma, \bar{x}_3 + \frac{z_{\alpha/2}}{\sqrt{n}} \sigma]] \quad (13)$$

$$\mu_F = [\mu_{F_1}, \mu_{F_2}] = [[\mu_{l1}, \mu_{l2}, \mu_{l3}], [\mu_{h1}, \mu_{h2}, \mu_{h3}]] \quad (14)$$

4. Fuzzy data with known fuzzy population standard deviation

Assuming a sample of size n with fuzzy data has been calculated, this fuzzy data has triangular membership function, so the sample mean (\bar{x}) will be TFN [12]

$$\bar{X}_F = [\bar{x}_1, \bar{x}_2, \bar{x}_3] \quad (15)$$

With triangular membership function $\mu_{\bar{X}_F}(x)$

$$\mu_{\bar{X}_F}(x) = \begin{cases} 0 & x \leq \bar{x}_1, x \geq \bar{x}_3 \\ \frac{x-\bar{x}_1}{\bar{x}_2-\bar{x}_1} & \bar{x}_1 \leq x \leq \bar{x}_2 \\ \frac{\bar{x}_3-x}{\bar{x}_3-\bar{x}_2} & \bar{x}_2 \leq x \leq \bar{x}_3 \end{cases} \quad (16)$$

Considering the population standard deviation (σ) is known and TFN

$$\sigma_F = [\sigma_1, \sigma_2, \sigma_3] \quad (17)$$

With triangular membership function $\mu_{\sigma_F}(x)$

$$\mu_{\sigma_F}(x) = \begin{cases} 0 & x \leq \sigma_1, x \geq \sigma_3 \\ \frac{x-\sigma_1}{\sigma_2-\sigma_1} & \sigma_1 \leq x \leq \sigma_2 \\ \frac{\sigma_3-x}{\sigma_3-\sigma_2} & \sigma_2 \leq x \leq \sigma_3 \end{cases} \quad (18)$$

Assuming the distribution is normal for the data calculated, then (1- α) % confidence interval for the mean of population (μ) can be calculated as

$$\mu_F = [\bar{X}_F - z\alpha/2 \frac{\sigma_F}{\sqrt{n}}, \bar{X}_F + z\alpha/2 \frac{\sigma_F}{\sqrt{n}}] \quad (19)$$

$$\mu_F = [[\bar{x}_1, \bar{x}_2, \bar{x}_3] - \frac{z\alpha/2}{\sqrt{n}} [\sigma_1, \sigma_2, \sigma_3], [\bar{x}_1, \bar{x}_2, \bar{x}_3] + \frac{z\alpha/2}{\sqrt{n}} [\sigma_1, \sigma_2, \sigma_3]] \quad (20)$$

$$\mu_F = [[\bar{x}_1 - \frac{z\alpha/2}{\sqrt{n}} \sigma_3, \bar{x}_2 - \frac{z\alpha/2}{\sqrt{n}} \sigma_2, \bar{x}_3 - \frac{z\alpha/2}{\sqrt{n}} \sigma_1], [\bar{x}_1 + \frac{z\alpha/2}{\sqrt{n}} \sigma_1, \bar{x}_2 + \frac{z\alpha/2}{\sqrt{n}} \sigma_2, \bar{x}_3 + \frac{z\alpha/2}{\sqrt{n}} \sigma_3]] \quad (21)$$

$$\mu_F = [\mu_{F_1}, \mu_{F_2}] = [[\mu_{l1}, \mu_{l2}, \mu_{l3}], [\mu_{h1}, \mu_{h2}, \mu_{h3}]] \quad (22)$$

5. Fuzzy data with unknown population standard deviation

Suppose that $x_1, x_2, x_3, \dots, x_n$ are the data of a fuzzy random sample of size n from a population normally distributed with unknown mean μ and unknown population standard deviation σ .

Assuming $x_1, x_2, x_3, \dots, x_n$ are TFNs with triangular membership functions $\mu_{x_1}(x), \mu_{x_2}(x), \mu_{x_3}(x), \dots, \mu_{x_n}(x)$.

Since the sample mean can be calculated as

$$\bar{X}_F = \frac{\sum_{i=1}^n x_i}{n} \quad (23)$$

Then \bar{X}_F will be TFN with triangular membership function $\mu_{\bar{x}}(x)$

$$\mu_{\bar{X}_F}(x) = \begin{cases} 0 & x \leq \bar{x}_1, x \geq \bar{x}_3 \\ \frac{x - \bar{x}_1}{\bar{x}_2 - \bar{x}_1} & \bar{x}_1 \leq x \leq \bar{x}_2 \\ \frac{\bar{x}_3 - x}{\bar{x}_3 - \bar{x}_2} & \bar{x}_2 \leq x \leq \bar{x}_3 \end{cases} \quad (24)$$

Since the standard deviation of population (σ) is unknown. So, the sample standard deviation (S) is provided as [11]

$$S_F = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{X}_F)^2} \quad (25)$$

The sample standard deviation (S) will be a TFN

$$S_F = [S_1, S_2, S_3] \quad (26)$$

With triangular membership function $\mu_{S_F}(x)$

$$\mu_{S_F}(x) = \begin{cases} 0 & x \leq S_1, x \geq S_3 \\ \frac{x - S_1}{S_2 - S_1} & S_1 \leq x \leq S_2 \\ \frac{S_3 - x}{S_3 - S_2} & S_2 \leq x \leq S_3 \end{cases} \quad (27)$$

Since the population standard deviation is unknown. Then, it has been calculated using t-student distribution with $(n-1)$ degree of freedom.

Then $(1-\alpha)$ % confidence interval for the population mean (μ) can be calculated as

$$\mu_F = [\bar{X}_F - t_{\alpha/2} \frac{S_F}{\sqrt{n}}, \bar{X}_F + t_{\alpha/2} \frac{S_F}{\sqrt{n}}] \quad (28)$$

$$\mu_F = [[\bar{x}_1, \bar{x}_2, \bar{x}_3] - \frac{t_{\alpha/2}}{\sqrt{n}} [S_1, S_2, S_3], [\bar{x}_1, \bar{x}_2, \bar{x}_3] + \frac{t_{\alpha/2}}{\sqrt{n}} [S_1, S_2, S_3]] \quad (29)$$

$$\mu_F = [[\bar{x}_1 - \frac{t_{\alpha/2}}{\sqrt{n}} S_3, \bar{x}_2 - \frac{t_{\alpha/2}}{\sqrt{n}} S_2, \bar{x}_3 - \frac{t_{\alpha/2}}{\sqrt{n}} S_1], [\bar{x}_1 + \frac{t_{\alpha/2}}{\sqrt{n}} S_1, \bar{x}_2 + \frac{t_{\alpha/2}}{\sqrt{n}} S_2, \bar{x}_3 + \frac{t_{\alpha/2}}{\sqrt{n}} S_3]] \quad (30)$$

$$\mu_F = [\mu_{F_1}, \mu_{F_2}] = [[\mu_{l1}, \mu_{l2}, \mu_{l3}], [\mu_{h1}, \mu_{h2}, \mu_{h3}]] \quad (31)$$

6. Fuzzy estimation of mean due to different significance levels

6.1. Crisp data with known crisp population standard deviation

Suppose that $x_1, x_2, x_3, \dots, x_n$ are the data of a random sample of size n from a population normally distributed with unknown mean (μ) and known population standard deviation σ .

The sample mean \bar{x} is calculated as equation (1) and it will be crisp.

Then $(1-\alpha)$ % confidence interval for the mean of population (μ) can be calculated as [11]

$$[\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}] \quad (32)$$

This method depends on calculating confidence interval of population mean based on changing the value of significance level α from 0.005 to 1. The starting point for α is arbitrary, it can start from 0.01, 0.05, 0.001, 0.005... etc.

The calculated confidence intervals are denoted as $[\mu_l(\alpha), \mu_h(\alpha)]$ for each α and place these confidence intervals one on top of the other, respectively from 0.005 to 1, to produce a TFN of population mean μ which α -cuts are the confidence intervals. For $\alpha=1$ the estimator will be a point instead of an interval.

The bottom of population mean (μ) must be completed to be a complete TFN. So, the graph of population mean (μ) is dropped straight down to complete its α -cuts. So that $\mu(\alpha) = [\mu_l(0.005), \mu_h(0.005)]$, for all $0 \leq \alpha \leq 0.005$, [15]

6.2. Crisp data with unknown population standard deviation

Suppose that $x_1, x_2, x_3, \dots, x_n$ are the data of a random sample of size n from a normal population with unknown mean (μ) and unknown population standard deviation σ .

The sample mean \bar{x} is calculated as equation (1) and it will be crisp.

The sample standard deviation (S) is provided as [13]

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (33)$$

The sample standard deviation (S) will be a crisp as the data is crisp. The t-student distribution is presented because of unknown population standard deviation.

Then $(1-\alpha)$ % confidence interval for the mean of population (μ) can be calculated as [13]

$$[\bar{x} - t_{\alpha/2} \frac{S}{\sqrt{n}}, \bar{x} + t_{\alpha/2} \frac{S}{\sqrt{n}}] \quad (34)$$

This method depends on calculating confidence interval of population mean based on changing the value of significance level α from 0.005 to 1. The starting point for α is arbitrary, it can start from 0.01, 0.05, 0.001, 0.005... etc.

The calculated confidence intervals are denoted as $[\mu_l(\alpha), \mu_h(\alpha)]$ for each α and place these confidence intervals one on top of the other, respectively from 0.005 to 1, to produce a TFN of population mean μ which α -cuts are the confidence intervals. For $\alpha=1$ the estimator will be a point instead of an interval.

The bottom of population mean (μ) must be completed to be a complete TFN. So, the graph of population mean (μ) is dropped straight down to complete its α -cuts. So that $\mu(\alpha) = [\mu_l(0.005), \mu_h(0.005)]$, for all $0 \leq \alpha \leq 0.005$, [16].

7. Advantages and disadvantages of using fuzzy algorithms

-The Fuzzy algorithms are often robust, in the sense that they are not very sensitive to changing environments and erroneous of forgotten rules.

-It gives the weight of each value.

-Simplicity and flexibility can handle problems with imprecise and incomplete data can model nonlinear functions of arbitrary complexity cheaper to develop, cover a wider range of operating conditions.

-Most people think that fuzzy is not accurate and rigorous. If we had Boolean (binary, non-fuzzy) brains, we would be seriously unable to survive.

8. Application on fuzzy estimation of the mean for electric load consumption

In this application the load consumption in a certain compound inside Cairo for one month has been measured. The data is considered to be in kilowatt. The mean of the load consumption has been calculated by choosing the fourth algorithm by assuming fuzzy load consumption data and the standard deviation is unknown. Also, the mean of load consumption with crisp data and unknown standard deviation for different significance levels has been estimated by choosing the fifth algorithm.

Only these two algorithms have been chosen for this application because they are more applicable due to the unknown of standard deviation.

A random sample data has been measured of the electric load consumption at January for 450 apartments in certain compound at Cairo. The apartments have the same area and the data was taken under the same conditions for all.

Crisp data \rightarrow TFN data: $X \rightarrow [X-k, X+k]$, where k is a constant.

Actual and TFN electricity load (kW) data with $k=10$		
Data number	Crisp data	TFN data
1	224	[214,224,234]
2	157	[147,157,167]
3	97	[87,97,107]
...
450	318	[308,318,328]

8.1. Fuzzy load consumption data with unknown standard deviation

Assuming the population is normal distribution. Calculating the sample mean

$$\bar{X}_F = \frac{\sum_{i=1}^n x_i}{n} = [146.28, 156.28, 166.28] \quad (35)$$

Then \bar{X}_F will be TFN with triangular membership function $\mu_{\bar{X}_F}(x)$

$$\mu_{\bar{X}_F}(x) = \begin{cases} 0 & x \leq 146.28, x \geq 166.28 \\ \frac{x-146.28}{156.28-146.28} & 146.28 \leq x \leq 156.28 \\ \frac{166.28-x}{166.28-156.28} & 156.28 \leq x \leq 166.28 \end{cases} \quad (36)$$

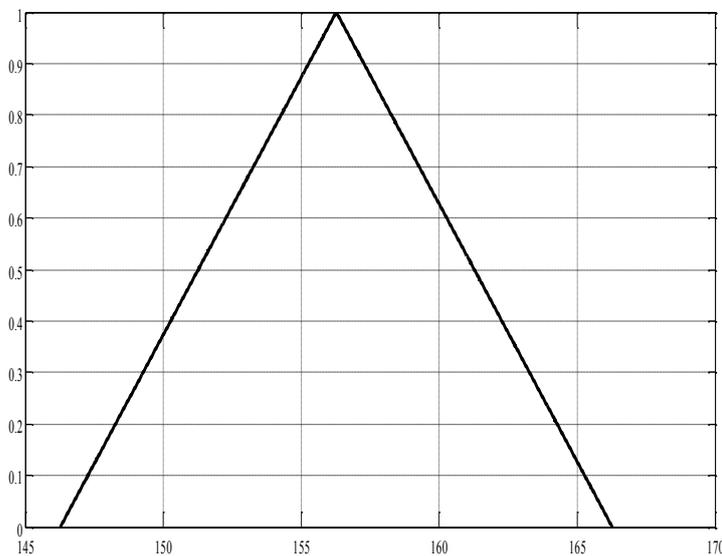


Figure 1. Membership function of sample mean $\mu_{\bar{X}_F}(x)$

Since the standard deviation of population (σ) is unknown. So, the sample standard deviation (S) is provided as

$$S_F = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{X}_F)^2} \quad (37)$$

The sample standard deviation (S) will be a TFN

$$S_F = [S_1, S_2, S_3] = [86.67, 96.67, 106.67] \quad (38)$$

With triangular membership function $\mu_{S_F}(x)$

$$\mu_{S_F}(x) = \begin{cases} 0 & x \leq 86.67, x \geq 106.67 \\ \frac{x-86.67}{96.67-86.67} & 86.67 \leq x \leq 96.67 \\ \frac{106.67-x}{106.67-96.67} & 96.67 \leq x \leq 106.67 \end{cases} \quad (39)$$

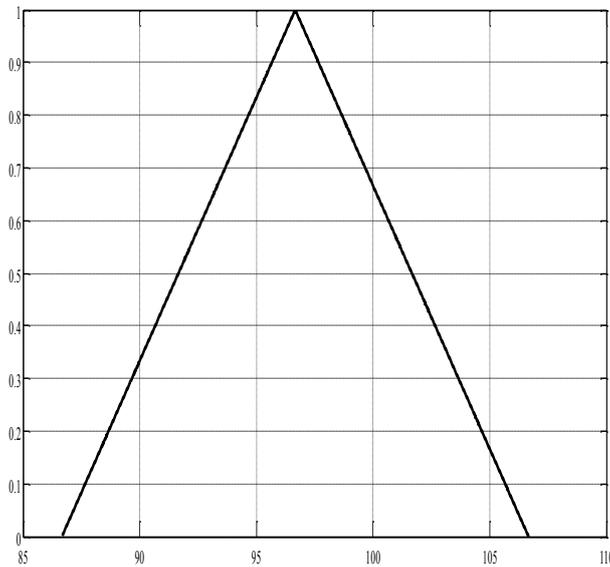


Figure 2. Membership function of sample standard deviation $\mu_{\bar{s}_F}(x)$

Since the population standard deviation is unknown. Then, it has been calculated using t-student distribution with 449 degree of freedom.

Then (1-0.01) % confidence interval for the population mean (μ) can be calculated

$$\mu_F = [\bar{X}_F - t_{0.005} \frac{S_F}{\sqrt{n}}, \bar{X}_F + t_{0.005} \frac{S_F}{\sqrt{n}}] = [[133.26, 144.48, 155.7], [156.84, 168.06, 179.28]]$$

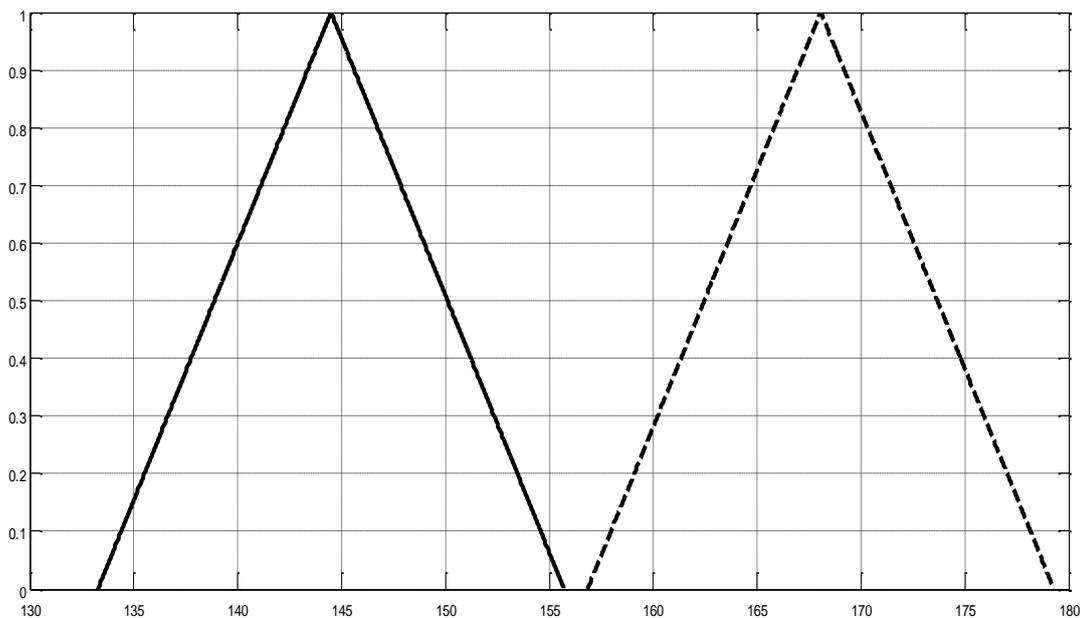


Figure 3. Membership function of fuzzy mean $\mu_F = [\mu_{F_1}, \mu_{F_2}]$

8.2. Crisp load consumption data with different significance levels

Assuming the population is normally distributed and the standard deviation of population is unknown. Calculating the sample mean

$$\bar{X} = \frac{\sum_{i=1}^n x_i}{n} = 156.28 \quad (40)$$

The sample standard deviation (S) is provided as

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2} = 96.67 \quad (41)$$

The sample standard deviation (S) will be a crisp as the data is crisp. The t-student distribution is presented because of unknown population standard deviation.

Then $(1 - \alpha)$ % confidence interval for the mean of population (μ) can be calculated as

$$[\bar{x} - t_{\alpha/2} \frac{S}{\sqrt{n}}, \bar{x} + t_{\alpha/2} \frac{S}{\sqrt{n}}] = [143.48, 169.06] \quad (42)$$

Calculating confidence interval of population mean based on changing the value of significance level α from 0.005 to 1. The calculated confidence intervals are denoted as $[\mu_l(\alpha), \mu_h(\alpha)]$ and place these confidence intervals one on top of the other, respectively from 0.005 to 1, to produce a TFN of population mean μ which α -cuts are the confidence intervals. For $\alpha = 1$ the estimator will be a point instead of an interval.

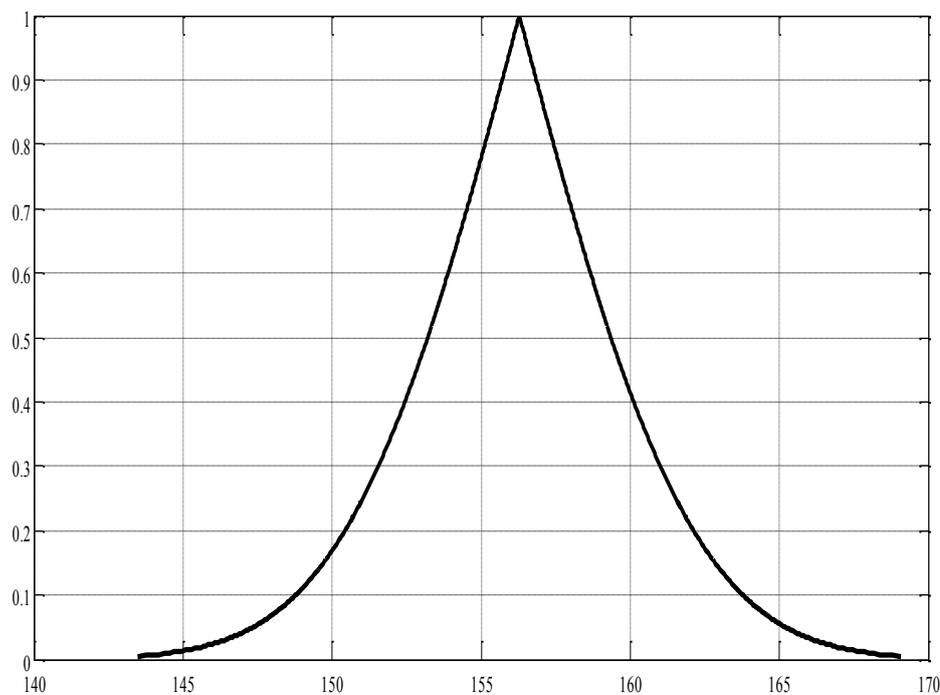


Figure 4. Membership function of fuzzy mean μ_F

9. Conclusion

Five new algorithms have been proposed to construct fuzzy confidence intervals for fuzzy and crisp parameters based on fuzzy and crisp observations. These algorithms recalled the ordinary methods of the theory of interval estimation in the classical estimation. There are some advantages and disadvantages of using fuzzy algorithms have been discussed. Each algorithm differs from the other depending on the time consumed and calculations complicity. Only two algorithms have been applied on fuzzy estimation of the mean for electric load consumption because they are more applicable due to the unknown of standard deviation parameter.

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